6

Network concepts

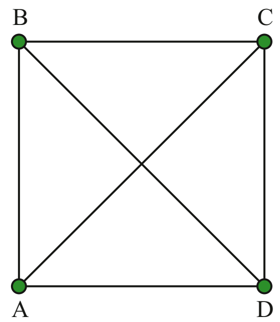
The main mathematical ideas in this chapter are:

- understanding the constituent parts of a network
- drawing network diagrams to represent information
- identifying walks, paths and cycles in a network
- finding trees and minimum spanning trees in a network
- using Kruskal's and Prim's algorithm to find a minimum spanning tree
- finding the shortest path between two vertices
- solving real-life problems using networks.

NETWORKS
MS-N2 Network Concepts

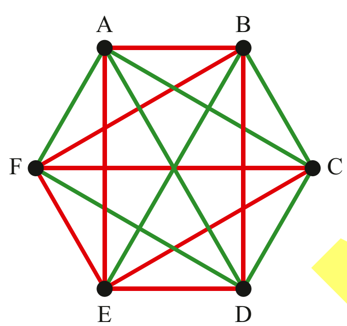
ARE YOU READY?

6A **1** How many line segments (straight lines between two points) are shown in the network?



A 4 **B** 6
C 7 **D** 8

Use the network below to answer questions **2** and **3**.



6A **2** Which of the following is a red triangle?

A ADF **B** FCE
C AED **D** ACD

6A **3** Which of the following is a green triangle?

A ADB **B** FCE
C AED **D** ACD

The following table contains the distance (in kilometres) between capital cities in mainland Australia. Use this table to answer questions **4** to **8**.

Adelaide						
2075	**Brisbane**					
1209	1267	**Canberra**				
3041	3435	4034	**Darwin**			
732	1813	651	3773	**Melbourne**		
2721	4434	3930	4037	3453	**Perth**	
1419	978	289	3971	876	3975	**Sydney**

6A **4** The distance between Brisbane and Melbourne is:

A 651 km **B** 1813 km
C 3435 km **D** 4434 km

6A **5** What is the total distance of a trip from Melbourne to Sydney, then Sydney to Canberra?

A 876 km **B** 940 km
C 1165 km **D** 1527 km

6A **6** What is the total distance of a trip from Melbourne to Canberra, then Canberra to Sydney?

A 876 km **B** 940 km
C 1165 km **D** 1527 km

6A **7** What is the smallest distance in the table?

A 156 km **B** 289 km
C 651 km **D** 4434 km

6A **8** A travelling salesman sets out from Melbourne to Brisbane. Along the way he must visit Sydney and Canberra. Which itinerary covers the shortest distance?

A Melbourne, Sydney, Canberra, Brisbane
B Melbourne, Canberra, Sydney, Brisbane
C They cover the same distance
D Impossible to tell

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your obook assess.

Q1–3 Support sheet 6A.1 Lines and line segments
Q4–8 Support sheet 6A.2 Interpreting tables

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6A Introduction to networks

These resources are available on your obook assess:
- **Video tutorial 6A:** Watch and listen to an explanation of Example 6A–3
- **Worksheet 6A:** Practise your skills with extra problems for networks
- **assess quiz 6A:** Test your skills with an auto-correcting multiple-choice quiz

network
a collection of objects that are related to each other in some way

vertex
a visual representation of some object, usually drawn as a point; sometimes called a node

edge
a line between two vertices, which indicates a relationship between the vertices; sometimes called an arc

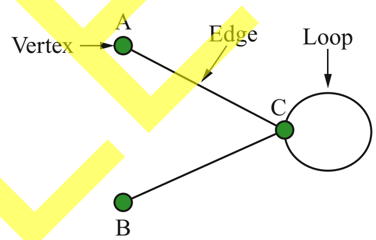
loop
an edge that starts and ends at the same vertex

degree
(of a vertex) the number of edges connected to a vertex

Networks deal with the idea of optimisation, which involves finding the *best possible* solution to a problem. Such as finding the *minimum* spanning tree, or the shortest path between two places.

Before we begin solving practical problems, we need to introduce some terminology. A **vertex** is a point, and an **edge** is a line drawn between two vertices. An edge drawn from a vertex back to itself is called a **loop**. The **degree** of a vertex is the number of edges connected to it (a loop adds 2 to the degree of a vertex).

Together vertices and edges form a network.



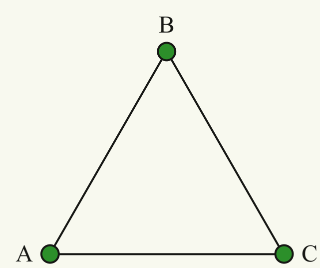
EXAMPLE 6A–1 Counting the number of vertices, edges and the degree of each vertex

For the network on the right, count:

a the number of vertices

b the number of edges

c the degree of each vertex.



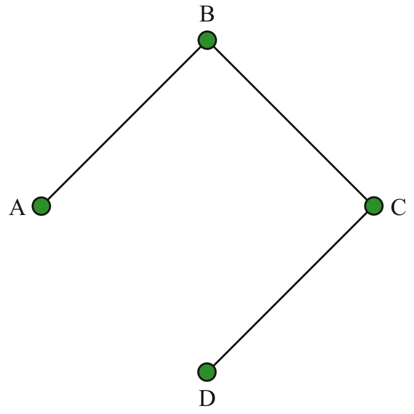
	Solve/Think	Apply
a	There are three vertices: A, B and C.	Vertices are the points in the network. They are denoted by a single letter. Edges are the lines joining two vertices together. They are denoted by the letters at either end of the edge, or by their own name. The degree of a vertex is the number of edges connected to it.
b	There are three edges: AB, BC and CA.	
c	Vertex A is connected to two edges: AB and CA. It has a degree of 2. Vertex B is connected to two edges: AB and BC. It has a degree of 2. Vertex C is connected to two edges: BC and CA. It has a degree of 2.	

EXERCISE 6A Introduction to networks

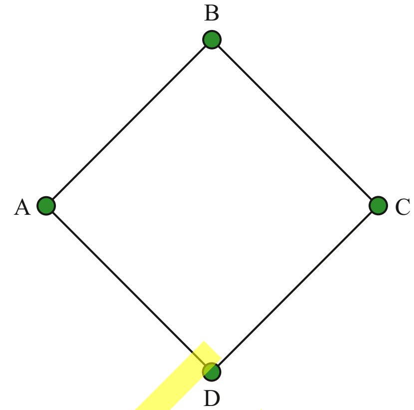
UNDERSTANDING, FLUENCEY AND COMMUNICATING

1 Count the number of vertices in each network.

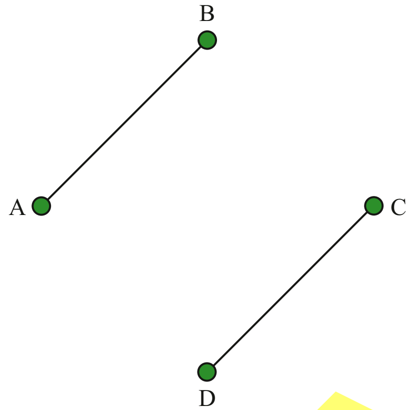
a



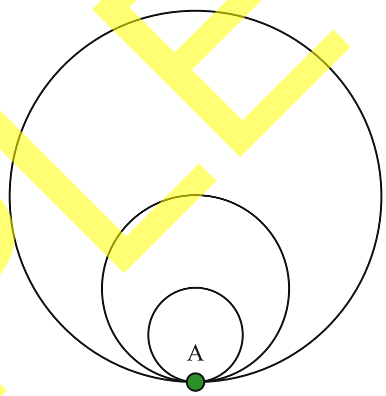
b



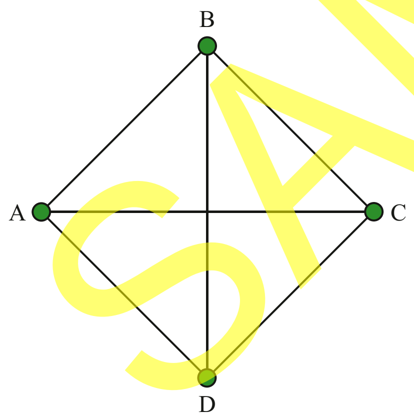
c



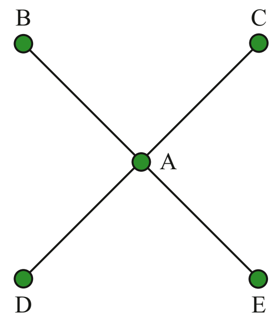
d



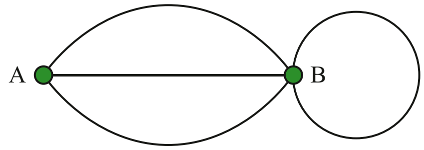
e



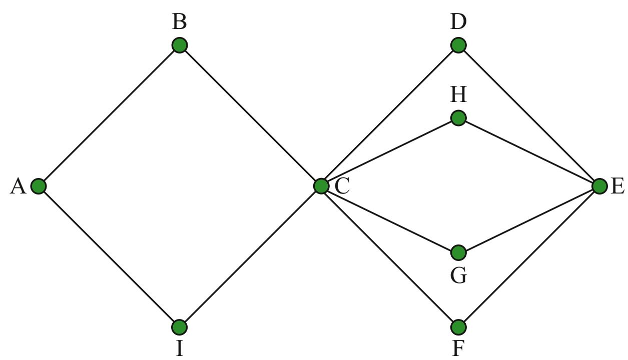
f



g



h

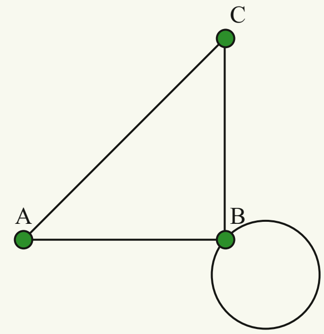


2 Count the number of edges in each network in question **1**.

NETWORKS

EXAMPLE 6A–2 Determining the sum total of the vertex degrees of a network

a Make a table of the vertex degrees for the network on the right.
b How many edges are in this network?
c What is the sum total of the vertex degrees?



	Solve/Think	Apply
a	<table><tr><td>**Vertex**</td><td>A</td><td>B</td><td>C</td></tr><tr><td>**Degree**</td><td>2</td><td>4</td><td>2</td></tr></table>	The loop contributes 2 to the vertex degree of B.
b	The number of edges is 4.	
c	The sum of vertex degrees is $2 + 4 + 2 = 8$.	

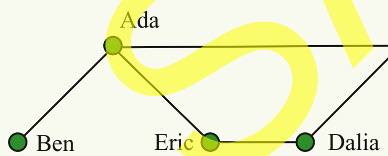
Note that the sum of the vertex degrees (8) is twice the number of edges (4). This is the case for all networks.

3 **i** Make a table of the vertex degrees for each network in question **1**.
ii What is the sum total of the vertex degrees for each network? Verify that the sum total of the vertex degrees is always twice the number of edges.

EXAMPLE 6A–3 Drawing a network from a table

Here is a list of people and their network of friends. Represent this information as a network.

Person	Ada	Ben	Carlos	Dalia	Eric
Friends	Ben, Carlos, Eric	Ada	Ada, Dalia	Carlos, Eric	Ada, Dalia

Solve	Think/Apply
	This information can be represented visually as a network by using a vertex to represent each person, and an edge to indicate that they are friends.

Note that this is just one of many ways to draw the information in the table. Another way would be to draw the vertex for Ben inside the loop.

4 Complete the table of the vertex degrees from Example 6A–3.

Vertex	Ada	Ben	Carlos	Dalia	Eric
Degree	3		2		

5 The following table represents airports and the destinations serviced by that airport. Draw a network that represents this information.

Airport	Albury (ABX)	Bathurst (BHS)	Coffs Harbour (CFS)	Dubbo City Regional (DBO)	Evans Head (EVH)
Destinations	BHS, DBO	ABX, CFS, DBO	BHS, EVH	ABX, BHS	CFS

6 **a** Draw a network that represents the following people and their friends.

Person	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
Friends	Vanessa, Zhang	Wasim, Uma	Vanessa, Zhang, Yvonne, Xavier	Wasim, Yvonne	Wasim, Xavier	Uma, Wasim

b Complete the table of vertex degrees.

Vertex	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
Degree						

c Uma is telling Vanessa a story about what happened to a friend of a friend of hers. Use your answer to part **a** to find out who the story is about.

d Yvonne is telling Wasim a story about what happened to a friend of a friend of hers. Use your answer to part **a** to find out who the story is *not* about.

directed network a network in which the edges have a direction; the orientation of the edge is represented by an arrow

weighted edge edges with a weight that represents a numerical quantity associated with the relationship between the vertices

!

A **directed network** is a network in which the edges are given a direction indicated by an arrow. A directed edge indicates a *one-sided* relationship between the vertices, such as the direction of a one-way street or the direction of the flow of water.

A **weighted edge** is an edge with a certain numerical value, such as the length of the street or the size of the flow of water in litres per second. In practice, the meaning of the edge direction and weight is determined by the context of the network.

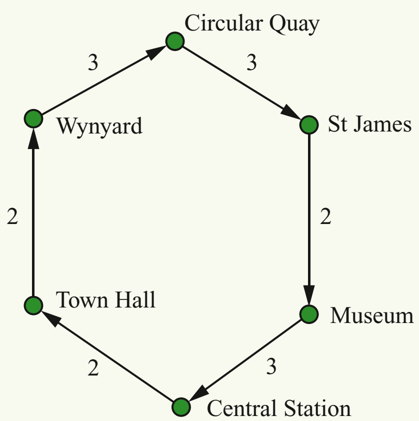
EXAMPLE 6A–4 Determining the travel time in a network with weighted edges

The Sydney CBD stations are connected by an underground rail network called the City Circle. Trains start at Central Station and travel in the City Circle clockwise (via Town Hall) or anticlockwise (via Museum). The following table shows the stations in the City Circle via the Town Hall line, and the time it takes to travel between them.

Station	Central	Town Hall	Wynyard	Circular Quay	St James	Museum
Next stop	Town Hall	Wynyard	Circular Quay	St James	Museum	Central
Time (minutes)	2	2	3	3	2	3

a Represent the City Circle via Town Hall line as a directed network with weighted edges.

b How long does it take to travel from Central to St James using the City Circle via the Town Hall line?

Solve/Think	Apply
a 	Each station is a vertex. The connections between the stations are represented by directed edges and detailed with the time it takes to travel between them.

	Solve/Think	Apply
b	It takes 2 + 2 + 3 + 3 = 10 minutes to travel from Central Station to St James.	Travel along this network is only permitted in the direction of the arrows, so although it would be shorter to travel from Central to St James via Museum, the direction of the edges in this network do not permit this route.

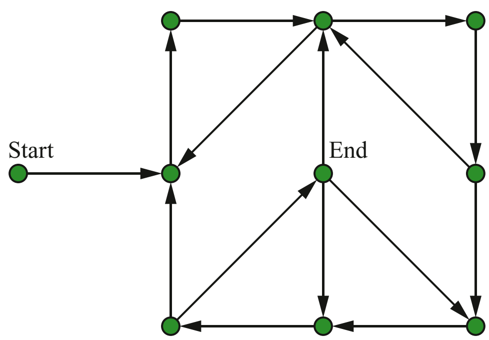
7 Using the network from Example 6A–4, find the time it takes to travel from Town Hall to Museum using the City Circle via the Town Hall line.

! Vertices in a directed network have an indegree and an outdegree, instead of a degree. The **indegree** is the number of edges pointing towards the vertex, and the **outdegree** is the number of edges pointing away from the vertex.

indegree the number of edges pointing towards a vertex in a directed network

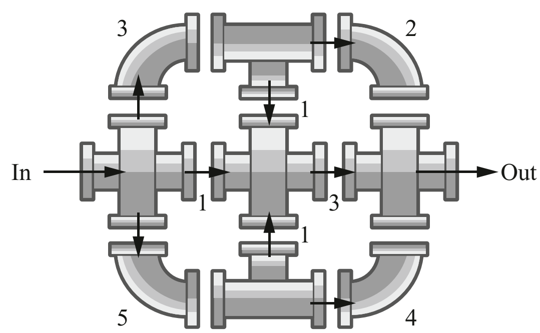
outdegree the number of edges pointing away from a vertex in a directed network

8 Find your way to the centre of the labyrinth. You can only travel along edges in the direction of the arrows.

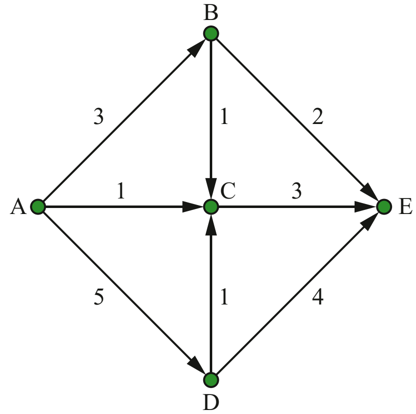


a How many vertices did you touch on your way (including the start and end)?
b How many edges did you travel along?
c What is the indegree of the End vertex?
d What is the outdegree of the End vertex?
e Is there more than one way from the Start to the End?

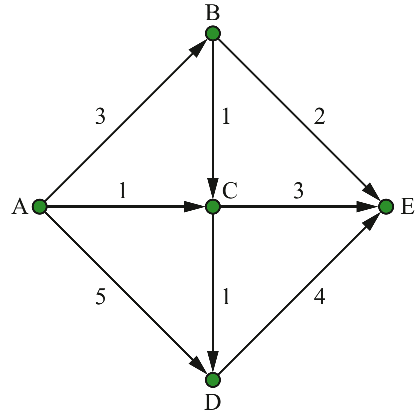
9 A network of pipes flows from junction to junction as shown. The vertices A, B, C, D and E represent the intersection points of the pipes.
Which of the diagrams below shows the direction of the flow and the amount of the flow, in litres per second, as a directed and weighted network?



