2

Practicalities of measurement

The main mathematical ideas in this chapter are:

- understanding approximations and significant figures
- expressing numbers in scientific notation
- converting between metric units of measurement
- using prefixes for units of measurement
- calculating error in measurement
- understanding accuracy of measurement.

ARE YOU READY?

2A **1** What does the first non-zero digit in the number 5083 represent?

A ones **B** tens
C hundreds **D** thousands

2A **2** What is the result when 482 096 is rounded to the nearest 1000?

A 480 000 **B** 482 000
C 483 000 **D** 48 000

2A **3** How many decimal places does 56.08 have?

A 1 **B** 2
C 3 **D** 4

2A **4** What is the result when 0.639 74 is rounded to three decimal places?

A 0.63 **B** 0.64
C 0.639 **D** 0.640

2A **5** Which decimal value is equivalent to the fraction $\frac{7}{8}$?

A 0.0875 **B** 0.78
C 0.87 **D** 0.875

2B **6** What is the result of 3.74 × 1000?

A 374 **B** 3740
C 37 400 **D** 374 000

2B **7** What is the result of 5.031 ÷ 100?

A 0.050 31 **B** 0.0531
C 0.5031 **D** 0.005 031

2C **8** Which length measurement is equivalent to 2354 mm?

A 2.354 m **B** 23.54 m
C 235.4 m **D** 0.2354 m

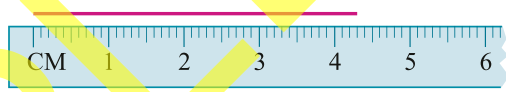
2C **9** Which area measurement is equivalent to 3 m^2?

A 300 cm^2 **B** 3000 cm^2
C 30 000 cm^2 **D** 300 000 cm^2

2C **10** Which volume measurement is equivalent to 5 cm^3?

A 50 mm^3 **B** 500 mm^3
C 5000 mm^3 **D** 50 000 mm^3

2E **11** The ruler below is marked in centimetres. To the nearest centimetre, what length does the line segment show?



A 3 cm **B** 4 cm
C $4\frac{1}{2}$ cm **D** 5 cm

2F **12** What is the area of a rectangle that is 12 cm long and 7 cm wide?

A 19 cm^2 **B** 38 cm^2
C 84 cm^2 **D** 168 cm^2

2F **13** What is the perimeter of the rectangle in question **12**?

A 19 cm **B** 38 cm
C 84 cm **D** 168 cm

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your obook assess.

Q1–2 **Support sheet 2A.1 Understanding place value in decimals**
Q3–4 **Support sheet 2A.2 Decimal places and rounding**
Q5 **Support sheet 2A.3 Converting between fractions and decimals**
Q6–7 **Support sheet 2B.1 Multiplying and dividing by 10, 100, 1000, etc.**
Q8 **Support sheet 2C.1 Length conversions**
Q9 **Support sheet 2C.2 Area conversions**
Q10 **Support sheet 2C.3 Volume conversions**
Q11 **Support sheet 2E.1 Accuracy in measurement**
Q12 **Support sheet 2F.1 Area of a rectangle**
Q13 **Support sheet 2F.2 Finding perimeter**

2A Significant figures

These resources are available on your obook assess:
- **Video tutorial 2A:** Watch and listen to an explanation of Example 2A-1
- **assess quiz 2A:** Test your skills with an auto-correcting multiple-choice quiz

significant figures the number of digits in a number that indicate its accuracy

The **significant figures** in a number are the important or meaningful figures. A crowd of 61 348 is approximately 61 000, indicating that only the first two figures (digits) are important.

It is impossible to cut a piece of timber to a length of 1.333 333… m. The digits after the fourth figure, are completely meaningless in this case.

EXAMPLE 2A-1 Rounding to a given number of significant figures

The first significant figure in a number is the first non-zero digit, reading from left to right. Round each of the following to:

i one significant figure　　ii two significant figures　　iii three significant figures

a 293 568　　b 0.076 04

		Solve	Think
a	i	300 000	Locate the relevant significant figure and then round appropriately using the next digit. The first non-zero digit is 2. This is the first significant figure. The next digit (9) is bigger than 5; so, rounded to one significant figure, 293 568 ≈ 300 000. (This is the same as rounding 293 568 to the nearest 100 000 because the first significant figure is in the 100 000s column.)
	ii	290 000	The second significant figure is 9. The next digit (3) is smaller than 5; so, rounded to two significant figures, 293 568 ≈ 290 000. (This is the same as rounding 293 568 to the nearest 10 000 because the second significant figure is in the 10 000s column.)
	iii	294 000	The third significant figure is 3. The next digit is 5; so, rounded to three significant figures, 293 568 ≈ 294 000. (This is the same as rounding 293 568 to the nearest 1000 because the third significant figure is in the 1000s column.)
b	i	0.08	The first non-zero digit is 7. This is the first significant figure. The next digit (6) is bigger than 5; so, rounded to one significant figure, 0.076 04 ≈ 0.08. (This is the same as rounding 0.076 04 to two decimal places because the first significant figure is in the second place after the decimal point; or rounding to the nearest hundredth because the first significant figure is in the hundredths column.)
	ii	0.076	The second significant figure is 6. The next digit (0) is smaller than 5; so, rounded to two significant figures, 0.076 04 ≈ 0.076. (This is the same as rounding 0.076 04 to three decimal places because the second significant figure is in the third place after the decimal point; or rounding to the nearest thousandth because the second significant figure is in the thousandths column.)
	iii	0.0760	The third significant figure is 0. The next digit (4) is smaller than 5; so, rounded to three significant figures, 0.076 04 ≈ 0.0760. (This is the same as rounding 0.076 04 to four decimal places because the third significant figure is in the fourth place after the decimal point; or rounding to the nearest ten-thousandth, because the third significant figure is in the ten-thousandths column.)

EXERCISE 2A Significant figures

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Complete the following to round:

a 5368 to three significant figures.
The first non-zero digit is ____. This is the first significant figure. The third significant figure is ____. The digit after this is smaller than/bigger than/equal to 5, indicating that, when rounded, the number is closer to ____ than to ____. So, rounded to three significant figures, 5368 ≈ ____.

b 0.062 53 to three significant figures.
The first non-zero digit is ____. This is the first significant figure. The third significant figure is ____. The digit after this is smaller than/bigger than/equal to 5, indicating that, when rounded, the number is closer to ____ than to ____. So, rounded to three significant figures, 0.062 53 ≈ ____.

2 Round each of the following to one significant figure.

a 42 600 **b** 59 **c** 4.6 **d** 108
e 0.6529 **f** 0.0082 **g** 0.025 **h** 990

3 Round each of the following to two significant figures.

a 290 365 **b** 3960 **c** 24.9 **d** 2653
e 8.63 **f** 0.0487 **g** 0.000 162 8 **h** 0.003 97

4 Round each of the following to three significant figures.

a 3688 **b** 20 657 **c** 154 299 **d** 813.4
e 14.294 **f** 0.003 508 1 **g** 0.039 14 **h** 1.999

5 Round each of the following to:

i one significant figure **ii** two significant figures **iii** three significant figures.

a 17.256 **b** 0.450 72 **c** 521 500 **d** 0.002 095

6 Express $3\frac{2}{7}$ as a decimal correct to four significant figures.

7 Express each of the following numbers as a decimal correct to three significant figures and arrange the numbers in ascending order (from smallest to largest).
$\sqrt{2}$, $1.\dot{4}$, $1\frac{2}{5}$, $\frac{10}{7}$

8 A town's average rainfall in summer over six successive years is 246.5 mm, 237.6 mm, 366.9 mm, 287.4 mm, 412.8 mm and 348.2 mm. Calculate the average rainfall for the town over this six-year period, correct to four significant figures.

9 A car travels for $3\frac{1}{2}$ hours at 71 km/h and then for $2\frac{1}{4}$ hours at 75 km/h.

a Find the total distance travelled by the car, correct to four significant figures.

b Calculate the average speed for the whole trip, correct to two significant figures.

CHALLENGE

10 A satellite orbits the Earth at a height of 32 km above Earth's surface. The diameter of the Earth is 12 740 km.

a Find the radius, to five significant figures, of the satellite's orbit, assuming that orbit is circular.

b Calculate the length (l) of the satellite's orbit, using $l = 2\pi r$ where r is the radius. Write your answer to five significant figures.

c If the satellite travels at a speed of 26 000 km/h, calculate how long, in minutes, correct to three significant figures it will take to complete one orbit.

2B Scientific notation

These resources are available on your obook assess:

- **Interactive 2B:** Explore key ideas for scientific notation
- **Investigation 2B:** Investigate measurements in our solar system
- **assess quiz 2B:** Test your skills with an auto-correcting multiple-choice quiz

OI GI

scientific notation
a value written as a number from 1 up to, but not including, 10 (with any number of decimal places) multiplied by a power of 10

Scientific notation (or standard notation) is a convenient way of writing very large and very small numbers. A number written in scientific notation is written as the product of a number between 1 and 10 and a power of 10; that is, it is put in the form $A \times 10^n$ where A lies between 1 and 10, and n is an integer (whole number).

