MY MATHS 7

AUSTRALIAN CURRICULUM QUEENSLAND

IT'S MINE!

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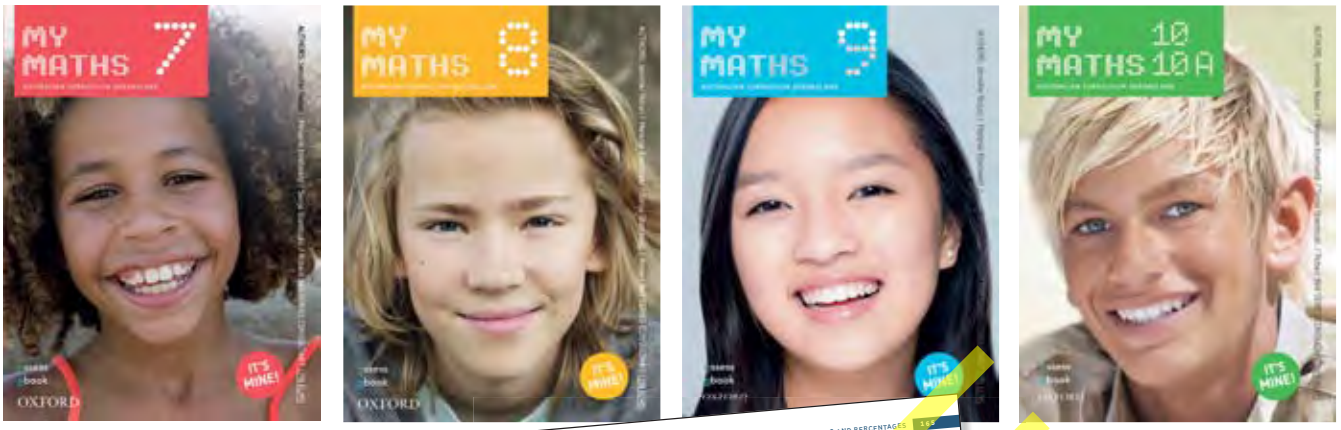
OXFORD

NUMBER AND ALGEBRA

MEASUREMENT AND GEOMETRY

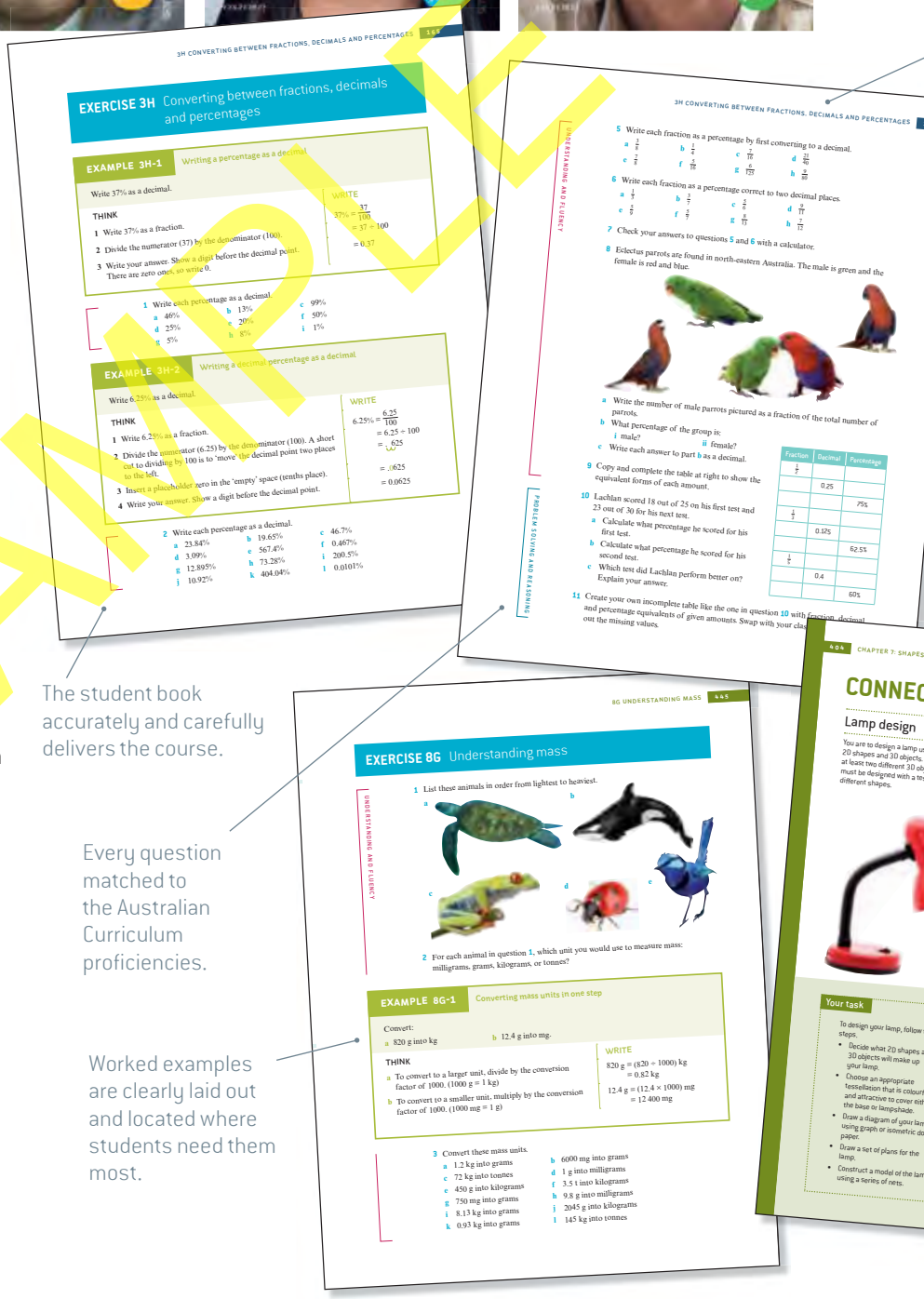
OXFORD MYMATHS FOR QUEENSLAND



Oxford MyMaths for Queensland has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