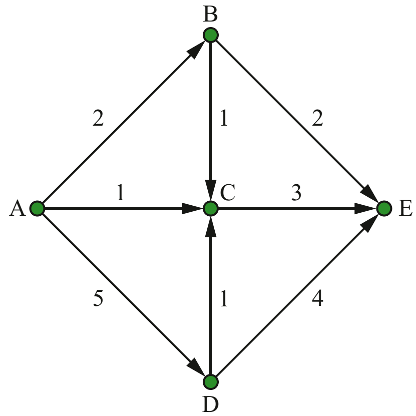
A



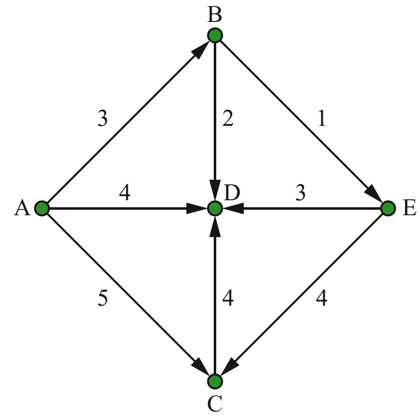
B



C



D



10 For the network in question **9**, find the total flow, in litres per second that:

a flows out of vertex A

b flows into vertex E

c flows into vertex B

d flows out of vertex C

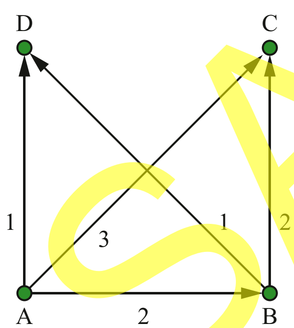
e flows out of vertex D.

11 Here is a table of vertices A, B, C and D, and the edge weights between these vertices.

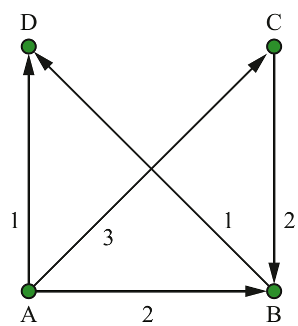
From vertex	To vertex			
	A	**B**	**C**	**D**
A	–	2	3	1
B	–	–	–	3
C	–	2	–	
D	–	–	–	–

Which of the following directed weighted networks represents this information?

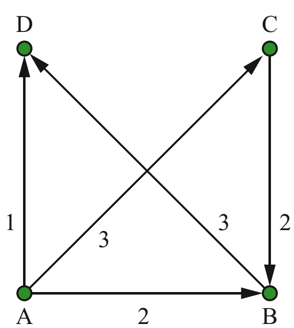
A



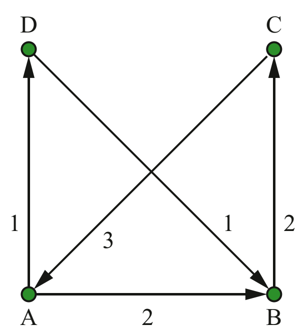
B



C



D



UNDERSTANDING, FLUENCEY AND COMMUNICATING

12 The table below shows the edge weights from question **9**. Fill in the blank edge weights.

From vertex	To vertex				
	A	B	C	D	E
A	–	3			–
B	–	–	1	–	
C	–	–	–	–	3
D	–	–	1	–	
E	–	–	–	–	–

13 Here is a table of vertices A, B, C and D, with the edge weights between these vertices. Draw a directed weighted network to represent this information.

From vertex	To vertex			
	A	B	C	D
A	–	–	2	5
B	1	–	7	–
C	–	–	–	4
D	–	3	–	–

PROBLEM SOLVING, REASONING AND JUSTIFICATION

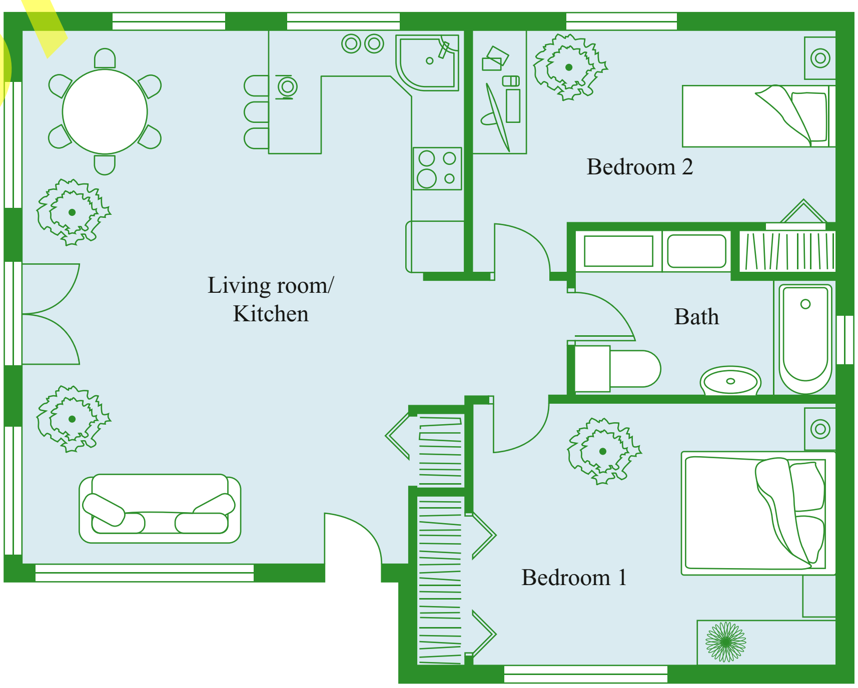
14 Imagine that the networks in question **1** represent people at a party. The people are vertices, and an edge indicates that the two people have shaken hands (a loop means that the person shakes their other hand, and so counts for two handshakes). How many people shook an odd number of hands in each network? Verify that there is always an even number of such people.

15 A house has a printer, computer, TV and gaming console. All these devices must be connected to a single router via ethernet cables.

a Draw a network diagram to represent this information. Use vertex names P, C, T, G and R for the printer, computer, TV, gaming console and router respectively, and use edges to represent an ethernet connection between the devices.

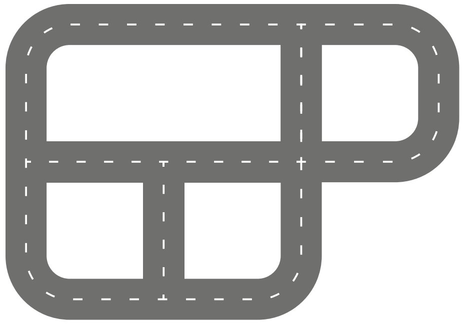
b How many ethernet connections are required at R?

16 Draw the following house plan as a network. Use vertices to represent the rooms and edges to represent the doorways that connect the rooms.

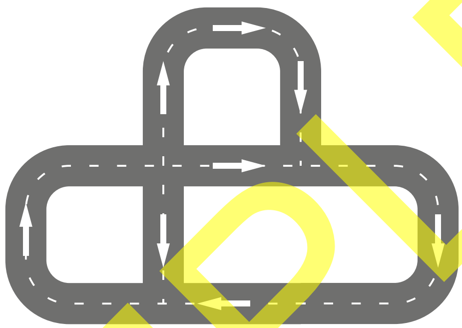


PROBLEM SOLVING, REASONING AND JUSTIFICATION

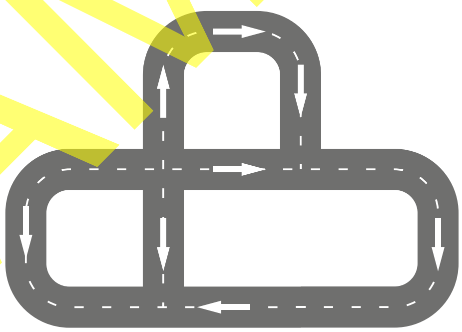
17 Redraw the roads below as a network using vertices to represent intersections, and edges to represent the span of road connecting two intersections.



18 Draw a directed network that represents the following street map. Represent the street intersections with vertices, and use directed edges to represent the span of road between intersections.

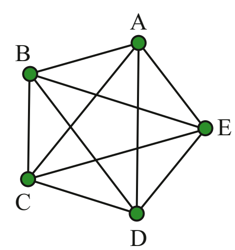


19 Here is the same road layout, except that the street orientation has changed. Represent this as a directed network, then use the notions of indegree and outdegree to describe the problem at each intersection.



20 Use a red and blue pen to colour over all the edges in the network on the right. Try to do this without drawing a red triangle or a blue triangle.

Hint: first colour as many edges in red as you can without drawing a triangle, then do the same for blue.

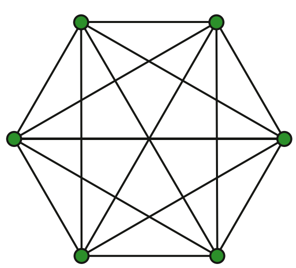


CHALLENGE

21 **a** Colour all the edges of the network on the right in blue or red.

b There will always be a red triangle or a blue triangle. See if you can find it.

c Suppose that the vertices represent people. The blue edges connect people who have met before, and the red edges connect people who have not met before. You are hosting a dinner party and want to ensure that three people will all have met before, or three people will have never met before. What is the smallest number of people you should invite?
Hint: compare your answers to questions **20** and **21b**.



NETWORKS

6B Paths and cycles

These resources are available on your obook assess:
- **Worksheet 6B:** Practise your skills with walks, paths and cycles
- **assess quiz 6B:** Test your skills with an auto-correcting multiple-choice quiz

OI
GI

walk
a sequence of vertices and the edges between them

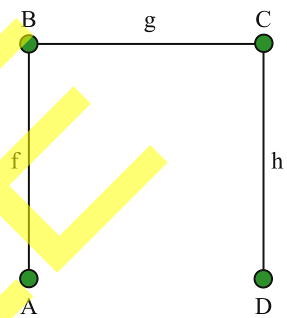
connected network
a network in which there is a walk between every pair of vertices

disconnected network
a network that is not connected

In this section we will explore walks through a network. A **walk** is a journey through a network that starts at a vertex and travels along the edges of the network, before ending at a vertex.

Trace your finger over the network on the right from vertex A to vertex D. The vertices and edges that you touched, in the order that you touched them, are:

A, f, B, g, C, h, D



This sequence of vertices and edges is a walk. It can be read as follows:

Starting from A, travel along edge f to vertex B, then along edge g to vertex C, then along edge h to vertex D. Vertex A is called the start vertex and vertex D is called the end vertex.

It is sometimes convenient to abbreviate the walk to just the vertices:

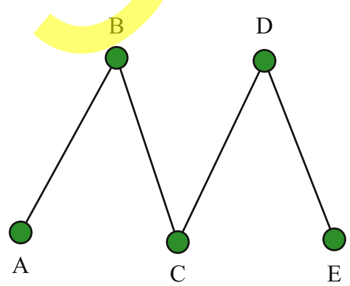
ABCD

The edges f, g and h are implicit in the walk that visits vertices A, B, C, D. This is convenient when the edges have no names, and we will use this style of abbreviation often in this chapter. Another style of abbreviation is to use only the edges:

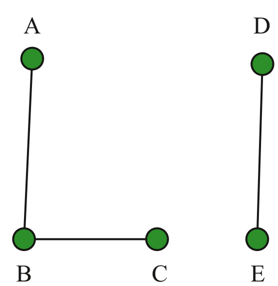
fgh

The vertices A, B, C and D are implicit in the walk with edges fgh. This is convenient when the edges do have names.

Connected networks are networks in which there is a walk between any pair of vertices. If there is a pair of vertices with no walk between them the network is said to be **disconnected**.



Connected network

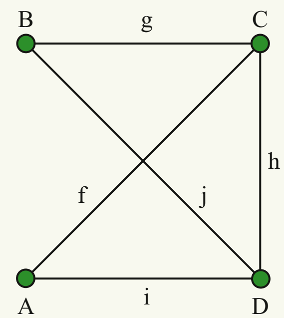


Disconnected network

EXAMPLE 6B–1 Finding a walk in a network

Which of the following is a walk from A to B?

A AB
B A, f, C, g, B
C A, f, C, h, D
D B, j, D, i, A

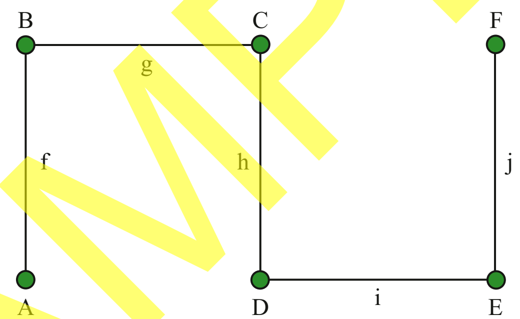


Solve/Think	Apply
Option B (A, f, C, g, B) is a walk from A to B.	Option A is not a walk since there is no edge between A and B. Option C is a walk from A to D. Option D is a walk from B to A.

EXERCISE 6B Paths and cycles

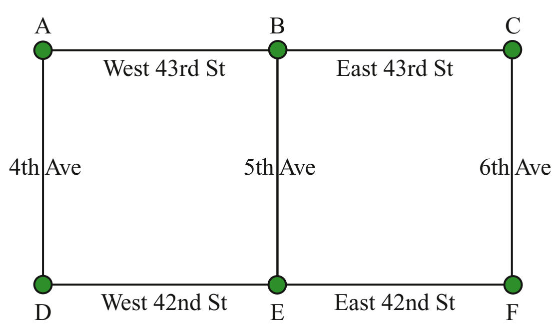
UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Trace out a walk from A to F in the network below. Write down each vertex and edge that you visit in order.



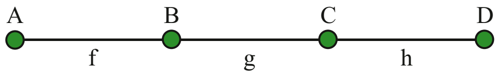
2 In the network from question 1, trace out the walk with edges ihg.
- **a** What is the start vertex?
- **b** What is the end vertex?

3 The streets of Manhattan are divided into East and West by 5th Avenue. The following network is a map of Manhattan between 4th and 6th avenues and 43rd and 42nd streets.



You are given the following directions: Start at corner A. Follow West 43rd St to corner B. Follow 5th Avenue to Corner E. Follow East 42nd St to Corner F. Then follow 6th Avenue to corner C. Are these valid directions? Where do they lead?

4 **a** Consider the walk fgh in the network below. What is the sequence of vertices visited by this walk? What are the start and end vertices?



b Consider the walk DCBA. What is the sequence of edges visited by this walk? What are the start and end vertices?

c Consider the walk ABCDCBA. What is the sequence of edges visited by this walk? What are the start and end vertices?

path
a walk that does not visit any vertex twice (or more)

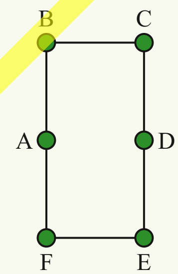
cycle
a walk that starts and ends at the same vertex, but does not visit any other vertices twice (or more)

A **path** is a walk that doesn't visit any vertex more than once. A **cycle** is a walk with the same start and end vertex, which doesn't visit any other vertex more than once. !

EXAMPLE 6B–2 Identifying walks, paths and cycles

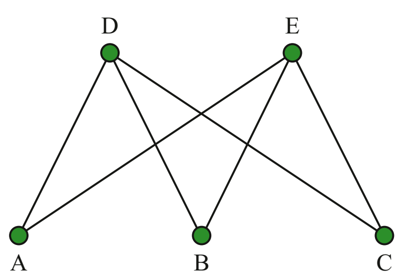
Identify the following as a walk, path, cycle, or other, giving reasons for your choice.

a ABCDED
b DEFA
c ABCDEFA
d AD



	Solve/Think
a	This is a walk because it follows the edges of the network. It cannot be a path because it visits vertex D twice.
b	This is a path because it follows the edges of the network but does not visit any vertex more than once.
c	This is a cycle because it follows the edges of the network, and the start vertex and end vertex are the same.
d	There is no edge between A and D, hence this is something other than a walk, path or cycle.
	Apply
	In this example, the walks are referred to as a sequence of vertices. This is common practice when the edges do not have labels.

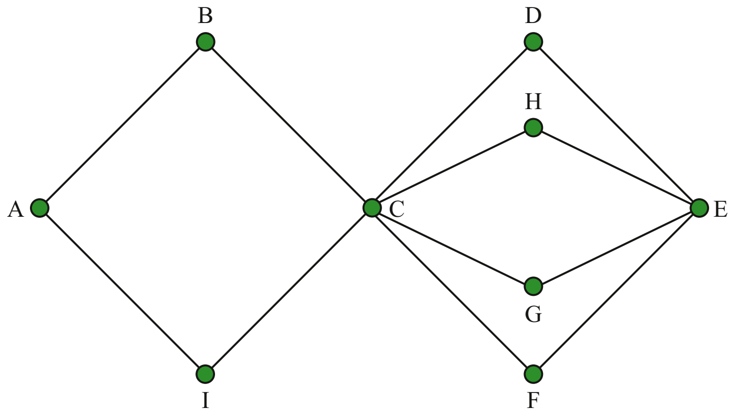
5 Which of the following is not a path from A to C?



A ABC **B** ADC **C** AEC **D** AEBDC

6 Which of the paths from question **5** visits every vertex?

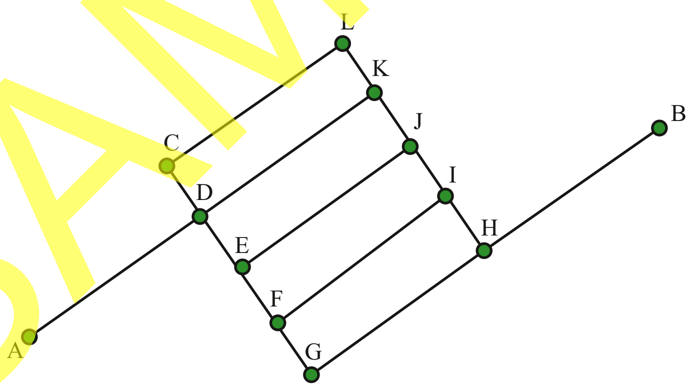
Use the network below to answer questions **7** and **8**.