EXAMPLE 2B–1 Identifying numbers written in scientific notation

State whether the following numbers are expressed in scientific notation.

a 5.3×10^7 **b** 78×10^5 **c** 4.9×10000 **d** 3×10^{-4} **e** 294 000

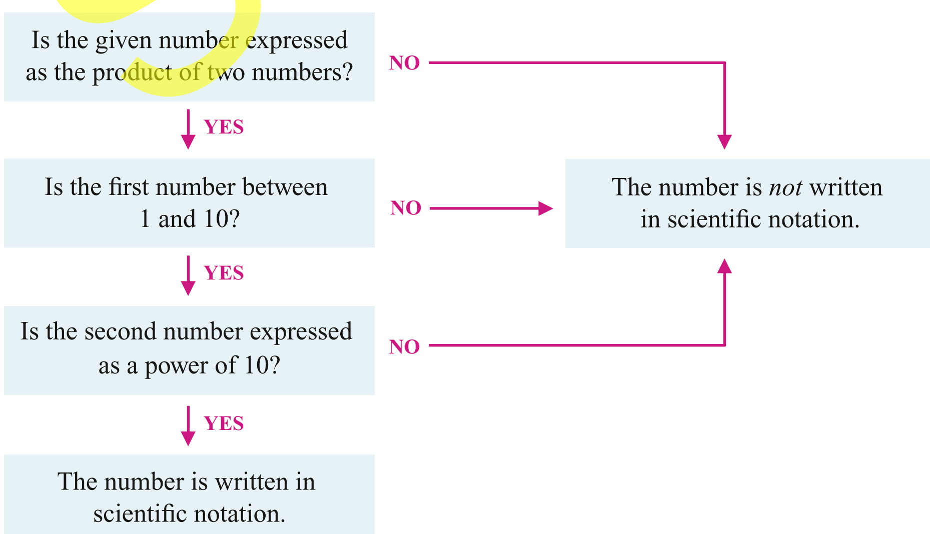
	Solve	Think	Apply
a	Yes	The first number in the product (5.3) is between 1 and 10, the second number (10^7) is a power of 10.	A number is written in scientific notation if it is written as the product of a number from 1 up to 10 and a power of 10.
b	No	The first number (78) is not between 1 and 10.	
c	No	The second number (10 000) is not written as a power of 10.	
d	Yes	The first number in the product (3) is between 1 and 10, the second number (10^{-4}) is a power of 10.	
e	No	294 000 is not written as a product of two numbers.	

EXERCISE 2B Scientific notation

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Use the flow diagram below to determine whether the given numbers are expressed in scientific notation.

a 2.91×10^{-17} **b** 53×10^6 **c** 3.8×100000 **d** 3^{26}



2 State whether the following numbers are written in scientific notation.

a 3.6×10^5 **b** $5.2 \times 10\,000$ **c** 21×10^5 **d** 2.87×10^{-6}
e $6.07 \times \frac{1}{1\,000\,000}$ **f** 594×10^{-5} **g** 70×10^8 **h** 3.06×10^{-9}

EXAMPLE 2B–2 Writing numbers in scientific notation

Write these numbers in scientific notation.

a 138 000 **b** 0.000 486

	Solve	Think	Apply
a	1.38×10^5	Move the decimal point so that it is positioned between the first and second digits. This always produces a number between 1 and 10, in this case 1.380 00. Count the number of places back to the original position of the decimal point: 1.38000 Number of places = five to the right = +5 This becomes the power of 10: $138\,000 = 1.38000 \times 10^5$ $= 1.38 \times 10^5$ (leave off the zeros)	Move the decimal point so that it is positioned between the first two digits. This produces a number between 1 and 10. Count the number of places back to the original position of the decimal point. This becomes the power of 10. *Note:* When counting back to the original position of the decimal point, counting to the right produces a positive power of 10 and counting to the left produces a negative power of 10.
b	4.86×10^{-4}	Move the decimal point so that it is positioned between the first and second digits, in this case 4.86. Count the number of places back to the original position of the decimal point: 00004.86 Number of places = four to the left = −4 This becomes the power of 10: $0.000\,486 = 4.86 \times 10^{-4}$	

3 Complete the following to write each number in scientific notation.

a 243 000

Position the decimal point between the first two digits → ________.

The number of places to the original position of the decimal point = _____ to the _____

= +_____.

So, $243\,000$ = _____ × 10□

b 0.000 586

Position the decimal point between the first two digits → ________.

The number of places to the original position of the decimal point = _____ to the _____

= −_____.

So, $0.000\,586$ = _____ × 10□

4 Use the method in Example 2B-2 to write each of these numbers in scientific notation.

a 526 000 **b** 28 000 **c** 7 000 000 **d** 49 800
e 28 000 000 **f** 603 000 000 **g** 910 000 **h** 13 200 000 000

5 Use the method in Example 2B-2 to write each of these numbers in scientific notation.

a 0.000 43 **b** 0.008 21 **c** 0.000 007 **d** 0.000 029
e 0.065 **f** 0.000 387 **g** 0.000 008 2 **h** 0.000 06

EXAMPLE 2B–3 Changing from scientific notation to an ordinary number

Write the following as ordinary (or basic) numbers.

a 4.83×10^7 **b** 9.2×10^{-6}

	Solve	Think	Apply
a	48 300 000	Because the power of 10 is +7, the decimal point is moved 7 places to the right. 48 300 000 so $4.83 \times 10^7 = 48\,300\,000$	The magnitude of the power of 10 tells us how many places to move the decimal point. If the power of 10 is positive, move the decimal point to the right. If the power of 10 is negative, move it to the left.
b	0.000 009 2	Because the power of 10 is −6, the decimal point is moved 6 places to the left. 0.000 009 2 so $9.2 \times 10^{-6} = 0.000\,009\,2$	

6 Complete the statements to write each of these numbers as an ordinary number.

a 5.48×10^6 Move the decimal point ____ places to the ____. 5.48×10^6 = ________.

b 3.09×10^{-5} Move the decimal point ____ places to the ____. 3.09×10^{-5} = ________.

7 Express these as ordinary numbers.

a 3.4×10^6 **b** 8.3×10^8 **c** 2.94×10^7 **d** 2.58×10^5

e 5.26×10^5 **f** 3.02×10^{12} **g** 2.9×10^7 **h** 8.75×10^8

8 Write the basic number for:

a 5.9×10^{-4} **b** 3.2×10^{-6} **c** 7.1×10^{-8} **d** 2×10^{-3}

e 8×10^{-7} **f** 2.64×10^{-5} **g** 8.67×10^{-9} **h** 2.97×10^{-6}

EXAMPLE 2B–4 Calculating numbers in scientific notation

Use your calculator to find:

a $(3.5 \times 10^7) \times (2.4 \times 10^9)$ **b** $(6.4 \times 10^8) \div (2.5 \times 10^{-6})$

c $\sqrt{2.4 \times 10^{10}}$ **d** $(1.5 \times 10^7)^3$

	Solve	Think	Apply
a	8.4×10^{16}	Possible steps using a Casio calculator are: Press 3.5 [$\times 10^x$] 7 [$\times$] 2.4 [$\times 10^x$] 9 [=] *Answer*: 8.4×10^{16}	If the answer is not displayed in scientific notation, you could use the SCI function on the calculator to express the answer in this form.
b	2.56×10^{14}	Press 6.4 [$\times 10^x$] 8 [$\div$] 2.5 [$\times 10^x$] −6 [=] *Answer*: 2.56×10^{14}	
c	1.55×10^5	Press [$\sqrt{\blacksquare}$] 2.4 [$\times 10^x$] 10 [=] *Answer*: 1.55×10^5 to three significant figures	
d	3.375×10^{21}	Press 1.5 [$\times 10^x$] 7 [$x^\blacksquare$] 3 [=] *Answer*: 3.375×10^{21}	

PROBLEM SOLVING, REASONING AND JUSTIFICATION

9 Calculate the following, correct to three significant figures. Write the answers in scientific notation.

a $(2.6 \times 10^8) \times (4.1 \times 10^7)$

b $(5.8 \times 10^9) \times (8.2 \times 10^{12})$

c $(9.2 \times 10^{24}) \times (3.5 \times 10^{-8})$

d $(5.8 \times 10^{-6}) \times (2.4 \times 10^{-9})$

e $(8.4 \times 10^{18}) \div (2.5 \times 10^7)$

f $(5.25 \times 10^{12}) \div (4.2 \times 10^{-8})$

g $(1.82 \times 10^{-6}) \times (2.9 \times 10^{-10})$

h $(2.8 \times 10^8)^4 \times (1.6 \times 10^6) \div (2.1 \times 10^{15})$

i $\sqrt{5.76 \times 10^{16}}$

j $\sqrt{6.8 \times 10^{17}}$

k $\sqrt{(3.1 \times 10^8)^5}$

l $(8 \times 10^{-10})^6$

10 Express the following numbers in scientific notation.

a The distance of Mars from the Sun is approximately 229 000 000 km.

b The diameter of the hydrogen atom is 0.000 000 000 025 4 m.

c The Sun produces the same amount of light as 3 000 000 000 000 000 000 000 000 000 candles.

d There are approximately 130 000 hairs on a person's head.

e There are approximately 10 000 000 000 000 cells in the human body.