7 **a** Find a path from A to E.
b Find a cycle that starts from vertex A.
c Find a path from F to B that includes vertex D.
d Find a cycle that starts from vertex H and includes vertex F.

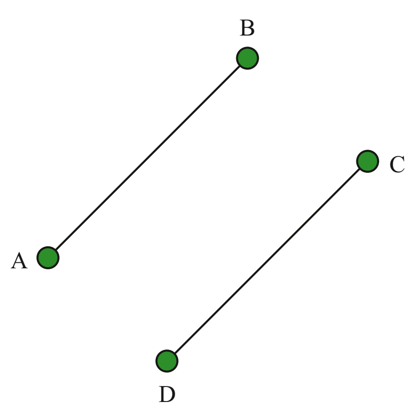
8 Find a walk that visits every edge exactly once. Is your walk a cycle? Is your walk a path?

9 The following map shows a network of roads between Broken Hill High School (A) and the local park (B).
a If it takes one minute to walk down the short side of a block, and three minutes to walk down the long side, how long does it take to walk from A to B?
b How many different routes from A to B take this amount of time?

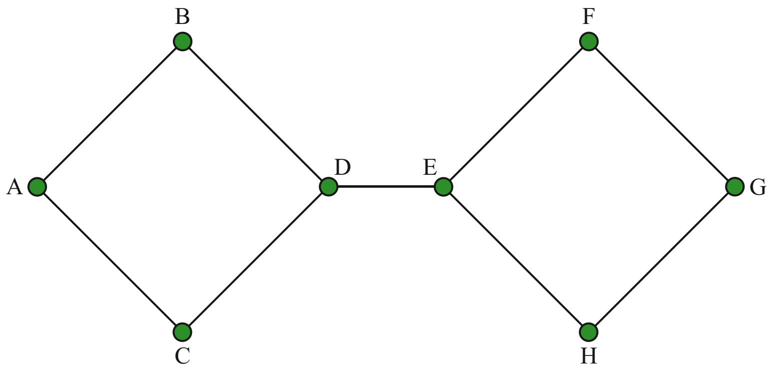


10 The following network is not connected because there is no path between which vertices?

A A and B
B D and C
C A and C
D C and C



Use this network to answer questions **11** and **12**.

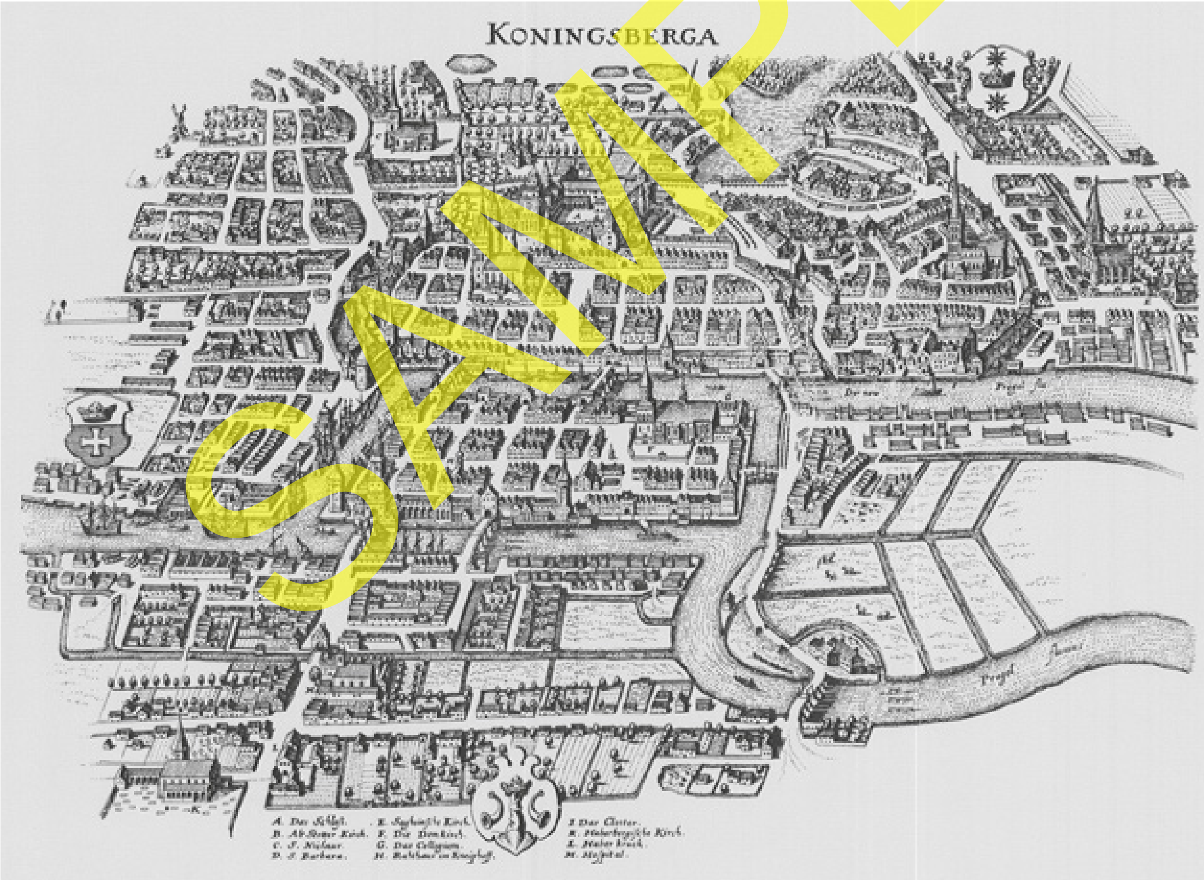


11 Which single edge should be removed to disconnect the network?

12 Find a walk in the network that visits every edge exactly once.

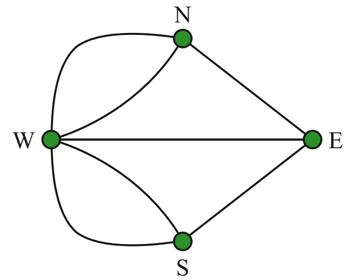
Historically, the first network paths problem is known as the **Seven Bridges of Königsberg**.

The old city of Königsberg lies on a river connected by seven bridges, as shown below. The problem is to plan a walk through Königsberg that crosses every bridge exactly once.



We can represent the city on each side of the river, and the two islands, by vertices. The bridges that connect the city can be represented by edges. This allows us to present a simplified version of the map that contains

only the relevant information. The question can then be rephrased as:
Is there a walk that visits every edge exactly once?



The answer to the seven bridges of Königsberg problem is that there is no such walk. In 1736 Leonhard Euler solved the Königsberg problem, showing that it has only to do with the number of vertices with odd degree. If there are more than two vertices with odd degree, then no walk can visit every edge exactly once.

Let's look at the vertex degrees of this network.

Vertex	N	E	S	W
Degree	3	3	3	5

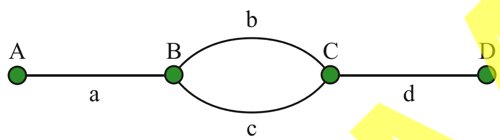
The network for this problem contains four vertices of odd degree, which is two too many! So it is not possible to find a walk that traverses each edge exactly once.

To find a walk in a network

If there are exactly two vertices of odd degree, then these vertices must be the start and end.

If there are no vertices of odd degree, then the start and end vertex are the same.

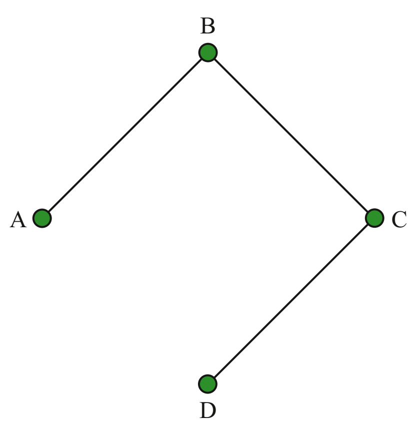
13 There are two walks from A to D in the network below.



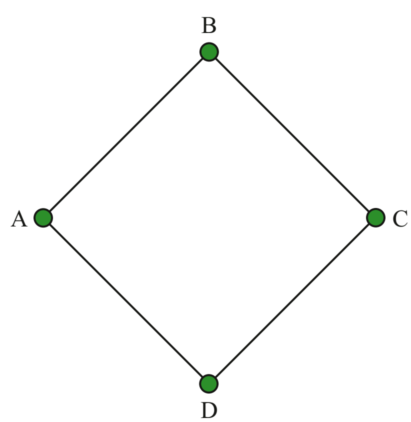
- **a** Write down a walk from A to D as a sequence of vertices and edges.
- **b** Write your answer to part **a** as a sequence of edges only.
- **c** Write your answer to part **a** as a sequence of vertices only.
- **d** Now find a different walk from A to D. Write down your walk as a sequence of vertices and edges.
- **e** Write down your answer to part **d** as a sequence of edges only.
- **f** Write down your answer to part **d** as a sequence of vertices only.
- **g** Compare your answers to parts **c** and **f**. In this example, is it a good idea to abbreviate the walks to a sequence of vertices?

14 For each of the networks below, first draw a table of vertex degrees and then decide if the network has a walk that traverses every edge exactly once. If such a walk exists then list the vertices of the walk.

a

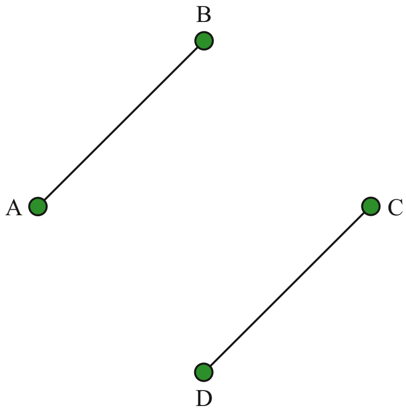


b

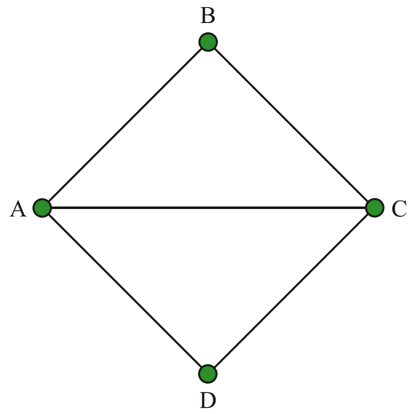


UNDERSTANDING, FLUENCEY AND COMMUNICATING

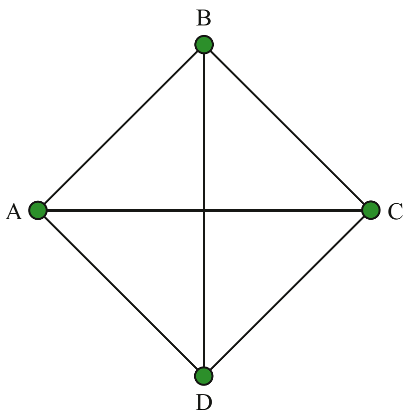
c



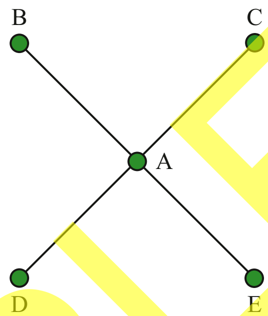
d



e



f



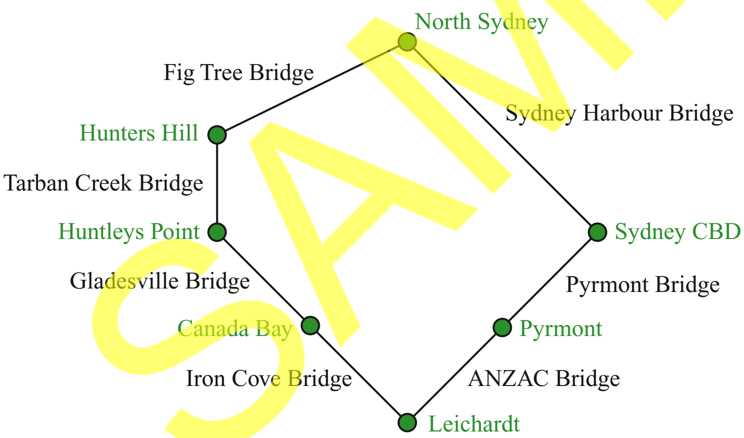
15 For each of the networks in question **14** parts **a** to **d**, how many different paths can you find from A to D? Remember that a path cannot visit the same vertex twice.

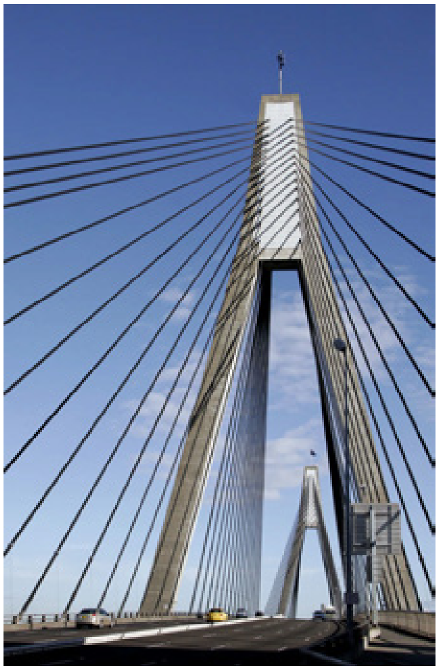
PROBLEM SOLVING, REASONING AND JUSTIFICATION

16 Design a garbage truck route that visits every street in the map below exactly once, or explain why this task is impossible.

17 Design a garbage truck route that visits every street in the map below exactly once, or explain why this task is impossible.

18 In 2017 the 'Seven Bridges Walk' toured some of the scenic bridges of Sydney.





Count the vertex degrees, then state why it is possible to cross the seven bridges of Sydney without visiting the same bridge more than once.

6C Trees

These resources are available on your obook assess:
- **Worksheet 6C:** Practise your skills with extra problems for trees and forests
- **assess quiz 6C:** Test your skills with an auto-correcting multiple-choice quiz

tree
a connected network with no cycles

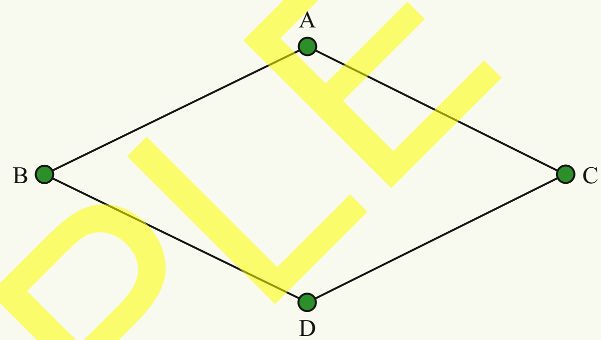
A **tree** is a special kind of network in which there is exactly one path between any two vertices. As such, a tree must be a connected network with no cycles.

EXAMPLE 6C–1 Identifying trees

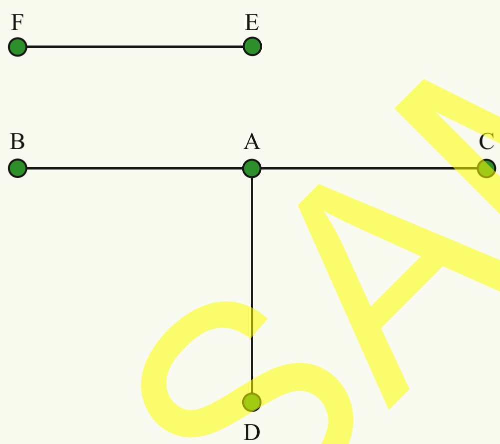
Which of the following networks are trees?

a

b



c



	Solve/Think	Apply
a	This is a tree because there is exactly one path from A to B: AB, and exactly one path from B to A: BA.	A more visual way to identify a tree is to use the alternative criteria: a tree is a connected network with no cycles. So, **a** is a tree because it has no cycles and is connected, **b** is not a tree because it has a cycle ABCDA, and **c** is not a tree because it is not connected.
b	This is not a tree because there are two paths from A to D: ABD and ACD.	
c	This is not a tree because there is no path from A to E.	

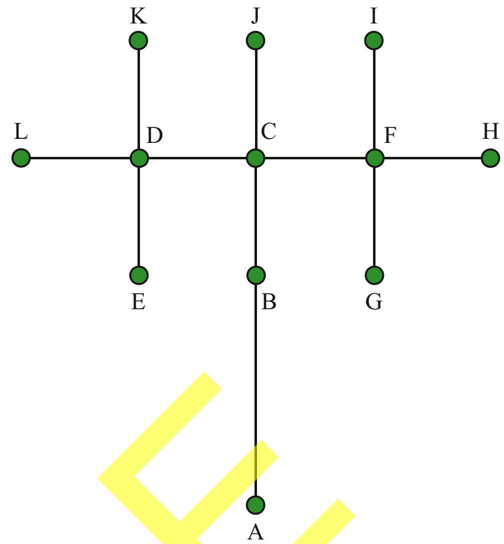
EXERCISE 6C Trees

UNDERSTANDING, FLUENCEY AND COMMUNICATING

1 The network given here is a tree, because there is exactly one path between any two vertices.

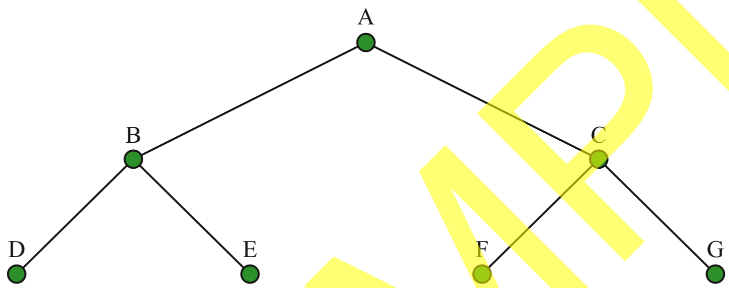
Find the unique path from:

a A to L
b L to E
c B to H
d G to K.

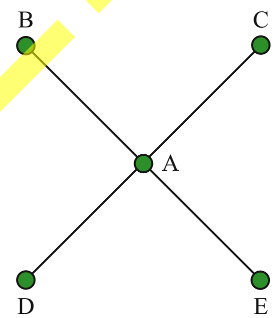


2 Which of the following networks are also trees?

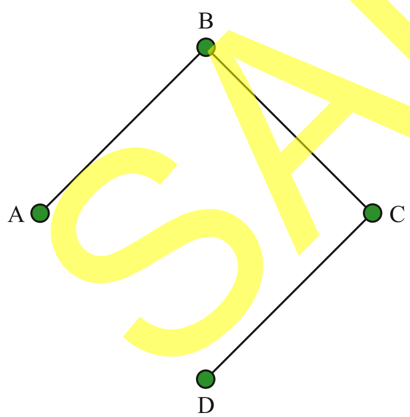
A



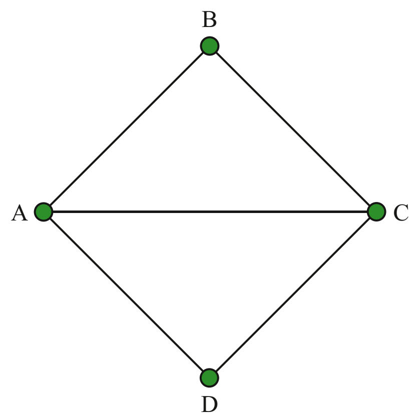
B



C



D



3 Count the number of edges and vertices in each of the trees from question **2**.

4 Draw the tree on right as a network. Use vertices to represent the end of a branch (in network terminology, such a vertex is called a **leaf**), as well as the joins between branches. Draw edges to represent the branches. Is your network a tree?

leaf
in a tree, any vertex of degree 1

NETWORKS

For a connected network, a **spanning tree** is a tree that contains all the vertices of the original network, and some of the edges. It is helpful to think of a spanning tree as a tree that is carved out of a larger network. Every connected network has at least one spanning tree. A spanning tree can be constructed by following these three steps:

Step 1: Find a cycle.
Step 2: Remove any edge from that cycle.
Step 3: Repeat steps 1 and 2 until there are no more cycles.

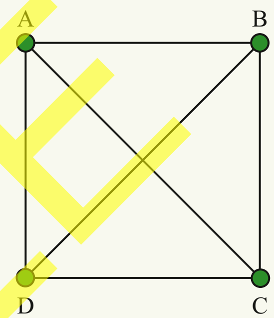
The remaining network will be a spanning tree. Note that a spanning tree always has one less edge than the number of vertices.

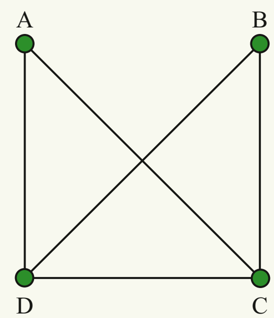
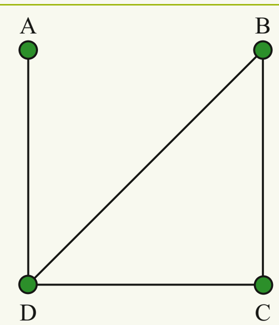
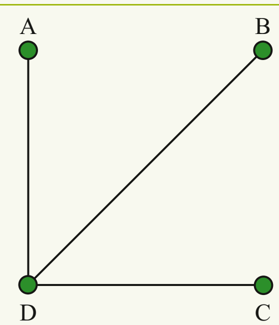
!

spanning tree
a tree made from all of the vertices and some of the edges of the original network

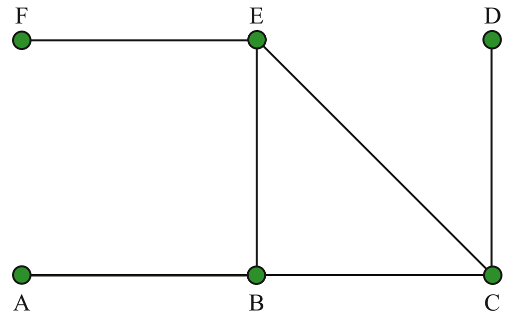
EXAMPLE 6C–2 Constructing a spanning tree

Construct a spanning tree for the following network.



Step	Solve/Think	Apply
1.1	Identify that ABCA is a cycle.	
1.2	Remove any edge from the cycle. In this example we have chosen to remove AB.	
2.1	Identify that ACDA is a cycle.	
2.2	Remove any edge from the cycle. In this example we have chosen to remove AC.	
3.1	Identify that BCDB is a cycle.	
3.2	Remove any edge from the cycle. In this example we have chosen to remove BC.	
Stop	There are no more cycles.	

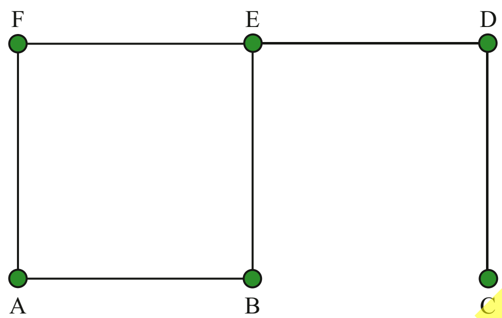
5 Which edge should be removed to transform this network into a tree?



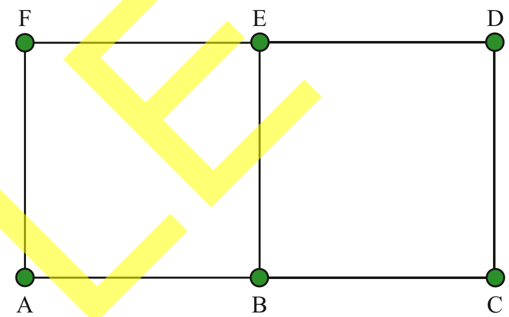
A AB **B** BC **C** CD **D** EF

6 Find a spanning tree for the following networks.

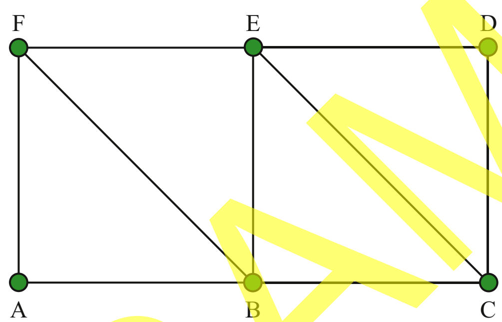
a



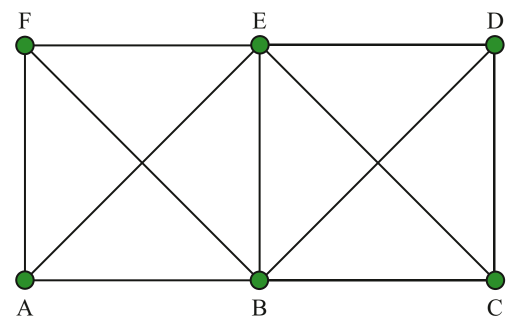
b



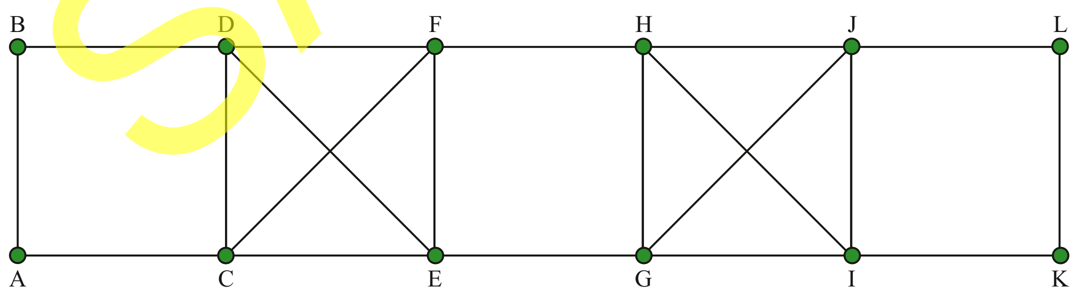
c



d



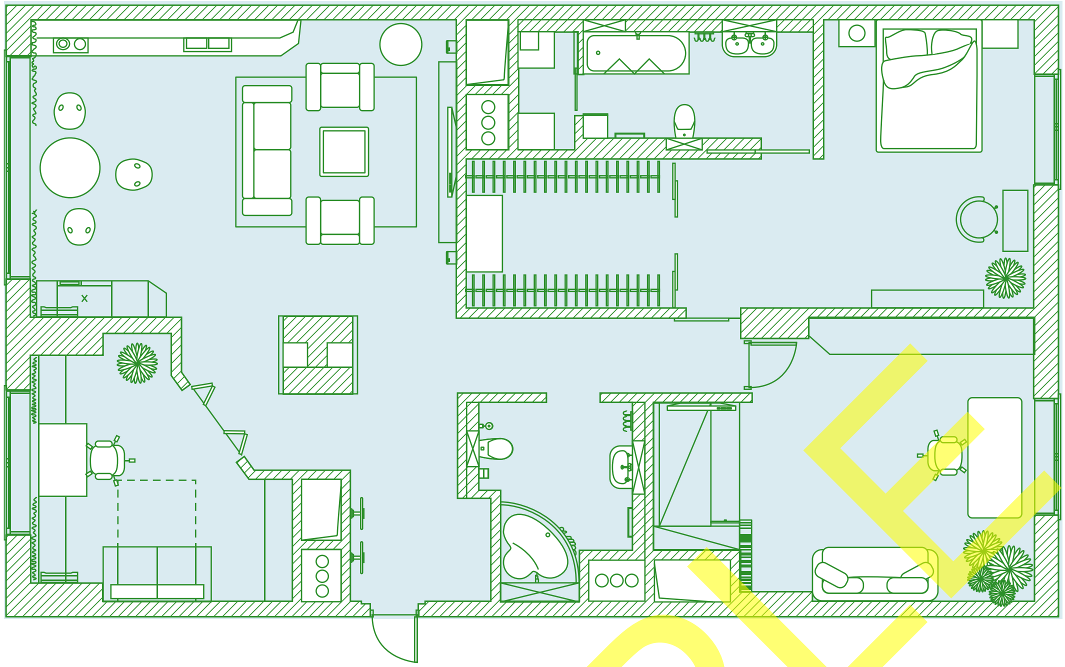
e



7 Find a different spanning tree to the one you found for question **6a** and **6b**.

8 **a** Draw a network that represents the following house plan. Use vertices to represent the rooms and edges to join rooms connected by a door.

b Is the network a tree? Does that make it a tree house?



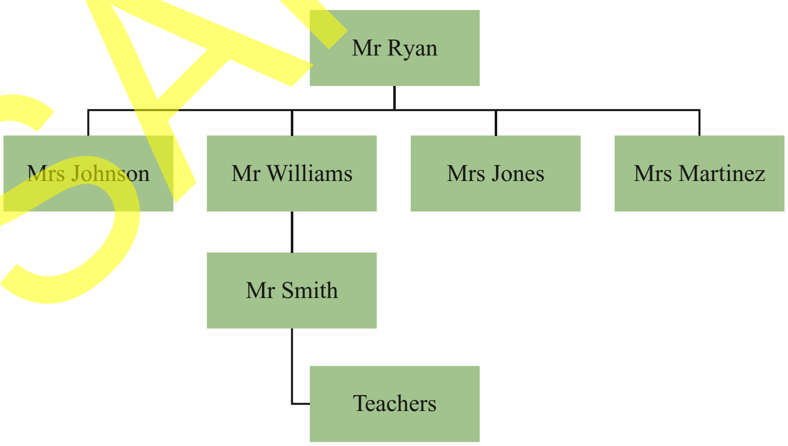
9 The following organisational chart shows the hierarchy within Beach Road Secondary School.

a Draw the organisational chart as a network.

b Is the network a tree?

c A teacher reports a serious incident to Mr Smith. How many steps will it take for this report to be handed up to Mr Ryan?

Beach Road Secondary School
Organisation Chart

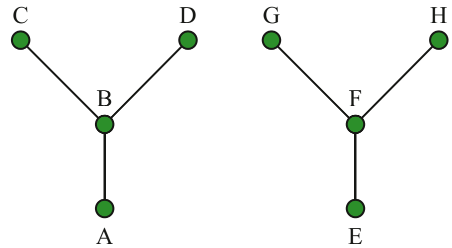


> A forest is a network with no cycles. In other words, a forest is a network made up of separate trees. !

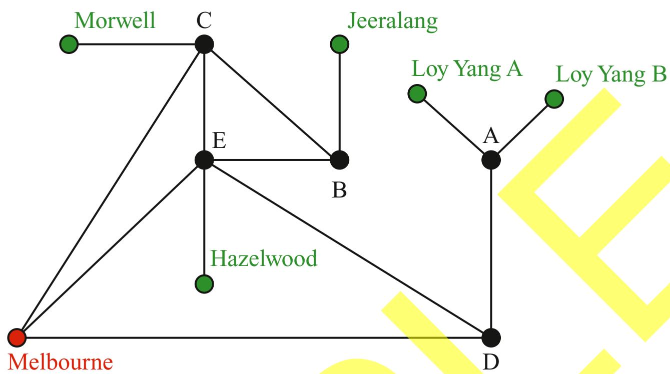
10 Which network in Example 6C–1 is a forest, but not a tree?

PROBLEM SOLVING, REASONING AND JUSTIFICATION

11 Identify the two trees in the forest on the right. Show that the network is not connected by stating two vertices with no path between them, and then turn the forest into a tree by adding an edge between those vertices.

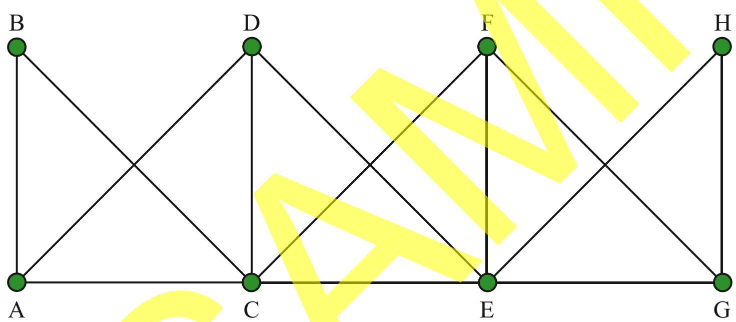


12 The following network shows several power stations in the Latrobe Valley, Victoria. The power stations, represented by green vertices, are connected to terminals represented by black vertices. Power is transmitted to Melbourne across several transmission lines, represented by edges. Remove as many transmission lines as you can without disconnecting any power station from Melbourne.



13 Suppose a spanning tree contains 10 vertices. How many edges are in the spanning tree?

14 For the network shown, find a spanning tree that is also a path.



15 For the network in question **14**, find a spanning tree that is not a path.

16 Can a single tree be a forest? Explain your thinking.

17 Consider a network with five vertices and no edges. Is this a forest?

18 Consider a network with five vertices and exactly one edge.

a If the edge is a loop, is this a forest?

b If the edge is not a loop, is this a forest?

CLASS ACTIVITY

19 Working in pairs, take turns in removing a single edge from the network in question **14**, without disconnecting the network. The winner is the first person to correctly identify when the network becomes a tree.

20 Draw a network with at most 10 vertices, and challenge one of your classmates to a game of Network Jenga.

6D Minimum spanning trees

These resources are available on your obook assess:

- **Video tutorial 6D:** Watch and listen to an explanation of Example 6D–2
- **Worksheet 6D:** Practise your skills with extra problems for minimum spanning trees
- **assess quiz 6D:** Test your skills with an auto-correcting multiple-choice quiz

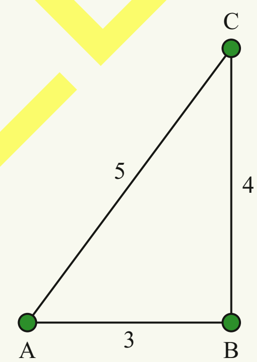
OI GI

minimum spanning tree the spanning tree with the smallest total edge weight

For a network without weighted edges there is no notion of a 'good' or 'bad' spanning tree. But if the network is weighted, then the spanning trees also have weight, and some trees might be preferable to others. The spanning tree with the smallest total edge weight is called the **minimum spanning tree**.

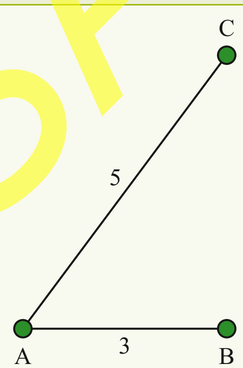
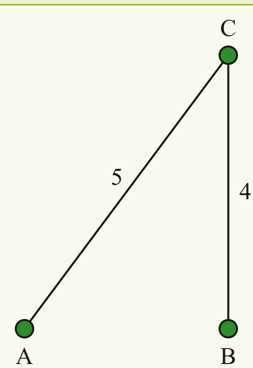
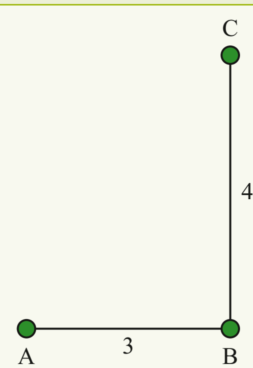
EXAMPLE 6D–1 Determining the minimum spanning tree by inspection

The network on the right shows the distances (in kilometres) between the three solar power plants A, B and C. The power plants must be connected by transmission lines. Design the shortest possible transmission network that connects power plants A, B and C.