11 Change the numbers in the following to ordinary numbers.

a There are approximately 3.16×10^7 s in a year.

b A molecule's diameter is 8.9×10^{-7} mm.

c The number of different possible hands in the card game Poker is approximately 2.6×10^6.

d Swarms of locusts have been known to contain as many as 3×10^{10} locusts.

e The size of the influenza virus is approximately 2.6×10^{-4} mm.

12 a Light travels at a velocity of 300 000 km/s. Express 300 000 in scientific notation.

b How far does light travel in:

i 1 minute? **ii** 1 hour? **iii** 1 day? **iv** 1 year?

c If light takes 4.1 min to reach Earth from Mars, what is the distance from Earth to Mars?

13 The radius of the Earth is approximately 6400 km.

a Calculate the area of the Earth's surface, to two significant figures. (Use $A = 4\pi r^2$.)

b Calculate the volume of the Earth, to two significant figures. (Use $V = \frac{4}{3}\pi r^3$.)

14 The radius of the Earth's orbit around the Sun is approximately 1.49×10^8 km. Assuming that the orbit is circular, calculate to two significant figures the distance travelled by the Earth in one orbit. (Use $C = 2\pi r$.)

15 a The human brain contains about 10^{10} cells. Write this as an ordinary number.

b Each human brain cell is about 2.8×10^{-5} m long. If all the brain cells could be placed next to each other, in a straight line, how long would this line be?

16 Measure your pulse to determine the number of times your heart beats in a minute. If you live to be 75 years of age, how many times will your heart have beaten in this time? (Assume your pulse remains constant.)

CHALLENGE

17 Light travels at approximately 3×10^5 km/s and sound travels at 330 m/s.

a After the starter fires the starting gun, how long does it take:

i the sight of the smoke from the gun to reach a timekeeper standing at the end of a 100 m running track. Give your answer in scientific notation to four significant figures.

ii the sound of the gun to reach the timekeeper, in scientific notation to four significant figures.

b What is the difference between the times from your answers to part **a**?

c If the timekeeper's stopwatch measures time to the nearest hundredth of a second, does it matter if she uses the sight of the smoke or the sound of the gun to start the stopwatch?

2C Metric units of measurement

These resources are available on your obook assess:

- **Interactive 2C.1:** Explore key ideas for converting units of length
- **Interactive 2C.2:** Explore key ideas for converting units of area and volume
- **assess quiz 2C:** Test your skills with an auto-correcting multiple-choice quiz

The metric system of measurement uses base units for quantities such as length, mass, capacity, area and volume.

The prefix of each unit name indicates the factor of 10 by which the base unit is multiplied.

EXERCISE 2C Metric units of measurement

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Complete this conversion diagram for length.

EXAMPLE 2C–1 Converting units of length

Convert these lengths.

a 2.5 km = ___ m **b** 430 mm = ___ m.

	Solve	Think	Apply
a	2.5 km = (2.5 × 1000) m = 2500 m	1 km = 1000 m, so conversion factor is 1000. Multiply by the conversion factor since we are converting from km to the smaller unit of m.	To convert to a smaller unit, multiply by the conversion factor.
b	430 mm = (430 ÷ 10) cm = 43 cm = (43 ÷ 100) m = 0.43 m	First convert from mm to cm (conversion factor of 10) and then convert from cm to m (conversion factor of 100). In each case, divide by the conversion factor because we are converting to a larger unit.	To convert to a larger unit, divide by the conversion factor.

2 Convert these lengths.

a 3.6 km = ___ m **b** 8.4 m = ___ mm **c** 34.82 m = ___ cm **d** 0.56 km = ___ m
e 2.9 m = ___ cm **f** 0.964 m = ___ mm **g** 0.658 m = ___ cm **h** 45.2 cm = ___ mm
i 15.68 km = ___ m **j** 3.69 cm = ___ mm **k** 16.37 m = ___ mm **l** 4.265 km = ___ m

3 **a** **i** Complete: 1 km = ___ cm. **ii** Express the answer in scientific notation.
b **i** Complete: 1 km = ___ mm. **ii** Express the answer in scientific notation.

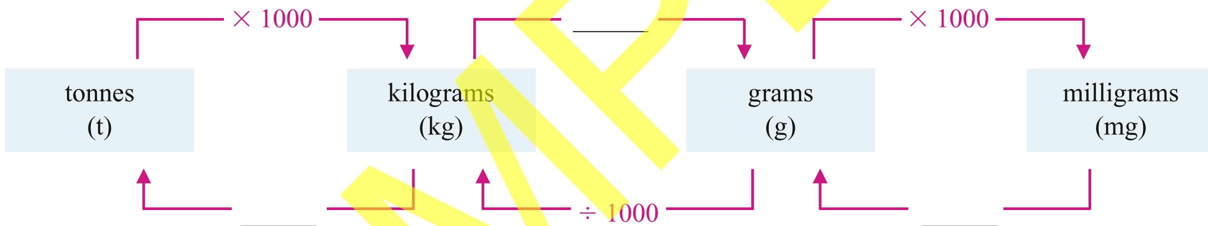
4 Convert these lengths.

a 7000 m = ___ km **b** 594 cm = ___ m **c** 8930 m = ___ km **d** 6000 mm = ___ m
e 40 mm = ___ cm **f** 85 m = ___ km **g** 800 cm = ___ m **h** 328 mm = ___ cm
i 620 mm = ___ m **j** 14 300 m = ___ km **k** 86 cm = ___ m **l** 630 m = ___ km
m 94 mm = ___ cm **n** 70 mm = ___ m **o** 24 895 m = ___ km **p** 23 000 mm = ___ m
q 14 960 mm = ___ m **r** 16 270 cm = ___ m **s** 3600 cm = ___ m **t** 72 945 mm = ___ m

5 What would be a convenient unit (millimetres, centimetres, metres, kilometres) to use for measuring the following?

a width of the classroom
b length of a textbook
c height of a student
d length of a baby
e length of your foot
f length of a matchstick
g distance from Sydney to Brisbane
h length of material for a dress
i length of a driveway
j distance between railway stations

6 Complete this conversion diagram for mass.



7 Convert these masses.

a 3.6 t = ___ kg **b** 7.1 g = ___ mg **c** 17.84 kg = ___ g **d** 0.63 t = ___ kg
e 4.8 kg = ___ g **f** 2.465 t = ___ kg **g** 21.59 t = ___ kg **h** 0.6 g = ___ mg
i 6.06 kg = ___ g **j** 0.489 kg = ___ g **k** 1.07 t = ___ kg **l** 0.03 g = ___ mg

8 **a** **i** Complete: 1 t = ____ g. **ii** Write the answer in scientific notation.
b **i** Complete: 1 t = ____ mg. **ii** Write the answer in scientific notation.
c **i** Complete: 1 kg = ____ g. **ii** Write the answer in scientific notation.