Solve/Think

The power plants can be connected by a spanning tree. There are three possible spanning trees.

	Option 1	Option 2	Option 3
Spanning tree			
Total distance	$5 + 3 = 8$	$5 + 4 = 9$	$3 + 4 = 7$

The spanning tree in option 3 covers the least total distance, and hence it is the minimum spanning tree.

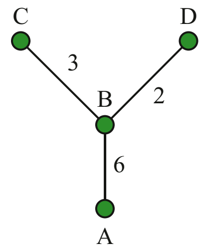
Apply

For a small network, we can list all possible spanning trees to determine the minimum spanning tree.

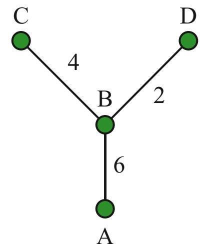
EXERCISE 6D Minimum spanning trees

1 What is the total weight of the following trees?

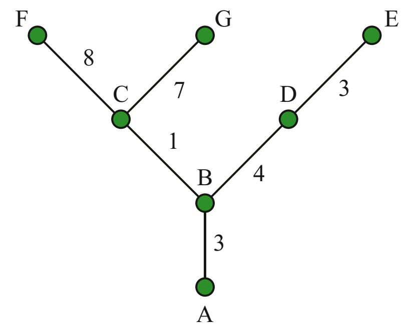
a



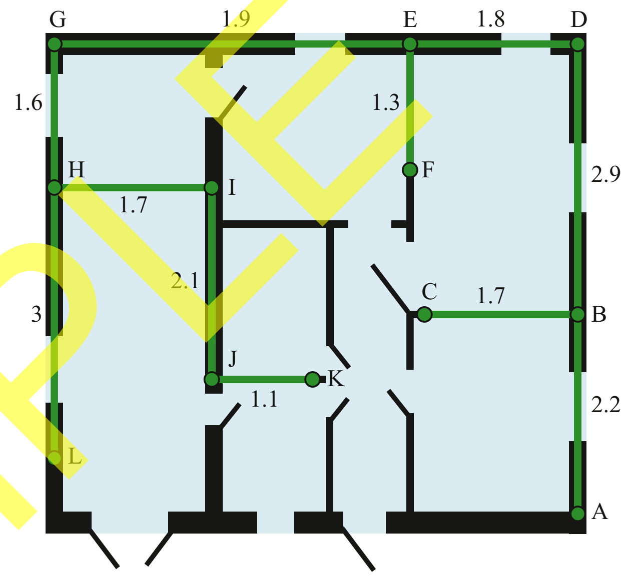
b



c

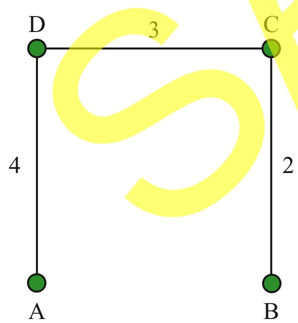


2 The image on the right shows a house plan and the electrical cables that connect the electricity meter at A to the power outlets throughout the house. The length of wire is shown against each edge. What is the total length of wire required?

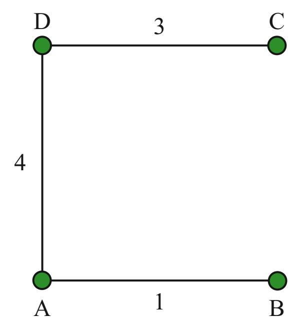


3 The four vertices A, B, C and D can be connected in four possible configurations. What is the total weight of each configuration below? Which configuration has the smallest total edge weight?

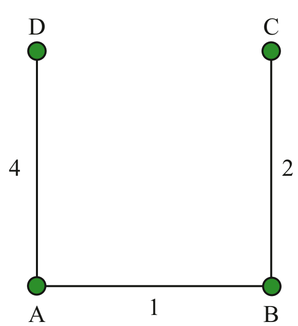
a



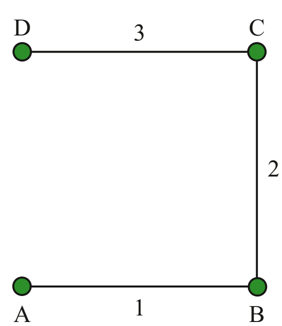
b



c



d



The strategy of finding all possible spanning trees and choosing the one with the least total edge weight becomes impractical for larger networks. Fortunately, there are two ways to find a minimum spanning tree that do not require listing every possible tree: Kruskal's algorithm and Prim's algorithm

Kruskal's algorithm was developed by Joseph Kruskal in 1956. Given a weighted network, a minimum spanning tree can be constructed as follows:

1. List all the edges of the network from smallest weight to largest.
2. Choose all of the vertices in the network, but none of the edges.
3. Proceed through your list of edges, starting with the edge of smallest weight. Add edges one by one, provided that your selection does not make a cycle.

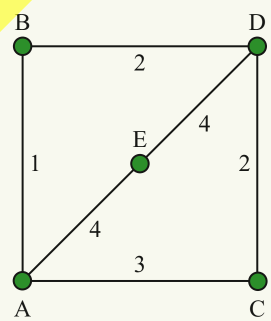
If your original network has n vertices, then you can stop after selecting n–1 edges. The outcome is guaranteed to be a spanning tree of minimum total edge weight.

Kruskal's algorithm starts with a forest of vertices, and adds edges starting with the edge of smallest weight. At every step, the network is a forest. The final step is when the forest grows into a single minimum spanning tree.

Kruskal's algorithm
a method for finding the minimum spanning tree of a connected, weighted network by selecting the required edges one at a time

EXAMPLE 6D–2 Using Kruskal's algorithm

Use Kruskal's algorithm to find the minimum spanning tree for the following weighted network.

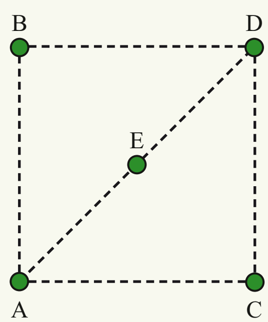


Step	Solve/Think
1	The list of edge weights is as follows.

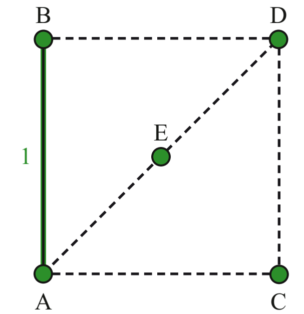
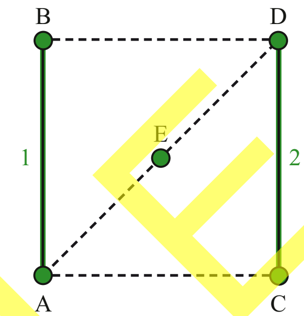
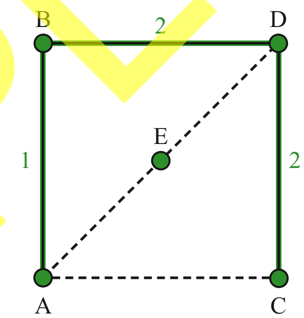
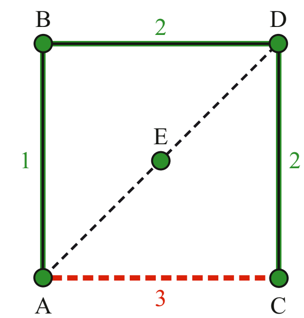
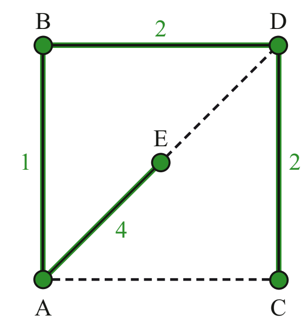
Edge	AB	DC	BD	CA	AE	ED
Weight	1	2	2	3	4	4

Proceed through this list, adding edges that do not form a cycle. The network has 5 vertices, so we stop once $5 - 1 = 4$ edges have been selected.

2 Choose all the vertices.



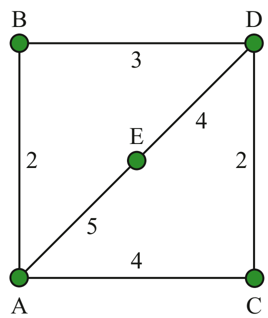
UNDERSTANDING, FLUENCEY AND COMMUNICATING

Step	Solve/Think	
3.1	Include the first edge AB.	
3.2	Include the second edge DC.	
3.3	Include the third edge BD.	
3.4	Skip the fourth edge CA, since ABDCA would be a cycle.	
3.5	Include the fifth edge AE.	
Stop	The selected vertices and edges form a minimum spanning tree.	

NETWORKS

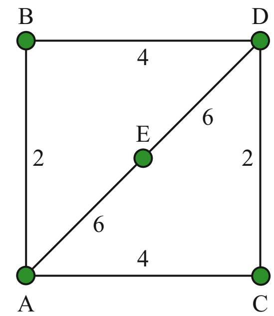
4 The edges from each network below are listed according to their edge weight. Fill in the remaining edge weights.

a



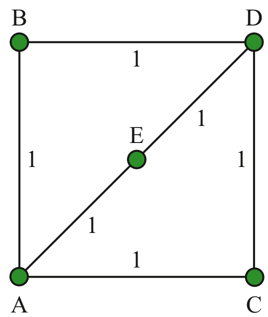
Edge	AB	DC	BD	DE	AC	AE
Weight	2		3			

b



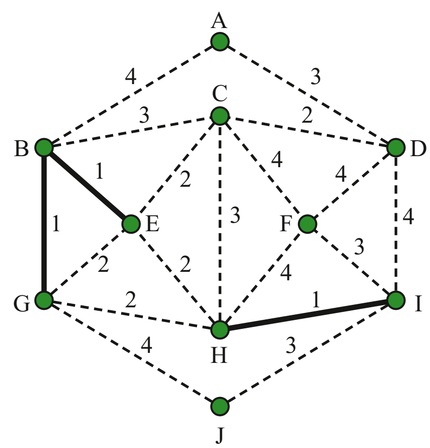
Edge	AB	DC	BD	AC	ED	AE
Weight		2				6

c



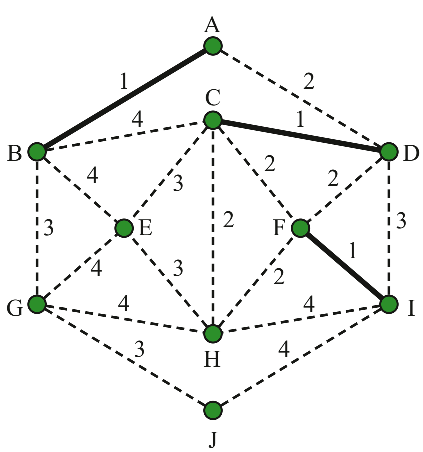
Edge						
Weight						

5 At every step of Kruskal's algorithm, the selected edges and vertices form a forest of trees. The first three steps of Kruskal's algorithm have been applied to the network on the right, resulting in all edges of weight 1 being added to the forest. Now proceed through all edges of weight 2, and add as many of them as possible to the forest (i.e. without creating a cycle). Then process all edges of weight 3, and then weight 4, until your forest grows into a single tree. The tree you created is a minimum spanning tree. Since this network has 10 vertices, your spanning tree should have 9 edges.

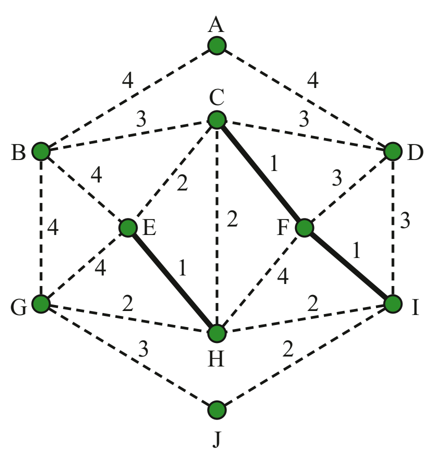


6 Repeat question **5** for the following networks.

a

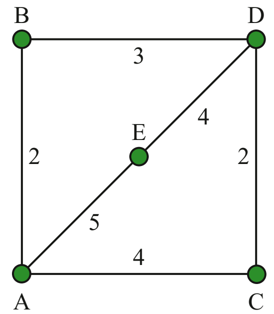


b

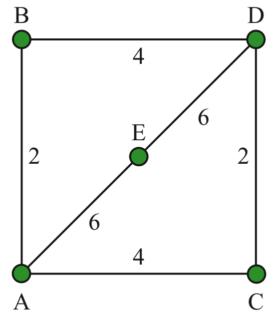


7 Use Kruskal's algorithm to find a minimum spanning tree for the following networks.

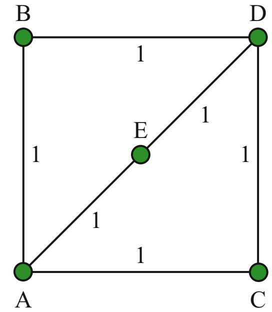
a



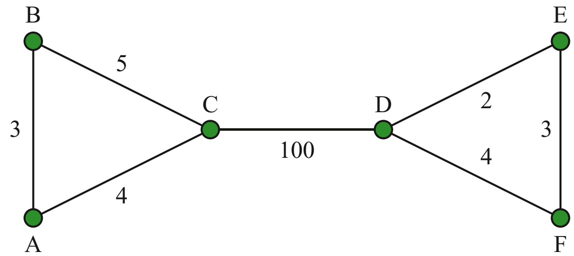
b



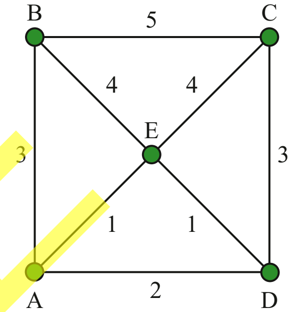
c



d



e



The second procedure for finding a minimum spanning tree is **Prim's algorithm**.

Starting with any vertex, Prim's algorithm selects the next 'best' edge and adds it to the tree without regard for the long-term consequences of that choice. Remarkably, this process still produces a minimum spanning tree.

The steps are as follows:

1. Choose any vertex as the start vertex. This is the start of the tree.
2. Of all the edges that join a vertex outside the tree to a vertex in the tree, choose the edge with *minimum weight* and add the vertex and the edge to the tree.
3. Repeat step 2 until the tree includes all vertices, which is when it becomes a spanning tree.

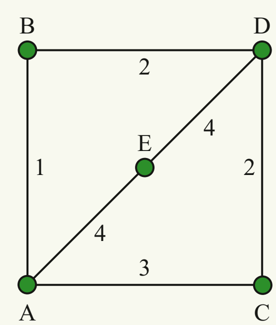
When there are many edges with the same minimum weight, any one of them can be chosen.

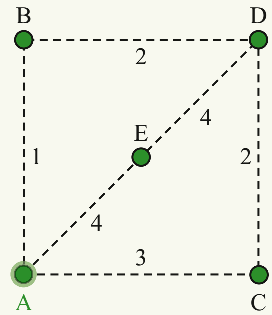
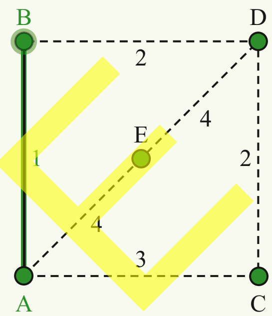
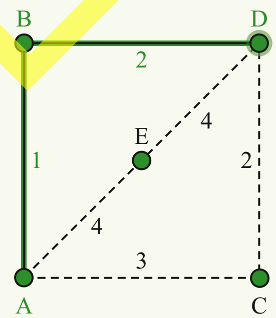
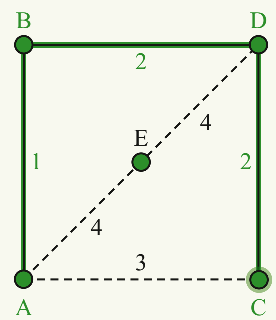
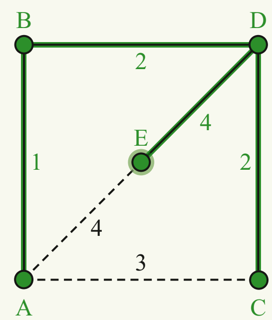
The selected edges and vertices form a tree at every step of Prim's algorithm. The algorithm continues until the tree includes every vertex. In contrast, Kruskal's algorithm contains every vertex right from the start. The two algorithms create minimum spanning trees, but in different ways.

Prim's algorithm a method for finding the minimum spanning tree of a connected weighted network by iteratively adding the edge of smallest weight

EXAMPLE 6D–3 Using Prim's algorithm

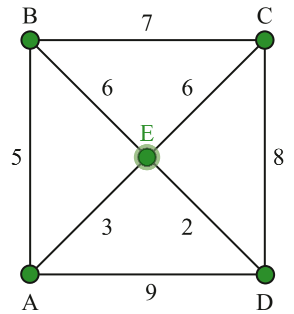
Use Prim's algorithm to find the minimum spanning tree for the weighted network on the right.