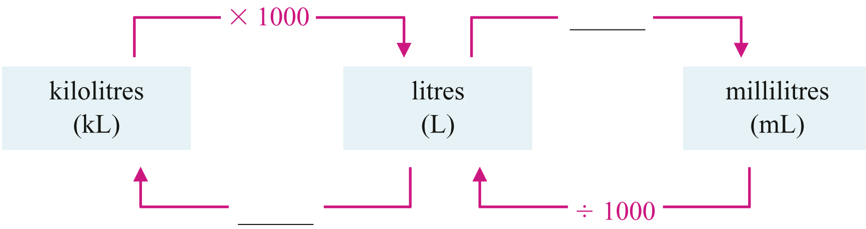
9 Convert the following.

a 8000 kg = ___ t **b** 4300 g = ___ kg **c** 2740 mg = ___ g **d** 690 g = ___ kg
e 65 mg = ___ g **f** 2320 g = ___ kg **g** 700 kg = ___ t **h** 460 mg = ___ g
i 80 g = ___ kg **j** 7 mg = ___ g **k** 9 kg = ___ t **l** 300 g = ___ kg

10 State a convenient unit to use (milligrams, grams, kilograms, tonnes) for measuring the mass of the following.

a a woman **b** a jar of honey
c a packet of sugar **d** a bag of cement
e a large SUV **f** a rhinoceros
g a headache tablet **h** a maths textbook
i a sewing needle **j** a tennis ball

11 Complete this conversion diagram for capacity.



12 Convert the following.

a 35 kL = ___ L **b** 15.9 L = ___ mL
c 1.65 L = ___ mL **d** 0.85 kL = ___ L
e 0.06 L = ___ mL **f** 1.08 kL = ___ L
g 0.015 L = ___ mL **h** 0.005 kL = ___ L

13 How many millilitres are there in 1 kL? Express your answer in scientific notation.

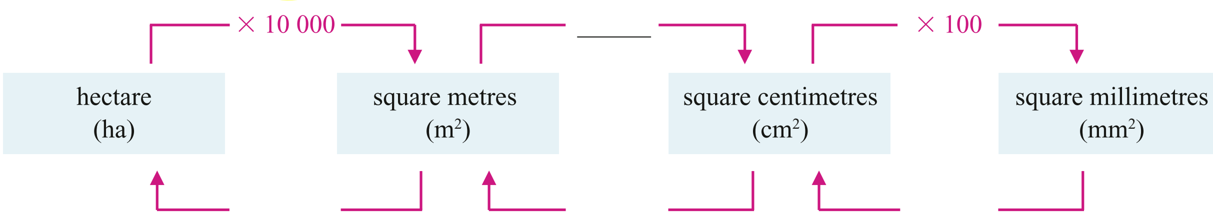
14 Convert the following.

a 15 000 mL = ___ L **b** 8000 L = ___ kL
c 7600 mL = ___ L **d** 800 mL = ___ L
e 9280 L = ___ kL **f** 725 L = ___ kL
g 95 mL = ___ L **h** 40 L = ___ kL

15 State an appropriate unit to use (millimetres, litres, kilolitres) for measuring the capacity of a:

a teaspoon **b** swimming pool
c bucket **d** fish tank
e laundry tub **f** farm dam
g car's petrol tank **h** kettle

16 Complete this conversion diagram for area.



17 Convert these areas.

a 2.6 ha = ___ m^2 **b** 4.9 m^2 = ___ cm^2 **c** 14 cm^2 = ___ mm^2 **d** 0.752 m^2 = ___ cm^2
e 1.65 ha = ___ m^2 **f** 24.8 cm^2 = ___ mm^2 **g** 8.294 km^2 = ___ m^2 **h** 5.671 km^2 = ___ cm^2

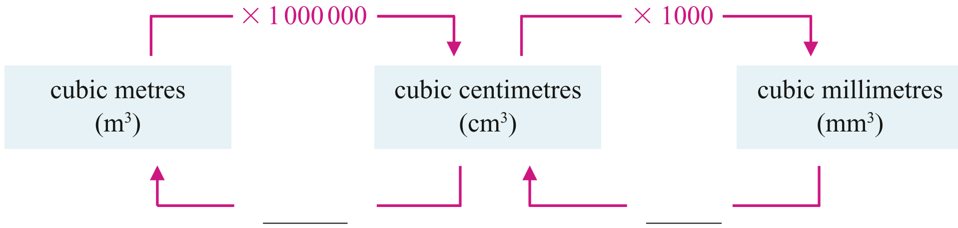
UNDERSTANDING, FLUENCY AND COMMUNICATING

18 Convert the following.

a $63\,000\text{ m}^2 = \_\_\_\text{ ha}$ **b** $127\,000\text{ cm}^2 = \_\_\_\text{ m}^2$ **c** $810\text{ mm}^2 = \_\_\_\text{ cm}^2$

d $45\,680\text{ cm}^2 = \_\_\_\text{ m}^2$ **e** $298\,000\text{ m}^2 = \_\_\_\text{ km}^2$ **f** $2.4\text{ km}^2 = \_\_\_\text{ ha}$

19 Complete this conversion diagram for volume.



20 Convert these volumes.

a $5\text{ cm}^3 = \_\_\_\text{ mm}^3$ **b** $3.9\text{ m}^3 = \_\_\_\text{ cm}^3$

c $25.6\text{ cm}^3 = \_\_\_\text{ mm}^3$ **d** $0.64\text{ m}^3 = \_\_\_\text{ cm}^3$

e $0.415\text{ cm}^3 = \_\_\_\text{ mm}^3$ **f** $7.39\text{ m}^3 = \_\_\_\text{ mm}^3$

21 Convert the following.

a $7\,400\,000\text{ cm}^3 = \_\_\_\text{ m}^3$ **b** $56\,700\text{ mm}^3 = \_\_\_\text{ cm}^3$

c $690\,000\text{ cm}^3 = \_\_\_\text{ m}^3$ **d** $4258.5\text{ mm}^3 = \_\_\_\text{ cm}^3$

22 **a** A bottle of wine has a mass of 1140 g. What would be the mass of a case of 1 dozen bottles if the cardboard packaging has a mass of 320 g. Write the answer in kilograms.

b If each bottle contains 750 mL of wine, what is the total capacity of one case. Answer in litres.

23 The catchment area of a dam is 25.6 km². If 25 mm of rain falls over the catchment area, what will be the increase in capacity of the dam, given that 1 m³ holds 1000 kL of water?

24 The average mass of an adult hippopotamus is 1.5 t. Baby hippopotami, which are born underwater, have an average mass of 37 kg. Express the mass of a baby hippo as a percentage of its mass as an adult.

CHALLENGE

25 A house brick has dimensions of 76 mm × 230 mm × 110 mm.

a When travelling to a work site, trucks typically carry 6 pallets each containing 500 bricks. Calculate the total space in cubic metres occupied by these 6 pallets.

b **i** Calculate the area of the largest face of a brick.

ii Assume that, when laid, each brick has a 10 mm thickness of mortar along one 230 mm edge and one 110 mm edge. What is the area covered by a brick plus the mortar?

iii How many bricks, surrounded by mortar, are needed to build a wall of area 60 m²?

MEASUREMENT

2D Prefixes for units of measurement

These resources are available on your obook assess:

- **Video tutorial 2D:** Watch and listen to an explanation of Example 2D-1
- **Investigation 2D:** Investigate prefixes used for file sizes of digital data
- **assess quiz 2D:** Test your skills with an auto-correcting multiple-choice quiz

Prefixes are used to indicate the factor of 10 by which the base metric unit of measurement is multiplied. For example, the unit 'kilogram' uses the name of the base unit for mass, the gram, with the prefix *kilo*, which indicates a multiplying factor of 1000: 1 kg = 1000 g.

This table summarises the most common prefixes used for very large and very small measurements.

Prefix	Multiplying factor
tera (T)	$10^{12} = 1\,000\,000\,000\,000$
giga (G)	$10^{9} = 1\,000\,000\,000$
mega (M)	$10^{6} = 1\,000\,000$
kilo (k)	$10^{3} = 1000$
centi (c)	$10^{-2} = 0.01$
milli (m)	$10^{-3} = 0.001$
micro (μ)	$10^{-6} = 0.000\,001$
nano (n)	$10^{-9} = 0.000\,000\,001$

EXAMPLE 2D–1 Converting length measurements

a Convert the following to metres.
  i 3.6 Gm **ii** 7 μm

b Convert 5.6 m to:
  i kilometres **ii** micrometres

	Solve	Think	Apply
a i	3.6 Gm = 3.6×10^9 m or 3 600 000 000 m	1 Gm = 1×10^9 m	Apply the multiplying factor for the prefix.
ii	7 μm = 7×10^{-6} m or 0.000 007 m	1 μm = 1×10^{-6} m	
b i	$5.6 \text{ m} = \frac{5.6}{10^3} \text{ km}$ $= 5.6 \times 10^{-3}$ or 0.0056 km	Divide 5.6 by the number of metres in a kilometre. $\frac{1}{10^3} = 10^{-3}$	Divide by the number of metres in the required unit.
ii	$5.6 \text{ m} = \frac{5.6}{10^{-6}} \text{ μm}$ $= 5.6 \times 10^{6}$ or 5 600 000 μm	Divide 5.6 by the number of metres in a micrometre. $\frac{1}{10^{-6}} = 10^{6}$	

EXERCISE 2D Prefixes for units of measurement

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 **a** Convert these to metres.
  i 5.7 Mm **ii** 9 cm **iii** 8 nm
b Convert these to grams.
  i 8 Gg **ii** 4.2 mg **iii** 5 μg
c Convert these to litres.
  i 3 TL **ii** 2.8 ML **iii** 7 mL

2 **a** Convert 4.9 m to:
  i kilometres **ii** micrometres **iii** nanometres
b Convert 2.4 g to:
  i megagrams **ii** teragrams **iii** micrograms
c Convert 6.5 L to:
  i megalitres **ii** millilitres **iii** gigalitres

EXAMPLE 2D–2 Converting mass measurements

Convert the following.
a 3.56 Tg = ___ Mg **b** 9.4 mg = ___ μg

	Solve	Think	Apply
a	$3.56\text{ Tg} = \frac{3.56 \times 10^{12}}{10^6}\text{ Mg}$ $= 3.56 \times 10^6$ or 3 560 000 Mg	Convert 3.56 Tg into grams and divide by the number of grams in a megagram.	Convert to grams and divide by the number of grams in the required unit.
b	$9.4\text{ mg} = \frac{9.4 \times 10^{-3}}{10^{-6}}\text{ μg}$ $= 9.4 \times 10^3$ or 9400 μg	Convert 9.4 mg into grams and divide by the number of grams in a microgram.	