<table>
<tr><th>Step</th><th colspan="2">Solve/Think</th></tr>
<tr><td>1</td><td>Select any vertex as the start vertex. In this example vertex A is chosen.</td><td>
</td></tr>
<tr><td>2.1</td><td>Edges AB, AC and AE will connect a new vertex to the tree.
<table><tr><th>Edge</th><th>AB</th><th>AC</th><th>AE</th></tr><tr><th>Weight</th><td>1</td><td>3</td><td>4</td></tr></table>
Connect B to the tree via edge AB because it has the smallest edge weight.</td><td>
</td></tr>
<tr><td>2.2</td><td>Edges BD, AC and AE will connect a new vertex to the tree.
<table><tr><th>Edge</th><th>BD</th><th>AC</th><th>AE</th></tr><tr><th>Weight</th><td>2</td><td>3</td><td>4</td></tr></table>
Connect D to the tree via edge BD because it has the smallest edge weight.</td><td>
</td></tr>
<tr><td>2.3</td><td>Edges AE, AC, DE and DC will connect a new vertex to the tree.
<table><tr><th>Edge</th><th>AE</th><th>AC</th><th>DE</th><th>DC</th></tr><tr><th>Weight</th><td>4</td><td>3</td><td>4</td><td>2</td></tr></table>
Connect C to the tree via edge DC because it has the smallest edge weight.</td><td>
</td></tr>
<tr><td>2.4</td><td>Edges AE and DE (but not AC) will connect a new vertex to the tree.
<table><tr><th>Edge</th><th>AE</th><th>DE</th></tr><tr><th>Weight</th><td>4</td><td>4</td></tr></table>
As both edges have the minimum weight, we are free to choose either. In this example we choose DE.
The tree now includes every vertex.</td><td>
</td></tr>
<tr><td>Stop</td><td colspan="2">The vertices and selected edges form a minimum spanning tree.</td></tr>
</table>

8 The first step of Prim's algorithm has been applied to the network below. In this case vertex E was chosen.

a Fill in the table of edges that connect a new vertex to the tree.



Edge	EB	EC	ED	EA
Weight	6			

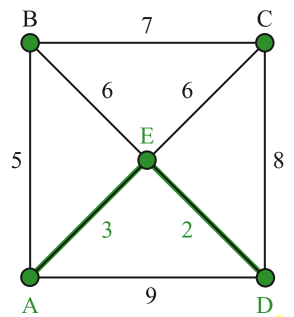
b Which of these edges has the smallest weight?

9 After a few more steps of Prim's algorithm, the selected tree has grown.

Only vertices B and C remain to be connected.

a Complete the table of edges that connect these vertices to the tree.

b Which of these edges has smallest weight?

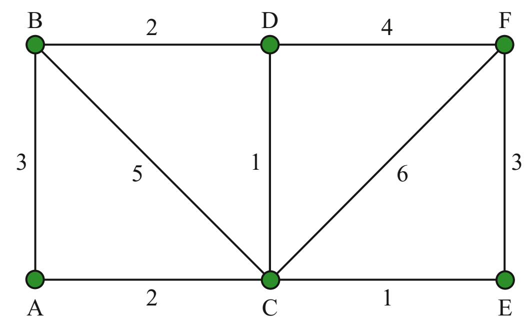


Edge	EB	EC	AB	CD
Weight				8

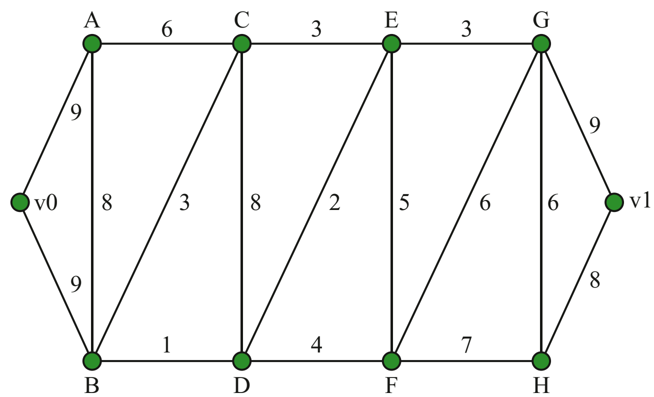
10 What is the minimum spanning tree for the network in question **8**?

11 Repeat question **7** with Prim's algorithm instead of Kruskal's.

12 Find a minimum spanning tree for this network using any method.



13 Find a minimum spanning tree for this network.

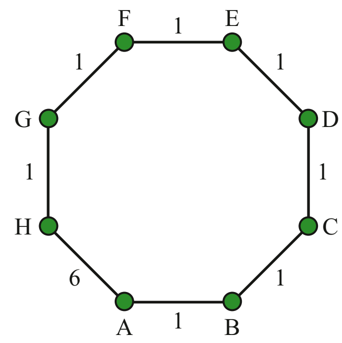


14 A network of security cameras must be connected by wires. The cost of connecting the cameras is shown, in dollars, as the edge weight in the network on the right.

a What is the cheapest way to connect all eight cameras?

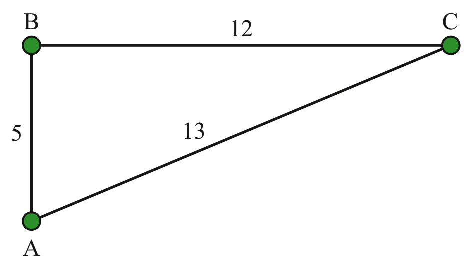
b What is the cheapest way to connect only cameras A and H?

c What is the cheapest way to connect only cameras A and H if the cost of wire between A and H is changed to 8?

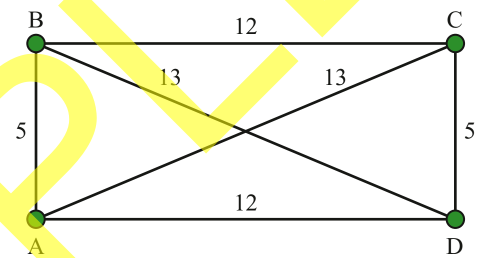


15 Power plants and the distances between them are shown in the networks below. The power plants must be connected by transmission lines. Design the shortest possible transmission network in each scenario.

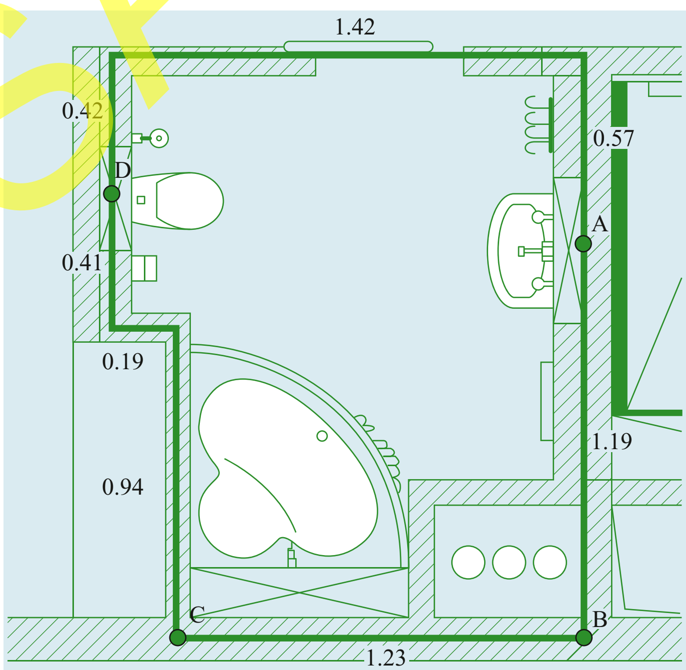
a



b

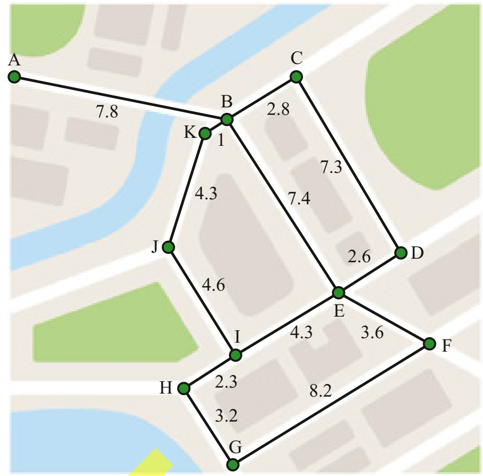


16 This question will help you decide where to place the water pipes in the following floor plan for a bathroom, so that the sink (at vertex A), bath (at vertex C) and toilet (at vertex D) are connected to the water main (at vertex B). The distances along possible pipelines are shown. First represent this information as a network (i.e. add the distances to find the total distances between A and D, and D and C), and then use Kruskal's algorithm to find a spanning tree that minimises the total distance covered by the pipes.



PROBLEM SOLVING, REASONING AND JUSTIFICATION

17 The network on the right shows the location of fibre optic nodes that form part of a national high-speed internet network. The possible location of the fibre optic cable is given as edges, and the length of the cable (in kilometres) is specified as the edge weight. Design the most cost-effective possible way of connecting each node (i.e. vertex) with fibre optic cable. How many kilometres of cable does your network require?



18 The table below shows the distances between towns A, B, C and D in kilometres.

	A	B	C	D
A		3.3	5.8	2.5
B	3.3		4.4	1.5
C	5.8	4.4		3.6
D	2.5	1.5	3.6	

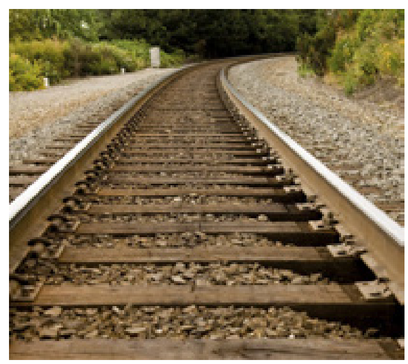
a Represent the information in the table as a network.

b What would be the most cost-effective way to connect the towns to each other by rail?

c If the cost per km of rail is $5 million, how much would the most cost-effective railway cost?

d A local politician suggests rail lines along AB, BC and CD. How much would this railway cost?

e What is the difference in price between the most cost-effective railway and the local politician's suggested railway?



CHALLENGE

19 The following table shows the distances (in kilometres) between mainland capital cities in Australia.

Adelaide						
2075	**Brisbane**					
1209	1267	**Canberra**				
3041	3435	4034	**Darwin**			
732	1813	651	3773	**Melbourne**		
2721	4434	3930	4037	3453	**Perth**	
1419	978	289	3971	876	3975	**Sydney**

a What is the cheapest way to construct a high-speed rail network that connects the mainland capital cities to each other?

b How many kilometres of rail is required?

c Given the information in tabular form, would you choose to use Kruskal's or Prim's algorithm?

6E The shortest path

shortest path (in a network diagram) a path between two vertices in a network for which the sum of the weights of its edges is minimised

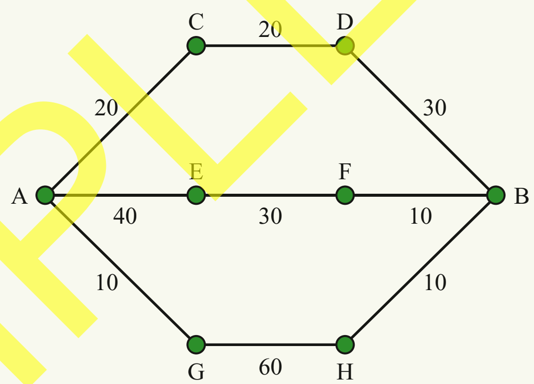
These resources are available on your obook assess:
- **Video tutorial 6E:** Watch and listen to an explanation of Example 6E–2
- **Worksheet 6E:** Practise your skills with extra problems for the shortest path
- **assess quiz 6E:** Test your skills with an auto-correcting multiple-choice quiz

OI GI

In a weighted network, the **shortest path** between two vertices is the path with the smallest total edge weight.

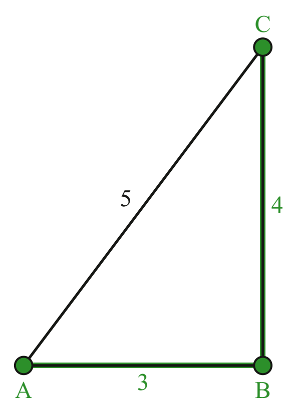
EXAMPLE 6E–1 Finding the shortest path by inspection

The network below shows cities connected by highways. The length of each highway is indicated by an edge weight. What is the shortest path between cities A and B?



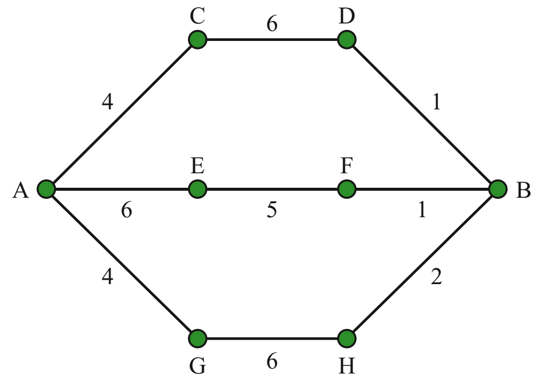
Solve/Think	Apply
There are three paths from A to B. 1 Path ACDB has a length of 20 + 20 + 30 = 70. 2 Path AEFB has a length of 40 + 30 + 10 = 80. 3 Path AGHB has a length of 10 + 60 + 10 = 80. The path with the shortest length is ACDB.	One way to find the shortest path is to calculate the length of every path, and then select the path with the shortest length. This method only works in practice when there is a small number of possible paths to consider.

The minimum spanning tree does not always provide the shortest path between two points. For example, in the network on the right the minimum spanning tree contains the path ABC (highlighted in green), whereas the shortest path between vertices A and C is AC.



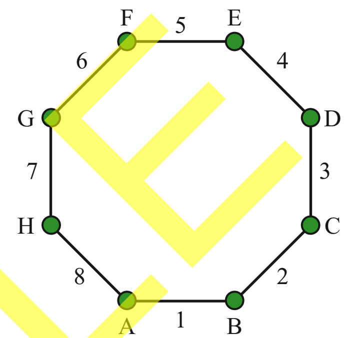
EXERCISE 6E The shortest path

1 Consider the following network.



a What is the shortest path from A to B?
b What is the shortest path from F to G?

2 Consider the network on the right.
a Which path from A to F is shorter: ABCDEF or AHGF?
b Which path from A to G is shorter: ABCDEFG or AHG?
c What is the minimum spanning tree for this network?



For larger networks it is difficult to consider all possible paths between two vertices. Fortunately, **Dijkstra's algorithm** gives us a method to construct a special spanning tree that contains all the shortest paths from a particular start vertex to every other vertex.

Starting with any vertex, let's call it A, Dijkstra's algorithm grows a tree of shortest paths from A. The steps are as follows:

1 Make a tree that only contains the vertex A.
2 Make a list of all paths starting from A that extend the tree by exactly one edge. Choose the path that has the *minimum total weight* and add the vertex and edge to the tree.
3 Repeat step 2 until the tree includes all vertices.

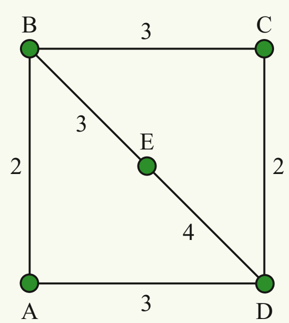
This is very similar to Prim's algorithm, except in step 2 you select a new edge based on the total distance from A rather than the individual edge weight.

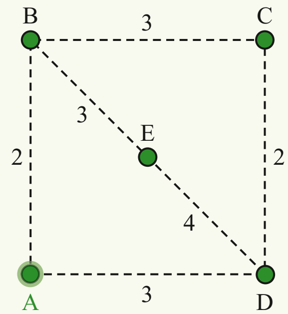
!

Dijkstra's algorithm
a method for finding the shortest path between any two vertices of a connected weighted network

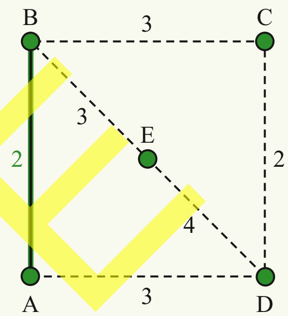
EXAMPLE 6E–2 Using Dijkstra's algorithm

Use Dijkstra's algorithm to find the shortest path from A to E in the following network.

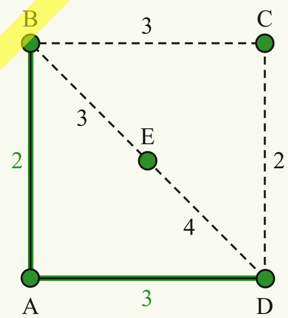


<table>
<tr><th>Step</th><th colspan="2">Solve/Think</th></tr>
<tr><td>1</td><td>Make a tree with only vertex A.</td><td>
</td></tr>
<tr><td>2.1</td><td>The tree contains only the vertex A. The list of all paths from A with exactly one vertex outside this tree is:

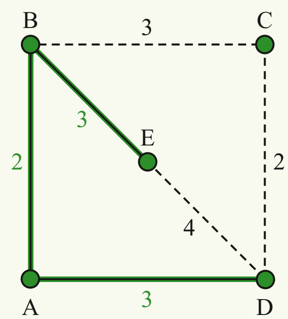
<table><tr><th>Path</th><td>AB</td><td>AD</td></tr><tr><th>Total weight</th><td>2</td><td>3</td></tr></table>

Path AB has minimum total weight, so add it to the tree.</td><td>
</td></tr>
<tr><td>2.2</td><td>The tree contains vertices A and B. The list of all paths starting at A with exactly one vertex outside this tree is:

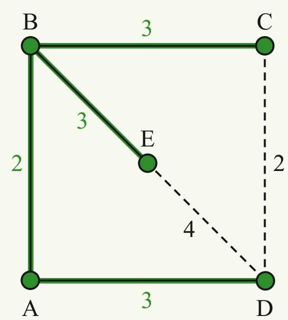
<table><tr><th>Path</th><td>AD</td><td>ABE</td><td>ABC</td></tr><tr><th>Total weight</th><td>3</td><td>5</td><td>5</td></tr></table>

Path AD has minimum total weight, so add it to the tree.</td><td>
</td></tr>
<tr><td>2.3</td><td>The tree contains vertices A, B and D. The list of all paths from A with exactly one vertex outside this tree is:

<table><tr><th>Path</th><td>ABE</td><td>ABC</td><td>ADE</td><td>ADC</td></tr><tr><th>Total weight</th><td>5</td><td>5</td><td>7</td><td>5</td></tr></table>

Paths ABE, ABC and ADC all have the same minimum total weight, so we are free to choose any one and add it to the tree. In this example we have chosen to add ABE.</td><td>
</td></tr>
<tr><td>2.4</td><td>The tree contains vertices A, B, D and E. Only vertex C remains outside the tree. The paths from A with exactly one vertex outside the tree are:

<table><tr><th>Path</th><td>ABC</td><td>ADC</td></tr><tr><th>Total weight</th><td>5</td><td>5</td></tr></table>

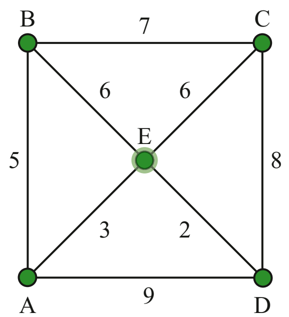
All paths have the same weight, so we are again free to choose any one and add it to the tree. In this example we have chosen to add ABC.</td><td>
</td></tr>
<tr><td>2.5</td><td colspan="2">The resulting tree is a spanning tree that contains the shortest path from A to any other vertex. In particular, the shortest path from A to E in this tree is ABE, which has total weight 2 + 3 = 5.</td></tr>
</table>

Apply

Follow the steps in Dijkstra's algorithm to create a spanning tree that contains all of the shortest paths in the network.
Note: we could have answered the question in step 2.3 when the path from A to E, ABE, was added to the tree of shortest paths.

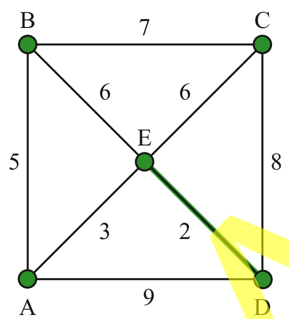
3 Use Dijkstra's algorithm to find the shortest path from E to B in the network below.

a The first step of Dijkstra's algorithm is to create a tree with just the start vertex E. Below is a list of paths with exactly one vertex outside this tree. Calculate the total weight of each of these paths.



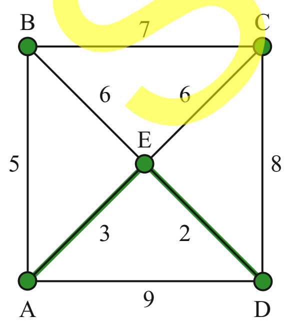
Path	EA	EB	EC	ED
Total weight	3			

b In the second step, the tree contains the vertices E and D, and the edge ED. Below is a list of all paths from E with exactly one vertex outside this tree. Calculate the total weight of each of these paths.



Path	EA	EB	EC	EDA	EDC
Total weight				11	

c In the third step, the tree contains vertices A, E and D, and the edges EA and ED. Below is a list of paths from E with exactly one vertex outside this tree. Calculate the total weight of each of these paths.



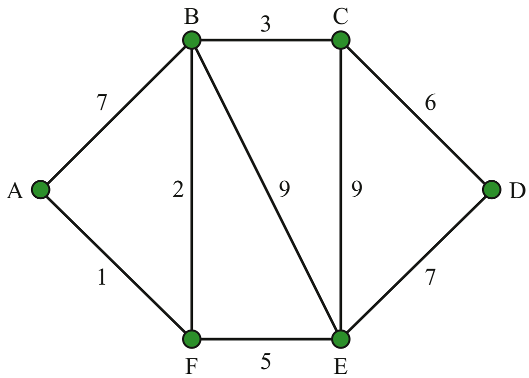
Path	EB	EC	EAB	EDC
Total weight			8	

i Which of the paths in part **c** has the smallest total weight?

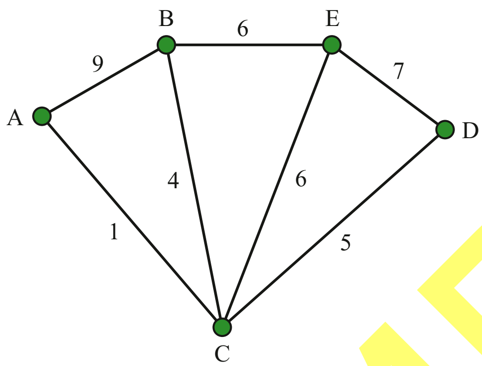
ii What is the minimum distance from E to B?

4 What is the shortest path from B to E in the networks for Exercise 6D question **7**?

5 What is the shortest path from A to D in the following network?



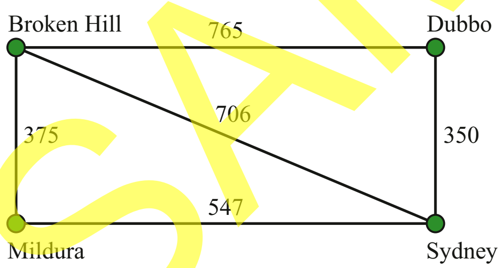
6 Find the shortest distance from A to E in the following network.



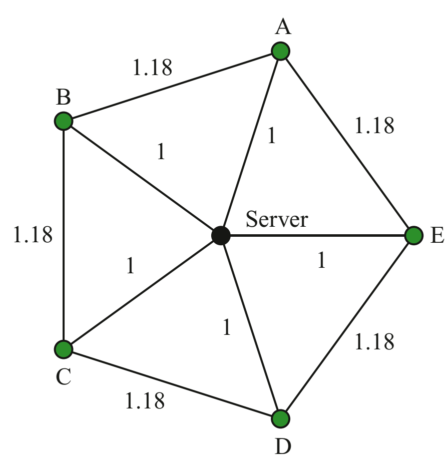
7 The cost of flying from Sydney to certain regional centres is shown below.

a What is the cheapest route to Broken Hill?

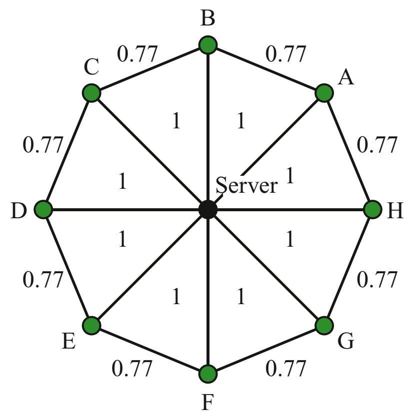
b Assuming the cheapest route is booked out, what is the second cheapest route?



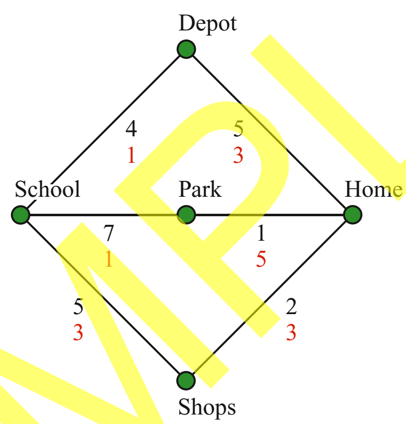
8 The network on the right shows the distances between the five computers A, B, C, D and E and a central server. Connect each computer to the server, so that the path from the server to the computer is as short as possible. Is your selected network also a minimum spanning tree?



9 Repeat question **8** for a network with eight computers. Is your selected network also a minimum spanning tree?



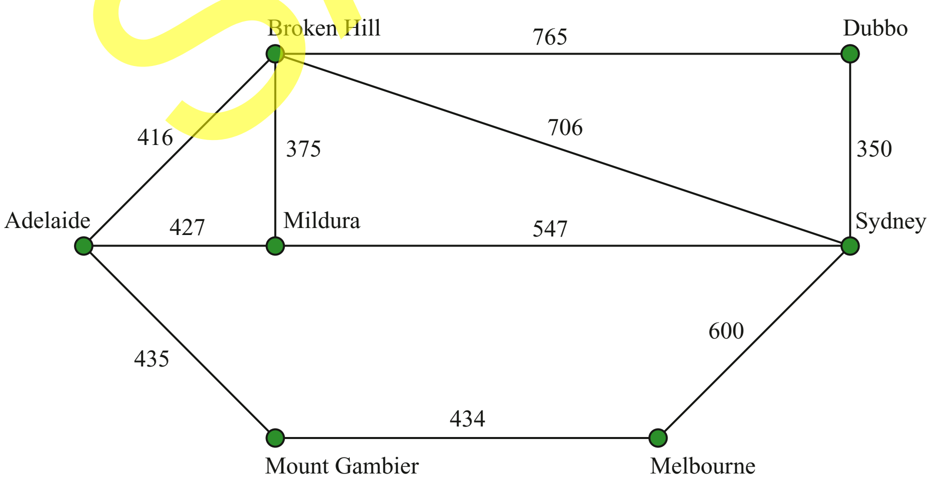
10 The following map shows the possible routes from school to home. The distance (in kilometres) is given by the black edge weight, and the level of danger is given by the red edge weight.



a What is the shortest route from school to home?

b What is the least dangerous route from school to home?

11 Here is a larger map of regional flights, along with the cost of each route. What is the cheapest route in the network from Dubbo to Mount Gambier?



CHAPTER 6 REVIEW NETWORK CONCEPTS

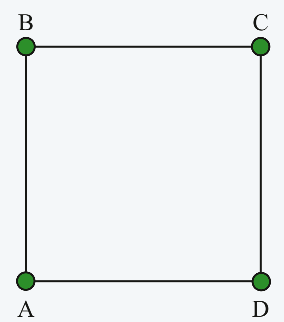
You should now be able to:

- ✔ convert a table of data into a network
- ✔ tabulate vertex degrees
- ✔ understand the difference between a walk, a path and a cycle
- ✔ determine if a network has a walk that visits every edge exactly once
- ✔ calculate the weight of a path in a network
- ✔ find a spanning tree
- ✔ find a minimum spanning tree using Kruskal's and Prim's algorithms
- ✔ find the shortest path between two vertices using Dijkstra's algorithm
- ✔ understand that a minimum spanning tree does not necessarily give the shortest path between two vertices.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

Use the diagram on the right to answer questions **1–3**.



6B **1** Which of the following walks is a path?

A ABCB **B** ABD **C** DAB **D** ABCDA

6B **2** Which of the following walks is a cycle?

A ABAD **B** ABCD **C** ABDA **D** ABCDA

6B **3** Which of the following walks is not a path and not a cycle?

A AB **B** ABA **C** BCDAB **D** ABAD

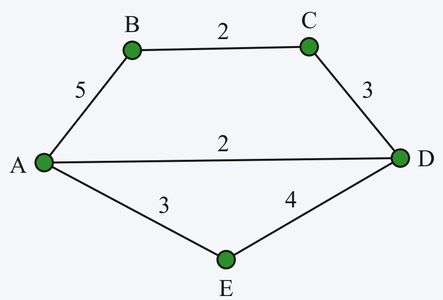
6C **4** A spanning tree contains five edges. How many vertices are in the network?

A 3 **B** 4 **C** 5 **D** 6

Use the diagram on the right to answer questions **5–9**.

The diagram shows the possible paths (in km) for laying gas pipes between various locations.

Gas is to be supplied from one location. Any one of the locations can be the source of the supply.



6D **5** What is the minimum total length of the pipes required to provide gas to all the locations?

A 10 km **B** 19 km **C** 12 km **D** 11 km

6B **6** What is the total weight of the cycle ADCBA?

A 7 **B** 9 **C** 10 **D** 12

6E **7** The shortest path from B to E is:

A BAE **B** BE **C** BCDE **D** BCDAE

6E **8** The shortest path from A to C is:

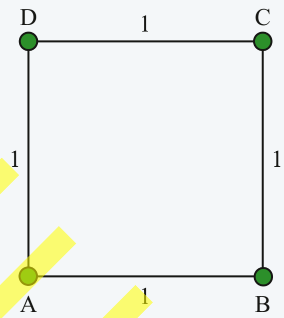
A AEDC **B** ABC **C** ADC **D** AC

6A **9** How many vertices have odd degree?

A none **B** one **C** two **D** more than two

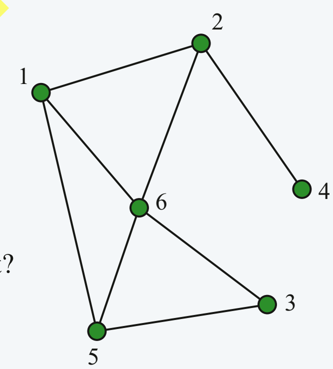
6D **10** How many different minimum spanning trees can you find for the following network?

A one **B** two
C three **D** four



REVIEW SET 1

Use the diagram on the right to answer questions **1–3**.



1 Complete the table of vertex degrees for the network.

Vertex	1	2	3	4	5	6
Degree						

2 Is there a walk in the network that visits every edge exactly once? Why or why not?

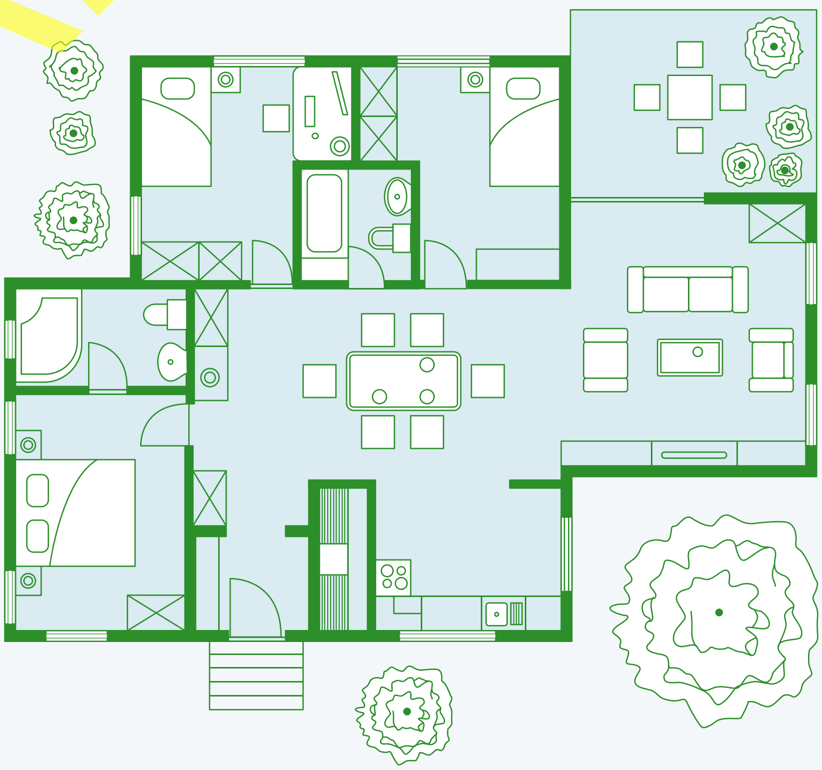
3 Find a walk in the network that visits every vertex exactly once.

Use the diagram of the floor plan to answer questions **4–6**.

4 Draw a network that represents the floor plan. Use vertices to represent rooms, and represent rooms connected by an internal door with an edge.

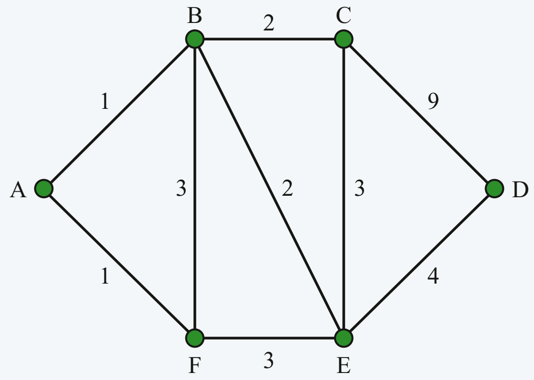
5 Is your network a tree?

6 Construct a table of vertex degrees for the network. Does there exist a walk through the house which passes through every internal door exactly once? Why or why not?



NETWORKS

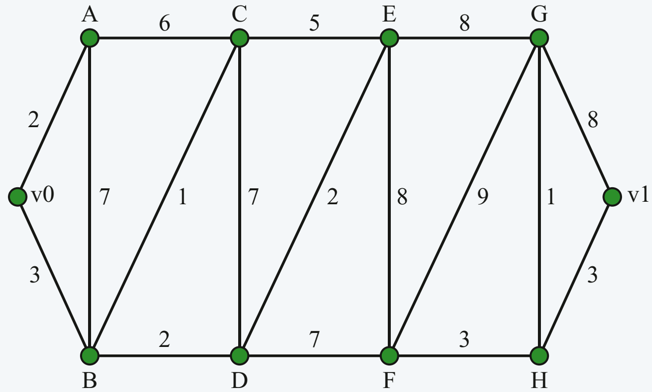
Use the diagram on the right to answer questions **7** and **8**.



7 Use Prim's algorithm to find the minimum spanning tree for the network.

8 What is the total weight of the path AFBCD for the network?

9 Find the shortest path from v0 to v1.

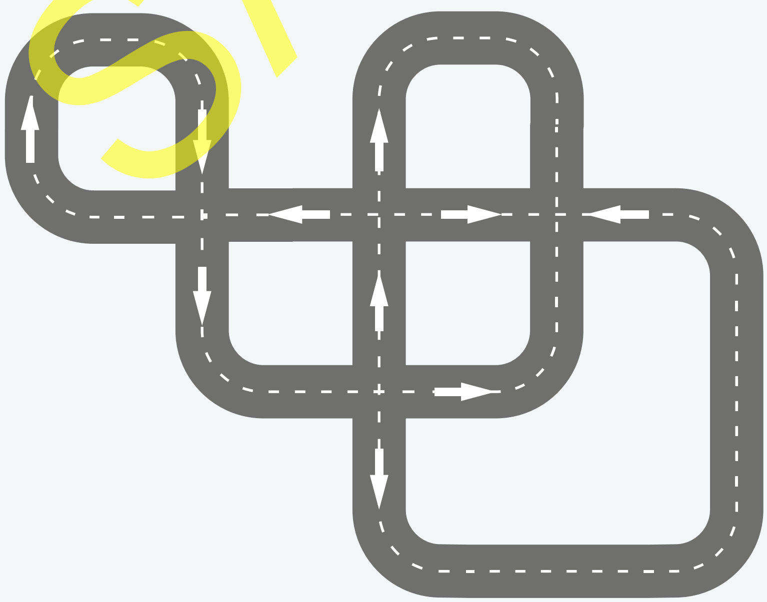


REVIEW SET 2

1 Represent the following table as a weighted network.

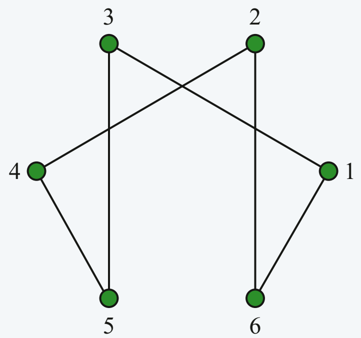
	A	B	C	D
A	–	2	8	9
B	2	–	7	4
C	8	7	–	1
D	9	4	1	–

2 Draw this system of one way streets as a directed network.



3 Using the concepts of indegree and outdegree, show that there is something wrong with the road design in question **2**.