3 Convert the following.
a 7.2 Gg = ___ Mg **b** 2.9 cg = ___ ng **c** 8 Tm = ___ km **d** 4.3 cm = ___ μm
e 8.8 TL = ___ ML **f** 9 kL = ___ mL **g** 5.3 ms = ___ ns **h** 1.2 μs = ___ ns

4 Light travels approximately 9.46×10^{15} m in a year. Express this in terametres.

5 Warragamba dam holds approximately 2 580 000 ML of water at full capacity. How many gigalitres is this?

6 The distance from Mars to the Sun is 0.228 Tm. Convert this distance to kilometres.

7 The mass of a hydrogen atom is 1.67×10^{-24} g. What is the mass in nanograms of 1 million hydrogen atoms?

8 A computer can access its memory in 24 ns. Convert this to microseconds.

CHALLENGE

9 Prefixes are also used for file sizes of digital data. You can access an extension task related to these file sizes (Investigation 2D) from your obook assess.

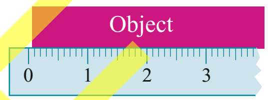
MEASUREMENT

2E Error and accuracy in measurement

These resources are available on your obook assess:
- **Investigation 2E:** Investigate different types of error
- **assess quiz 2E:** Test your skills with an auto-correcting multiple-choice quiz

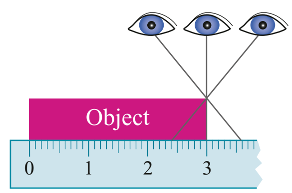
When physically measuring using a measuring instrument, there are several sources of possible error and uncertainty.

- Errors occur if the zero on the scale of the measuring instrument does not coincide with the end of the object or with the pointer on the measuring instrument.



- An error occurs if the end of the measuring instrument has been damaged. In this case start measuring from the 1, for example, instead of 0.

- Calibration error can occur if the scale is not accurately marked on the measuring instrument.
- Parallax error occurs if your eye is not directly above the scale on the measuring instrument.
- There is always an error due to the limit of reading the measuring instrument.



Repeating a measurement a number of times and averaging the values can reduce the effect of any errors.

As a result of the accumulating effect of errors when calculations are performed with measured values, the following conventions are usually applied.

- When adding or subtracting measured quantities, the degree of accuracy of the answer is limited by the measurement that is accurate to the least number of decimal places.
- When multiplying or dividing with measured quantities, the degree of accuracy of the answer is limited by the measurement with the least number of significant figures.

EXAMPLE 2E–1 Averaging measurements to find an approximation

John measured the width of his maths textbook five times using a ruler marked in millimetres. The results were 190 mm, 189 mm, 190 mm, 192 mm and 190 mm. Average these measurements to give an approximation for the width of the book, to the nearest millimetre.

Solve	Think	Apply
Average $= \frac{190 + 189 + 190 + 192 + 190}{5}$ $= 190.2$ mm $= 190$ mm (to the nearest millimetre)	Determine the average by finding the sum of the measurements and then dividing by the number of measurements. The answer is 190 mm, to the nearest millimetre, because 190.2 is closer to 190 than to 191.	Averaging measurements reduces the effects of any errors. The answer should be given to the same degree of accuracy as the given measurements (in this case, to the nearest millimetre). The differences in the measurements could have been caused by any of the errors discussed on the previous page.

EXERCISE 2E Error and accuracy in measurement

1 A student measured the length of his textbook using a ruler marked in millimetres. The results were 256 mm, 255 mm, 255 mm, 254 mm and 254 mm. Average these measurements to give an approximation of the length of the book, to the nearest millimetre.

2 Average the following to give an approximation of the true measurement. In each case, the measurements were taken to the same degree of accuracy.

a 83 mm, 85 mm, 84 mm, 85 mm, 85 mm, 84 mm

b 4.9 kg, 4.8 kg, 4.9 kg, 5.0 kg, 4.9 kg

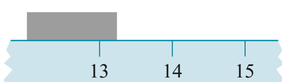
c 162 mL, 162 mL, 160 mL, 161 mL, 161 mL, 162 mL

d 22.49 s, 22.61 s, 22.54 s, 22.56 s, 22.52 s

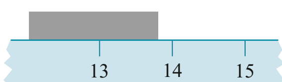
3 Ask five students to measure your height to the nearest centimetre. Average these measurements to give an approximation of your true height.

4 The diagrams show several steel rods being measured with a ruler divided into centimetres. Write the length of each rod, using the scale given on the ruler.

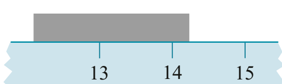
a



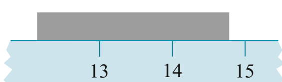
b



c



d



5 The length of a rod is measured using the ruler in question **4**, and the measurement is recorded as 14 cm.

a Would this be the exact length of the rod?

b Between what values would the actual length lie?

c What is the absolute possible error in stating that the length is 14 cm?

d How could we find a more accurate value for the length of the rod?

EXAMPLE 2E–2 Finding a sensible approximation for the result of an addition (or subtraction) calculation

Write a sensible approximation for the result of this calculation: 15.642 m + 8 m + 19.21 m.

Solve	Think	Apply
15.642 m + 8 m + 19.21 m = 42.852 m = 43 m (to the nearest metre)	15.642 m is accurate to three decimal places. 8 m is accurate to the nearest whole number. 19.21 m is accurate to two decimal places. The least precise measurement is 8 m (the nearest whole number), so the answer should be rounded to the nearest whole number.	When adding or subtracting measured quantities, the degree of accuracy of the answer is limited by the measurement with the least decimal place accuracy.

6 Complete the following to calculate 13.65 L + 10.9 L + 12.624 L.

13.65 is accurate to _____ decimal place(s).

10.9 is accurate to _____ decimal place(s).

12.624 is accurate to _____ decimal place(s).

The least precise measurement is ____ L to ____ decimal place(s).

Hence, 13.65 L + 10.9 L + 12.624 L = ____ L= ____ L to ____ decimal place(s).

7 Write a sensible approximation for the results of the following calculations.

a 9.87 m + 15.219 m + 11 m

b 27.3 L + 21.475 L + 16.54 L

c 6.132 km − 3.46 km

d 10.528 kg + 11.607 kg − 9.2 kg

EXAMPLE 2E–3 Finding a sensible approximation for the result of a multiplication (or division) calculation

Write a sensible approximation for the result of this calculation: 15.2 m × 9.8 m.

Solve	Think	Apply
15.2 m × 9.8 m = 148.96 m^2 = 150 m^2 (to two significant figures)	15.2 is accurate to three significant figures. 9.8 is accurate to two significant figures. The measurement with the least number of significant figures is 9.8 m (two significant figures), so the answer should be rounded to two significant figures.	When multiplying or dividing with measured quantities, the degree of accuracy of the answer is limited by the measurement with the least number of significant figures.

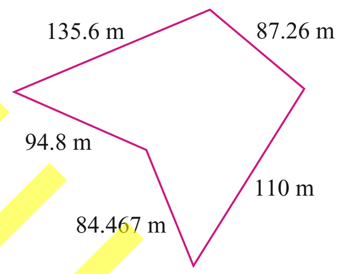
8 Complete the following to calculate 7.5 m × 12.3 m.
7.5 m is accurate to ____ significant figures.
12.3 m is accurate to ____ significant figures.
The least precise measurement is ____ m (to ____ significant figures).
So, 7.5 m × 12.3 m = ____ m^2 = ____ m^2 (to ____ significant figures)

9 Write a sensible approximation for the result of each of the following calculations.

a 23.6 m × 5.7 m
b 405.2 cm × 58.6 cm
c 88 cm^3 ÷ 65 cm^3
d 37.7 mm ÷ 12 mm

PROBLEM SOLVING, REASONING AND JUSTIFICATION

10 Five students are given the task of each measuring one of the sides of an irregular 5-sided block of land. Each student measured their own side to a different degree of accuracy, as shown on the figure on the right. Write a sensible approximation for the perimeter of the land.



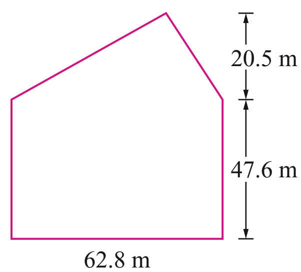
11 The diameter of a 20-cent coin is measured to be 2.3 cm. Write a sensible approximation for the area of the face of the coin.