4 Find a cycle with six edges in the following network.



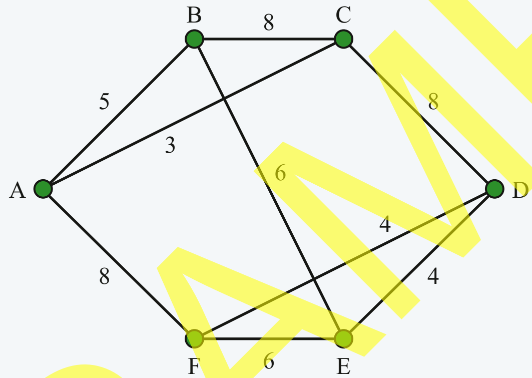
5 Transform the cycle in question **4** into a spanning tree by removing one or more edges.

6 Transform the spanning tree in question **5** into a forest of two trees by removing one or more edges.

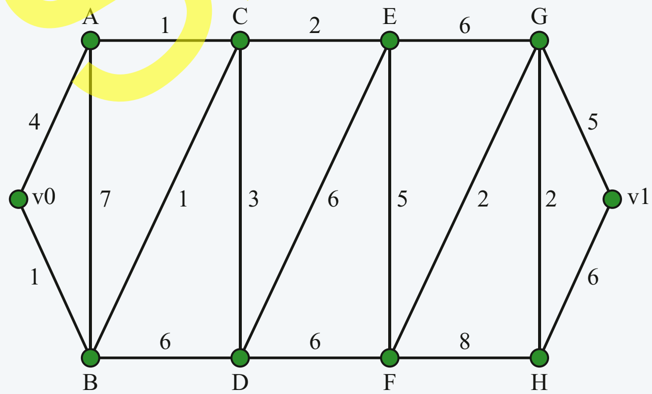
7 Is there a walk in the network from question **4** that visits every edge exactly once?

8 Is there a walk in the network from your answer to question **5** that visits every edge exactly once?

9 Find a minimum spanning tree for the following network.



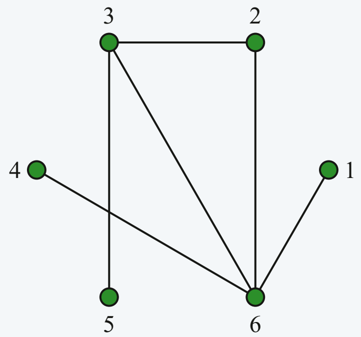
10 Find the shortest path from v0 to v1 for the following network.



NETWORKS

REVIEW SET 3

Use the diagram below to answer questions **1–2**.

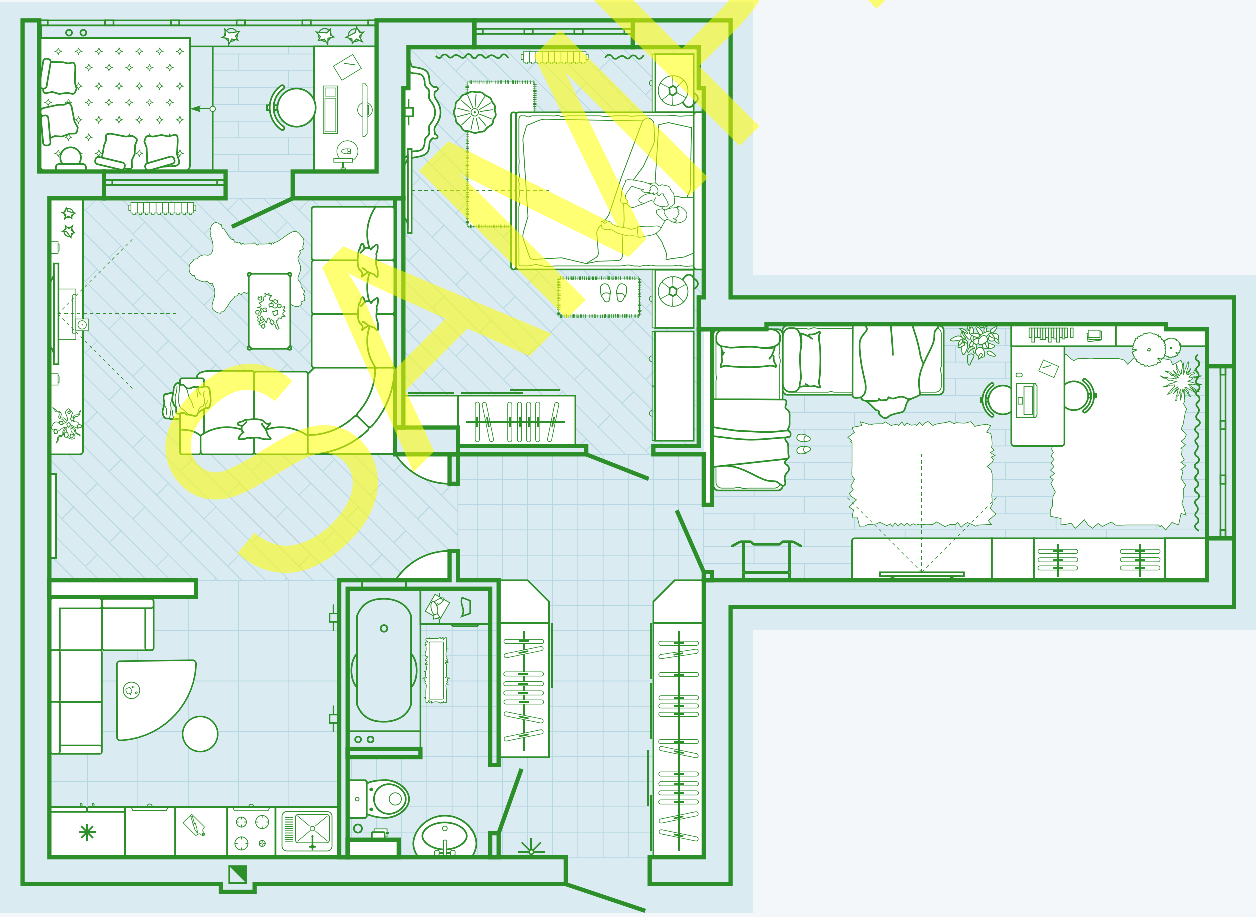


1 Complete the table of vertex degrees for the network.

Vertex	1	2	3	4	5	6
Degree						

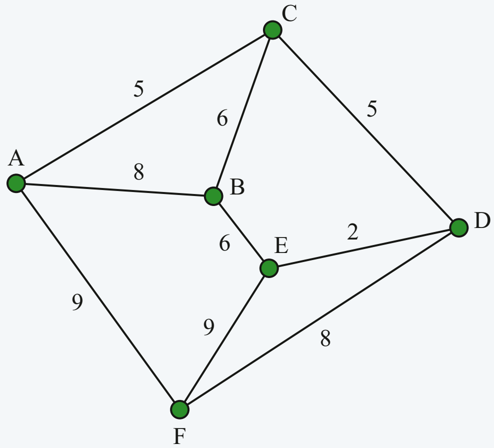
2 Is there a path in the network that visits every edge exactly once?

3 Draw a network that represents the following floor plan. Use vertices to represent rooms and represent rooms connected by a door with an edge.



4 Is the network from question **3** a tree?

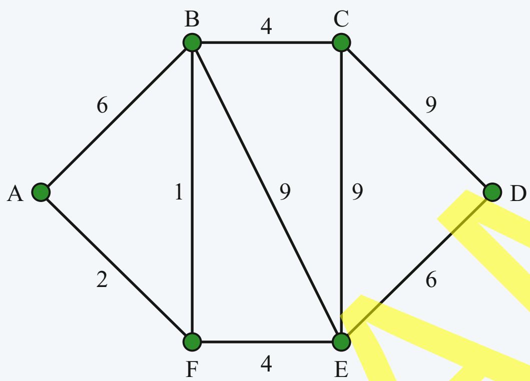
Use the diagram below to answer questions **5** and **6**.



5 Find a minimum spanning tree for the network.

6 What is the weight of the walk ABEDC?

Use the diagram below to answer questions **7**, **8** and **9**.



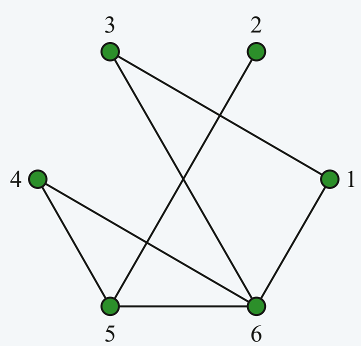
7 Find the shortest path from A to D in the network.

8 Find a walk that visits every edge of the network.

9 Use Kruskal's algorithm to find the minimum spanning tree for the network.

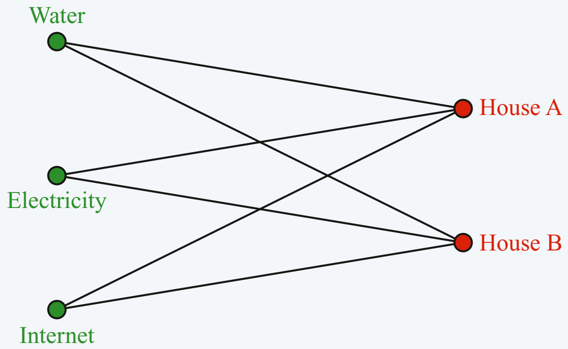
REVIEW SET 4

1 Complete the table of vertex degrees for the following network.



Vertex	1	2	3	4	5	6
Degree						

2 Three essential services (water, electricity and internet) are to be connected to house A and house B.

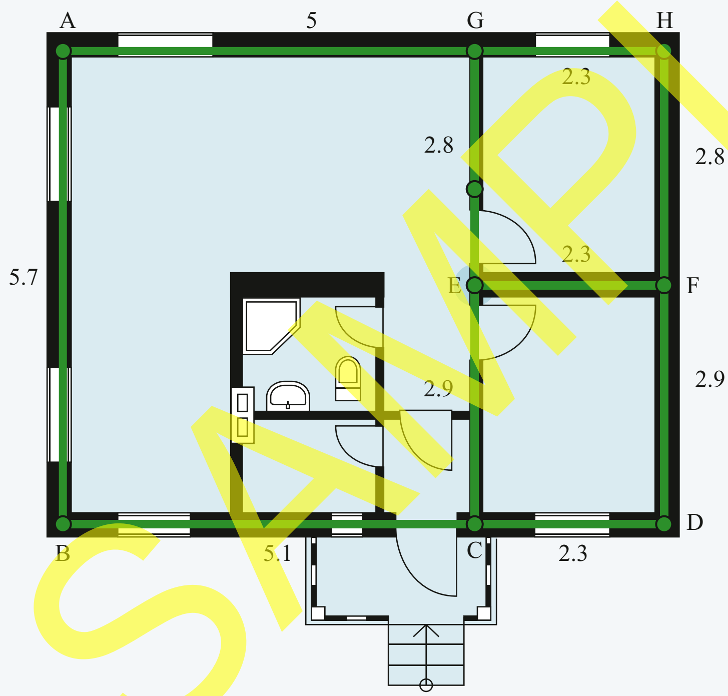


Re-draw this network so that the utility lines do not cross.

3 Find a walk through the network in question **2** that visits every edge exactly once.

Use the diagram below to answer questions **4–6**.

The following house plan shows the possible locations of power cables.

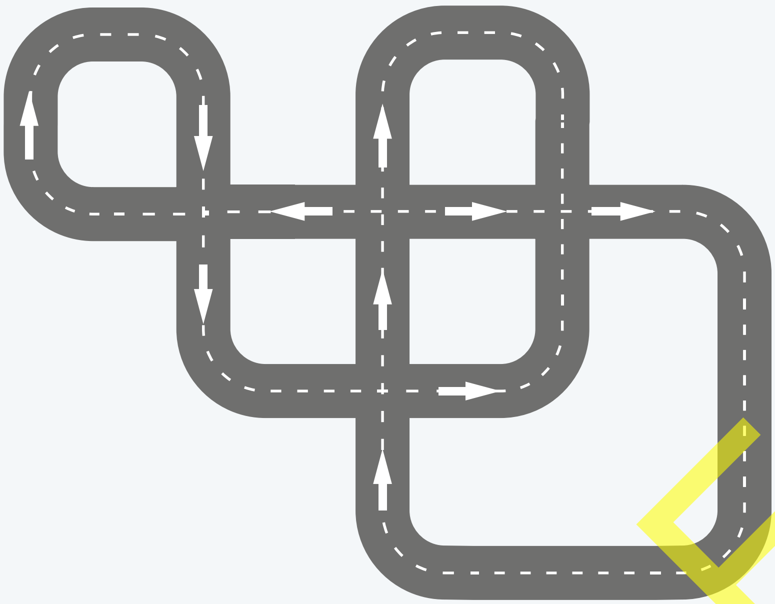


4 What is the most cost-effective way to connect the points A, B, C, D, E and F to the power meter at G?

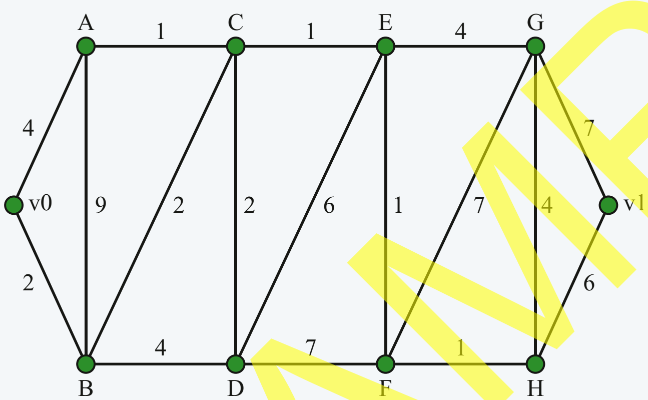
5 What is the weight of the path ABCDFEG for the network?

6 Use Prim's algorithm to find the minimum spanning tree for the network.

7 Draw this system of one way streets as a directed network.

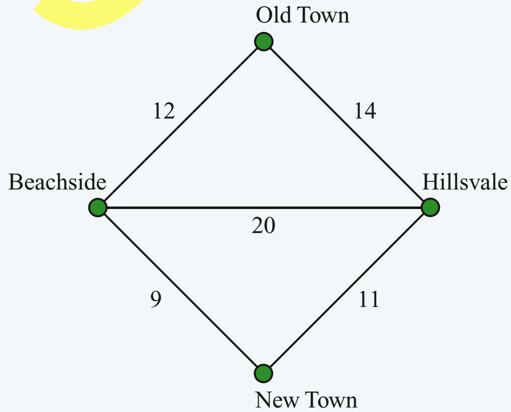


8 Find the shortest path from v0 to v1.



REVIEW PRACTICE EXAMINATION QUESTION

1 **a** This diagram shows the possible paths (in km) for laying gas pipes between various towns.



Gas is to be supplied from one location. Any one of the locations can be the source of the supply.

i What is the minimum total length of the pipes required to provide gas to all the locations?

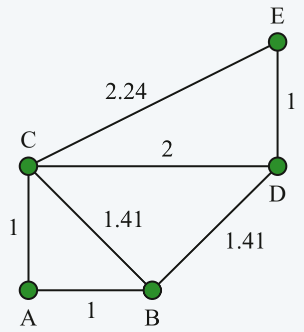
A 32 km **B** 33 km **C** 29 km **D** 40 km (1 mark)

ii How many different paths are there from New Town to Old Town? (1 mark)

iii What is the shortest path from Beachside to Hillsvale? (1 mark)

iv Find a walk that visits every edge exactly once. (1 mark)

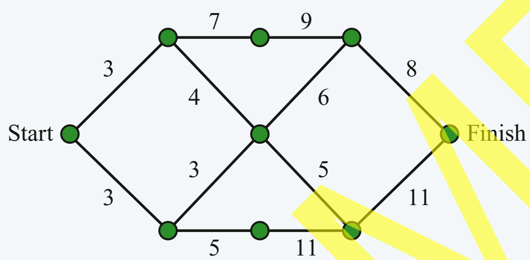
b This diagram shows the distance by road (in km) between towns.



i What is the shortest path from town A to town E? (1 mark)

ii What roads should be resurfaced, to minimise the total length of resurfaced road while still connecting each town with newly resurfaced road? (1 mark)

c The following network shows a maze. Distances are in metres.



i What is the shortest path from the start of the maze to the finish?

A 20 m **B** 30 m **C** 40 m **D** 50 m (1 mark)

ii Why does this network not contain a walk that visits every edge exactly once? (1 mark)

iii Find a minimum spanning tree for this network. (1 mark)

d Consider the road network below.

i Is the graph connected? (1 mark)

ii Which word best describes the road network?

A tree **B** path **C** cycle **D** directed (1 mark)

e Consider the following town map as a network of intersections (vertices) connected by roads (edges).

i How many vertices are in the network? (1 mark)

ii What is the highest vertex degree? (1 mark)

iii Can you design a bus route that visits every street exactly once? (2 marks)

TOTAL: 15 marks

NETWORKS