12 The base and perpendicular height of a triangle are measured to be 5.6 m and 3.89 m respectively. Write a sensible approximation for the area of the triangle.

13 The mass and height of a woman are measured to be 63.8 kg and 175 cm respectively. Write a sensible approximation for the body mass index (BMI) of the woman using the formula BMI $= \frac{\text{mass}}{(\text{height})^2}$ where mass is in kilograms and height is in metres.

CHALLENGE

14 Consider the diagram on the right, showing a composite figure made up of a rectangle and a triangle.



a Calculate a sensible approximation for the area of the rectangle.
b **i** How many significant figures are there in the answer to part **a**?
ii Has the answer to part **a** been rounded to the nearest whole square metre, 10 m^2 or 100 m^2?
c Calculate a sensible approximation for the area of the triangle.
d **i** How many significant figures are there in the answer to part **c**?
ii Has the answer to part **c** been rounded to the nearest whole square metre, 10 m^2 or 100 m^2?
e Calculate a sensible approximation for the area of the composite figure.

2F Absolute error and limits of accuracy

These resources are available on your obook assess:

- **Video tutorial 2F:** Watch and listen to an explanation of Example 2F-2
- **Investigation 2F:** Compare the relative size of errors in calculations
- **assess quiz 2F:** Test your skills with an auto-correcting multiple-choice quiz

precision smallest unit on a measurement instrument

absolute error equals plus or minus half the precision

lower bound of true measurement result of subtracting the absolute error from the given measurement

upper bound of true measurement result of adding the absolute error to the given measurement

In question **5** in the previous section, a steel rod was measured to the nearest centimetre because this was the smallest unit on the ruler: the length was closer to 14 cm than to 13 cm or 15 cm. The greatest possible error for this measurement is 0.5 cm, or half of the smallest scale unit (centimetre) on the ruler.

The actual length will lie between 13.5 cm and 14.5 cm; that is, between 14 − 0.5 cm and 14 + 0.5 cm. To obtain a more accurate measurement, we would need to use a more accurate ruler, one that has a scale marked in smaller units.

Because there is always some degree of error in a numerical value found by measurement, it follows that the results of any calculations involving this value will also contain a degree of error.

> The smallest unit on a measuring instrument is called the **precision** of the instrument.
>
> The **absolute error** when measuring a quantity (sometimes called the greatest possible error) is equal to plus or minus half the precision.
>
> The smallest and largest values between which the actual measurement lies are called the **lower and upper bounds of the true measurement**. These are the limits of accuracy of the measurement.

EXAMPLE 2F–1 Finding precision and absolute error

For each of the measurements below, find:

i the smallest unit of measurement (the precision) **ii** the absolute error.

a 18 cm **b** 2.4 kg

	Solve	Think	Apply
a i	The smallest unit of measurement is 1 cm; that is, the measurement has been made to the nearest centimetre. Precision = 1 cm	The last significant figure of the number is in the units column. So, the smallest scale on the measuring instrument is 1 cm.	The position of the last digit in the number determines the smallest scale on the measuring instrument used. This is the precision of the instrument. The absolute error is ± half the precision.
ii	Absolute error $= \pm\frac{1}{2} \times 1$ cm $= \pm 0.5$ cm	Absolute error $= \pm\frac{1}{2} \times$ precision	
b i	The smallest unit of measurement is 0.1 kg; that is, the measurement has been made to the nearest 0.1 of a kilogram. Precision = 0.1 kg	The last significant figure of the number is in the tenths column. So, the smallest scale on the measuring instrument is 0.1 kg.	
ii	Absolute error $= \pm\frac{1}{2} \times 0.1$ kg $= \pm 0.05$ kg	Absolute error $= \pm\frac{1}{2} \times$ precision	

EXERCISE 2F Absolute error and limits of accuracy

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Complete the following.

a For a measurement given as 138 cm, the last significant figure is in the ____ column.
So, the smallest scale on the measuring instrument is ____.
The measurement has been made to the nearest ____.
So, Precision = ____.
Absolute error = $\pm\frac{1}{2} \times$ _____ = _____

b For a measurement given as 11.7 s, the last significant figure is in the ____ column.
So, the smallest scale on the measuring instrument is ____.
The measurement has been made to the nearest ____.
So, Precision = ____.
Absolute error = $\pm\frac{1}{2} \times$ _____ = _____

2 For each of the following measurements, find:
- **i** the smallest unit of measurement (the precision)
- **ii** the absolute error.

a 16 cm **b** 286 g **c** 38 m **d** 16 L
e 3.6 kg **f** 15.3 s **g** 2.8 L **h** 3.76 m

EXAMPLE 2F–2 Finding absolute error and limits of accuracy

For each of the following measurements, find:
- **i** the smallest unit of measurement (the precision)
- **ii** the absolute error
- **iii** the lower and upper bounds of the true measurement.

a 16 s **b** 9.38 m

	Solve	Think	Apply
a i	The smallest unit of measurement is 1 s; that is, this measurement of time has been made to the nearest second. Precision = 1 s	Find the precision (1 s) and the absolute error (±0.5 s) as in Example 2F-1. Lower bound = 16 s − 0.5 s Upper bound = 16 s + 0.5 s	Find the precision and the absolute error. Lower bound = measurement − half the precision Upper bound = measurement + half the precision *Note:* The true measurement is greater than or equal to the lower bound, but is less than the upper bound; that is, lower bound ≤ true measurement < upper bound.
ii	Absolute error = $\pm\frac{1}{2} \times 1 = \pm 0.5$ s		
iii	Lower bound = 16 − 0.5 = 15.5 s Upper bound = 16 + 0.5 = 16.5 s True measurement is between 15.5 s and 16.5 s.		
b i	Smallest unit of measurement = 0.01 m; that is, this measurement of length has been made to the nearest 0.01 of a metre. Precision = 0.01 m	Find the precision (0.01 m) and absolute error (±0.005 m) as in Example 2F-1. Lower bound = 9.38 m − 0.005 m Upper bound = 9.38 m + 0.005 m	
ii	Absolute error = $\pm\frac{1}{2} \times 0.01$ m = ±0.005 m		
iii	Lower bound = 9.38 − 0.005 = 9.375 m Upper bound = 9.38 + 0.005 = 9.385 m True measurement is between 9.375 m and 9.385 m.		

MEASUREMENT

3 Complete the following for a measurement of 2.6 kg.

a Precision = ____ kg **b** Absolute error = ____ kg

c Lower bound of measurement = 2.6 − ____ kg. Upper bound of measurement = 2.6 + ____ kg

The true measurement lies between ____ and ____.

4 For each of the measurements below, find:

i the precision **ii** the absolute error in the measurement

iii the lower and upper bounds of the true measurement.

a 12 mm **b** 348 g **c** 375 mL **d** 8.2 km

e 18.4 s **f** 4.9 kg **g** 2.37 m **h** 5.81 L

5 The capacity of a container is given as 750 mL, to the nearest 50 mL. Complete the following.

a The measurement has been given to the nearest ____ mL. Precision = ____ mL.

b Absolute error = $\pm\frac{1}{2}\times$ ____ mL = ____ mL.

c Lower bound of measurement = 750 − ____ mL.

Upper bound of measurement = 750 + ____ mL.

The true measurement lies between ____ and ____.

6 For each of the measurements below, find:

i the smallest unit of measurement

ii the absolute error in the measurement

iii the lower and upper bounds of the true measurement.

a The mass of a can of soup is 420 g, to the nearest 30 g.

b The capacity of a drink bottle is 380 mL, to the nearest 20 mL.

c The crowd at a cricket match was 38 000, to the nearest 1000.

d The time taken for a plane flight was $6\frac{1}{2}$ hours, to the nearest $\frac{1}{2}$ hour.

EXAMPLE 2F–3 Finding absolute error as a percentage of the measurement

a Find the absolute error for the measurement 18 kg.

b Express the absolute error as a percentage of the measurement.

	Solve	Think	Apply
a	Smallest unit of measurement = 1 kg Absolute error = ±0.5 kg	Find the precision (1 kg) and absolute error (±0.5 kg) as in Example 2F-1.	The percentage error is the absolute error expressed as a percentage of the given measurement. Percentage error $= \frac{\text{absolute error}}{\text{measurement}} \times 100\%$
b	Percentage error $= \pm\frac{0.5}{18} \times 100\%$ = ±2.8% (to one decimal place)	Percentage error $= \pm\frac{0.5}{18} \times 100\%$	

7 For each of the measurements below, find:

i the absolute error

ii the percentage error.

a 10 cm **b** 32 s **c** 250 g **d** 14 min

e 6 L **f** 2.4 kg **g** 13.5 s **h** 12.56 m

8 Explain how to determine the percentage error for a measurement.

EXAMPLE 2F–4 Finding limits of true perimeter and maximum error

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively.

a Calculate the perimeter of the rectangle using these measurements.

b Find the lower and upper bounds of the rectangle's true perimeter.

c Hence find the maximum error in the answer to part **a**.

	Solve	Think	Apply
a	Perimeter $= 2 \times 8 + 2 \times 6$ $= 28$ cm	Perimeter (using measurements) $= 2 \times l + 2 \times b = 28$ cm	Calculate the perimeter using the measured length and breadth.
b	Now 7.5 cm $\le$ length $<$ 8.5 cm and 5.5 cm $\le$ breadth $<$ 6.5 cm. So, $2 \times 7.5 + 2 \times 5.5$ cm $\le$ perimeter $< 2 \times 8.5 + 2 \times 6.5$ cm. Thus 26 cm $\le$ perimeter $<$ 30 cm.	The absolute error of each measurement is ± 0.5 cm: length lies between 7.5 cm and 8.5 cm and breadth between 5.5 cm and 6.5 cm. Lower bound of perimeter $= 2 \times 7.5 + 2 \times 5.5 = 26$ cm Upper bound of perimeter $= 2 \times 8.5 + 2 \times 6.5 = 30$ cm	Determine the lower and upper bounds of each given measurement. Calculate the perimeter using the lower and upper bounds of length and breadth.
c	Maximum error $= 28$ cm $- 26$ cm (or 28 cm $-$ 30 cm) $= \pm 2$ cm	Maximum error = perimeter (using given measurements) $-$ lower bound of perimeter (or upper bound of perimeter).	Find the difference between the perimeter, calculated using the given measurements, and the perimeter using the lower (or upper) bound of each measurement.

9 The length and breadth of a rectangular recreation room are measured to be 7 m and 4 m, respectively. Complete the following.

a Using the given measurements,
Perimeter $= 2 \times$ ____ $+ 2 \times$ ____
$=$ ____ m

b Now 6.5 m $\le$ length $<$ ____ m
and ____ m $\le$ breadth $<$ 4.5 m
Lower bound of perimeter
$= 2 \times 6.5 + 2 \times 3.5$
$=$ ____ m
Upper bound of perimeter
$= 2 \times$ ____ $+ 2 \times$ ____
$=$ ____ m
So, ____ m $\le$ perimeter $<$ ____ m

c Maximum error $=$ ____ m $-$ ____ m
$= \pm$____ m

10 The length and breadth of a rectangle were measured to be 9 cm and 5 cm respectively.

a Calculate the perimeter of the rectangle using these measurements.

b Find the lower and upper bounds of the rectangle's true length and breadth.

c Hence, find the lower and upper bounds of the rectangle's true perimeter.

d Find the maximum error in the answer to part **a**.

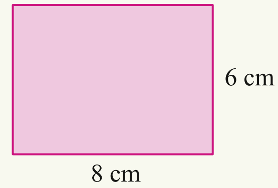
EXAMPLE 2F–5 Finding limits of true area and maximum error

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively.

a Calculate the area, using the measurements given.

b Find the lower and upper bounds of the true area.

c Hence, find the maximum error in the answer to part **a**.



	Solve	Think	Apply
a	Area = $8 \times 6 = 48$ cm^2	Using the given measurements: Area $= l \times b$ $= 48$ cm^2	Calculate the area using the measured length and breadth.
b	Now 7.5 cm $\leq$ length < 8.5 cm and 5.5 cm $\leq$ breadth < 6.5 cm. $(7.5 \times 5.5) \leq$ area $< (8.5 \times 6.5)$. So 41.25 cm$^2 \leq$ area < 55.25 cm^2.	The absolute error of each measurement is ±0.5 cm. So, the length lies between 7.5 cm and 8.5 cm and the breadth lies between 5.5 cm and 6.5 cm. Lower bound of area $= 7.5 \times 5.5$ $= 41.25$ cm^2 Upper bound of area $= 8.5 \times 6.5$ $= 55.25$ cm^2	Determine the lower and upper bounds of each given measurement. Calculate the area using the lower bounds of the length and breadth, and calculate the area using the upper bounds of the length and breadth.
c	$48 - 41.25 = 6.75$ cm^2 $48 - 55.25 = -7.25$ cm^2 Maximum error $= 7.25$ cm^2	Maximum error = area (using given measurements) − lower bound of area (or upper bound of area).	Find the difference between the area calculated using the given measurements and the area using the lower (or upper) bound of each measurement.

11 The length and breadth of a rectangle are measured to be 7 cm and 4 cm respectively. Complete the following.

a Using the given measurements: Area = (____ × ____) cm^2 = ____ cm^2

b Now 6.5 cm $\leq$ length < ____ cm and ____ cm $\leq$ breadth < 4.5 cm

Lower bound of area = (6.5×3.5) cm^2 = ____ cm^2

Upper bound of area = (____ × ____) cm^2 = ____ cm^2

So ____ cm^2 $\leq$ area < ____ cm^2

c ____ − lower bound of area = ____ cm^2

____ − upper bound of area = ____ cm^2

Maximum error = ±____ cm^2

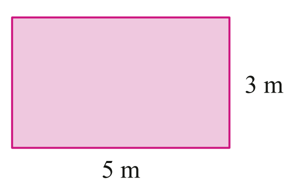
12 A rectangle was measured to be 5 m long by 3 m wide.

a Calculate the area of the rectangle using these measurements.

b Find the lower and upper bounds of the true length and width.

c What are the lower and upper bounds of the true area?

d Find the maximum error in the answer to part **a**.



PROBLEM SOLVING, REASONING AND JUSTIFICATION

13 Two pieces of timber were measured to be 164 cm and 128 cm respectively.
- **a** If the two pieces were placed end to end, what would be their total length, using the measurements given?
- **b** Find the lower and upper bounds of the true length of each piece.
- **c** Hence, calculate the lower and upper bounds of the true total length of these two pieces of timber.
- **d** Find the maximum error in the answer to part **a**.

14 The masses of two bags of sand were measured and found to be 47 kg and 52 kg.
- **a** What is the total mass of the two bags?
- **b** Find the lower and upper bounds of the true mass of each bag.
- **c** Calculate the lower and upper bounds of the true total mass.
- **d** What is the maximum error in the answer to part **a**?

15 Repeat question **14** given that the masses of the sand bags were 47.4 kg and 51.9 kg.

16 A rectangular room was measured to be 5.4 m long by 3.2 m wide.
- **a** Calculate the area of the room using these measurements.
- **b** Find the lower and upper bounds of the room's true length and width.
- **c** What are the lower and upper bounds of the room's true area?
- **d** Find the maximum error in the answer to part **a**.
- **e** Compare your answers to those obtained in question **12**.

17 The diameter of a circular pizza tray is measured to be 28.6 cm.
- **a** Calculate the area of the tray using the measurement given. (Use $A = \frac{\pi d^2}{4}$.)
- **b** What are the lower and upper bounds of the true length of the tray's diameter?
- **c** Find the lower and upper bounds of the true area of the tray.
- **d** What is the maximum error in the answer to part **a**?

CHALLENGE

18 A (rectangular) billiards table is measured to be 2.84 m by 1.42 m.
- **a** Write the absolute error of each measurement.
- **b** Find the percentage error of each measurement, to two significant figures.
- **c** Calculate the area of the table using the given measurements.
- **d** Calculate the lower and upper bounds of the table's true area.
- **e** What is the maximum error in the answer to part **c**?
- **f** Express the error stated in part **e** as a percentage of the calculated area in part **c**, to two significant figures.
- **g** Is the percentage error in part **f** the sum of the percentage errors in part **b**?

CHAPTER 2 REVIEW PRACTICALITIES OF MEASUREMENT

You should be able to:

- ✔ round numbers using significant figures
- ✔ express decimal numbers in scientific notation, and vice versa
- ✔ perform calculations with numbers expressed in scientific notation
- ✔ convert between the commonly used metric units for length, mass, capacity, area and volume
- ✔ understand the possible sources of error in measuring and how to reduce their effect
- ✔ determine the precision, the absolute error, the upper and lower bounds and the percentage error for a measurement
- ✔ find the maximum possible error when measurements are used in calculations
- ✔ make sensible approximations for the results of calculations using measurements.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

2A **1** When rounded to two significant figures, 3950.628 becomes:
A 3900 **B** 4000 **C** 39 **D** 3950.63

2B **2** Which of the following numbers is written in scientific notation?
A 5×10000 **B** 50000 **C** 5×10^4 **D** 50×10^3

2B **3** 7.06×10^{-6} is equivalent to
A 0.000 007 06 **B** 0.000 0706 **C** 706 000 **D** 7 060 000

2B **4** The result of the calculation $(4 \times 10^5) \div (8 \times 10^{-3})$ is:
A 5×10^8 **B** 5×10^7 **C** 5000 **D** 5×10^2

2C **5** Which one of the following is equivalent to 5.06 kg?
A 0.005 06 g **B** 5060 g **C** 0.0506 g **D** 506 g

2C **6** Which one of the following is *not* equivalent to 5.3 m?
A 530 cm **B** 5300 mm **C** 0.0053 km **D** 0.053 km

2C **7** The capacity of a drinking glass would be closest to:
A 2 mL **B** 20 mL **C** 200 mL **D** 2 L

2D **8** 6 ML is equivalent to:
A 60 000 kL **B** 6000 kL **C** 600 kL **D** 60 kL

2F **9** The absolute error in the measurement 3.6 L is:
A ±0.1 L **B** ±0.05 L **C** ±0.5 L **D** ±3.55 L

2F **10** The mass of a can of soup was 250 g, to the nearest 10 g. The percentage error in this measurement is:
A ±4% **B** ±2% **C** ±0.4% **D** ±0.2%

2F **11** The side length of a square was measured to be 8 cm. The maximum error in stating that the perimeter is 32 cm is:
A 0.5 cm **B** 1 cm **C** 2 cm **D** 4 cm

REVIEW SET 1

1 Round 3659.063 to:

a the nearest 100 **b** the nearest whole number **c** two decimal places
d three significant figures **e** five significant figures **f** one significant figure

2 State whether each of these numbers is written in scientific notation.

a 6×1000 **b** 15×10^7 **c** 2.04×10^{-6}

3 Express these numbers in scientific notation.

a 105 000 000 **b** 0.000 062 **c** 3179

4 Calculate the following, writing the answers in scientific notation.

a $(4.1 \times 10^8) \times (6 \times 10^5)$ **b** $(1.96 \times 10^{-3}) \div (1.4 \times 10^7)$
c $(8 \times 10^5)^4$ **d** $\sqrt{8.41 \times 10^{-12}}$

5 Convert the following.

a $5.6\ \text{cm}^2 =$ ____ mm^2 **b** $43\,000\ \text{m}^2 =$ ____ ha
c $2.9\ \text{m}^3 =$ ____ cm^3 **d** $5600\ \text{mm}^3 =$ ____ cm^3

6 Convert the following.

a 2.3 Gm = ____ m **b** 52 ML = ____ kL **c** 3 ms = ____ μs **d** 7.2 Tg = ____ mg

7 For each of the following measurements, find the:

i precision **ii** absolute error
iii lower and upper bounds of the true measurement **iv** percentage error (to one decimal place).

a 7.5 m **b** 280 g

8 The length and breadth of a rectangle were measured to be 6 cm and 4 cm.

a Calculate the rectangle's perimeter using these measurements.
b Write the lower and upper bounds of the true length and breadth of the rectangle.
c Find the lower and upper bounds of the rectangle's true perimeter.
d What is the maximum error in the answer in part **a**?
e Calculate the rectangle's area using the measurements given.
f Find the lower and upper bounds of the rectangle's true area.
g Find the maximum error in the answer to part **e**.

9 Write sensible approximations for the results of the following calculations.

a 17.3 m + 15.89 m **b** 17.3 m × 15.89 m

REVIEW SET 2

1 Round 1472.634 to:

a the nearest 10 **b** two significant figures **c** two decimal places.

2 Express each of the following in scientific notation.

a 749 000 **b** 0.000 003 **c** 0.0105

3 Calculate $(1.4 \times 10^7) \times (4.5 \times 10^8)$, expressing the answer in scientific notation.

4 Convert the following.

a $2.1\ \text{ha} =$ ___ m^2 **b** $780\ \text{mm}^2 =$ ___ cm^2 **c** $9\,500\,000\ \text{cm}^3 =$ ___ m^3 **d** $72\ \text{cm}^3 =$ ___ mm^3

5 Convert the following.

a 4.3 cm = ____ μm **b** 2 Tg = ____ Mg **c** 52 000 kL = ____ ML **d** 9.1 mm = ____ nm

6 Write sensible approximations for the results of the following calculations.

a 15.36 m + 9.7 m + 11.62 m **b** 16.5 cm × 4.7 cm

7 For each of the following measurements, find the:

i precision **ii** absolute error

iii lower and upper bounds of the true measurement **iv** percentage error (to one decimal place).

a 12.8 kg **b** 12.56 m

8 The length and breadth of a table were measured to be 154 cm and 80 cm, to the nearest centimetre.

a Calculate the table's area using the measurements given.

b Find the lower and upper bounds of the table's true area.

c Find the maximum error in the answer to part **a**.

d Express the maximum error from part **c** as a percentage of the area (from part **a**).

REVIEW SET 3

1 Round 0.005 06 to:

a two significant figures **b** two decimal places **c** one significant figure

2 State whether or not each of the following numbers is expressed in scientific notation.

a $4.9 \times 100\,000$ **b** 7.0×10^8 **c** 70×10^7

3 Convert each of the following numbers to scientific notation.

a 67 000 **b** 0.000 080 9 **c** 1230

4 Write the basic numeral for:

a 3.4×10^6 **b** 8.7×10^{-5} **c** 2.053×10^4

5 Convert the following.

a 11.2 cm^2 = ___ mm^2 **b** 129 000 m^2 = ___ ha

c 3.4 m^3 = ___ cm^3 **d** 73 000 mm^3 = ___ cm^3

6 Convert the following.

a 4.5 Mm = ___ m **b** 2 Mg = ___ t **c** 7 μs = ___ ns **d** 3.5 kL = ___ GL

7 The masses of two bags of potatoes were measured, to be 49 kg and 51 kg, to the nearest kilogram.

a What is the total mass of the two bags using these measurements?

b Write the lower and upper bounds of the true mass of each bag.

c Calculate the lower and upper bounds of the total mass of the two bags.

d Determine the absolute error in the answer to part **a**.

e Express the absolute error as a percentage of the total mass of the bags.

8 Write sensible approximations for the results of the following calculations.

a 43.2 kg − 8 kg **b** 125.345 L ÷ 0.85 L

REVIEW SET 4

1 Round 2.0695 to these numbers of significant figures:

a one significant figure **b** two significant figures **c** three significant figures **d** four significant figures.

2 Convert the following.

a 13.65 m = ___ cm **b** 3460 kg = ___ t **c** 276 mL = ___ L **d** 8.3 m = ___ mm

3 Convert the following.

a 13.65 ha = ___ m^2 **b** 1960 mm^2 = ___ cm^2 **c** 3 700 000 cm^3 = ___ m^3 **d** 6.8 cm^3 = ___ mm^3

4 Convert the following.

a 6 mL = ___ μL **b** 4.2 Gg = ___ kg **c** 8.1 μs = ___ ns **d** 560 ML = ___ TL

5 Express in scientific notation, correct to two significant figures.

a 643 700 000 **b** 0.000 000 304

6 Calculate the following, expressing the answer in scientific notation.

a $(1.08 \times 10^{-6}) \div (7.2 \times 10^{-5})$ **b** $\sqrt{1.96 \times 10^{20}}$

7 The base and perpendicular height of a triangle were measured to be 15.4 cm and 12.5 cm respectively.

a Find the area of the triangle using these measurements.

b Calculate the range within which the triangle's true area lies.

c What is the maximum error in using part **a** as the area?

d Express the maximum error from part **c** as a percentage of the area (from part **a**).

8 The length and breadth of a bed are measured to be 1.9 m by 0.84 m respectively. Write a sensible approximation for:

a the perimeter of the bed **b** the area of the bed.

REVIEW PRACTICE EXAMINATION QUESTION

1 **a** Write 0.001 306 in scientific notation, rounded to three significant figures. (2 marks)

b How many significant figures are there in the measurement 5.0×10^6 metres? (1 mark)

c Evaluate $\sqrt{6.724 \times 10^{11}}$. Write your answer in scientific notation. (1 mark)

d Convert each measurement:

i 94 630 mm = ___ m **ii** 2.5 m^3 = ___ cm^3. (2 marks)

e The scale on a thermometer measures temperature to the nearest 0.5°C.

i What is the absolute error in stating that the temperature is 19.5°C? (1 mark)

ii Calculate the percentage error for this measurement. (1 mark)

f Bottle A contains 2.64 L of saline solution and bottle B contains 2.88 L of the same solution. They are both poured into a large container.

i Calculate the volume of saline solution in the large container, using the measurements given. (1 mark)

ii Find the lower and upper bounds of the true volume of saline solution. (2 marks)

iii Calculate the maximum error in the answer to part **i**. (1 mark)

iv Express the error from part **iii** as a percentage of the answer in part **i**. (1 mark)

g The base and height of a triangle are measured to be 28.4 cm and 9.6 cm respectively. Write a sensible approximation for the area of the triangle, based on the accuracy of each measurement. (2 marks)

TOTAL: 15 marks