STUDENT BOOK + OBOOK/ASSESS

- Finely levelled exercises to ensure smooth progress
- Integrated worked examples – right where your students need them
- Learning organised around the 'big ideas' of mathematics
- Discovery, practice, thinking and problem-solving activities promote deep understanding
- A wealth of revision material to consolidate and prove learning
- Rich tasks to apply understanding
- Highly accessible and easy to navigate
- Comprehensive digital tutorials and guided examples to support independent progress



The student book accurately and carefully delivers the course.

Every question matched to the Australian Curriculum proficiencies.

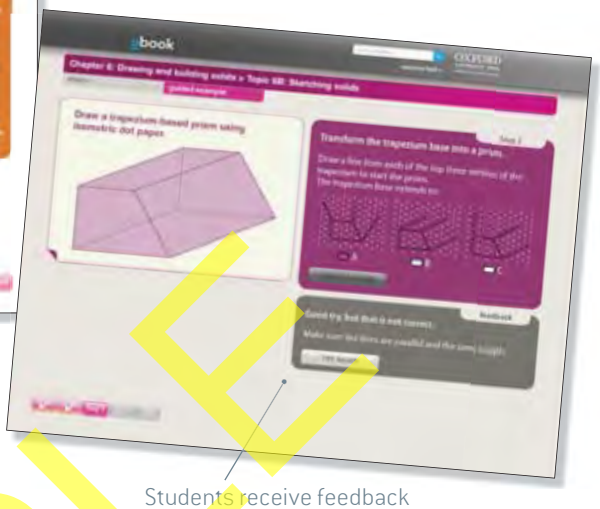
Worked examples are clearly laid out and located where students need them most.

24/7 LEARNING AND SUPPORT

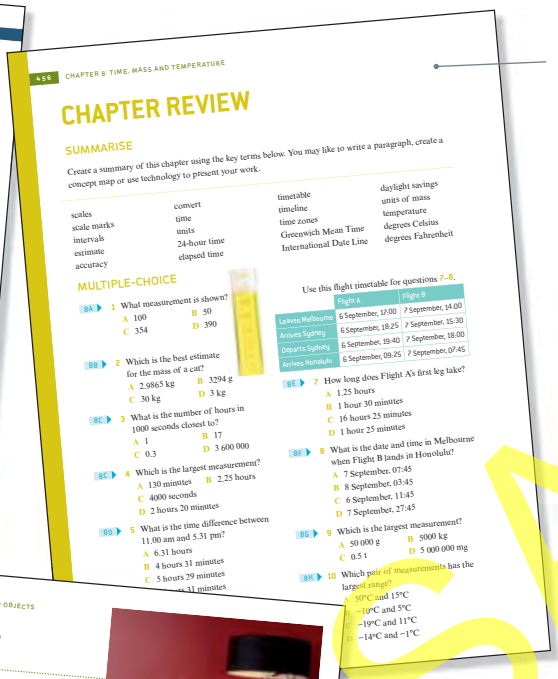
E-tutors scaffold understanding of key concepts and build confidence.

Self-discovery opportunities for students through guided exploration.

Guided examples support practice and fluency



Finely levelled content enables students to progress with ease

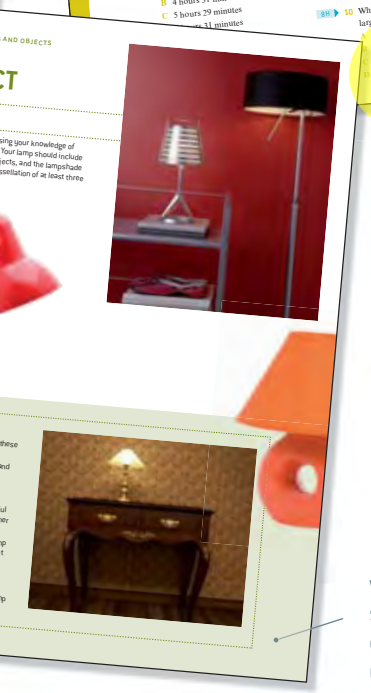


Ample revision to consolidate understanding and prove that learning has happened

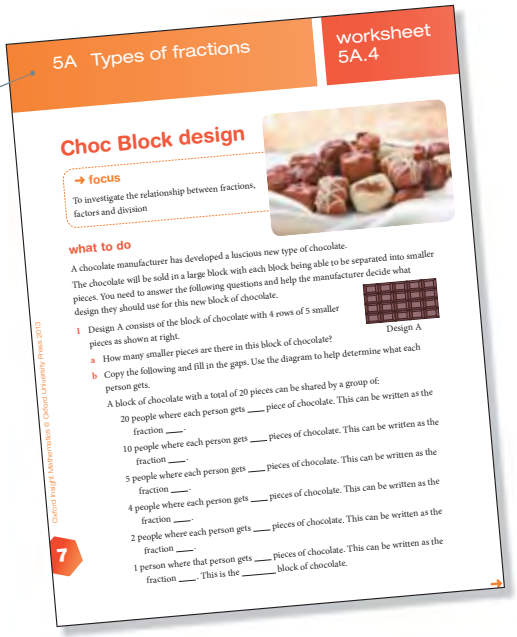
Students receive feedback for incorrect responses

Optimise understanding and performance.

Personalised learning: tailor the very best learning experiences for all.



Intervention and extension worksheets supplied for every topic.



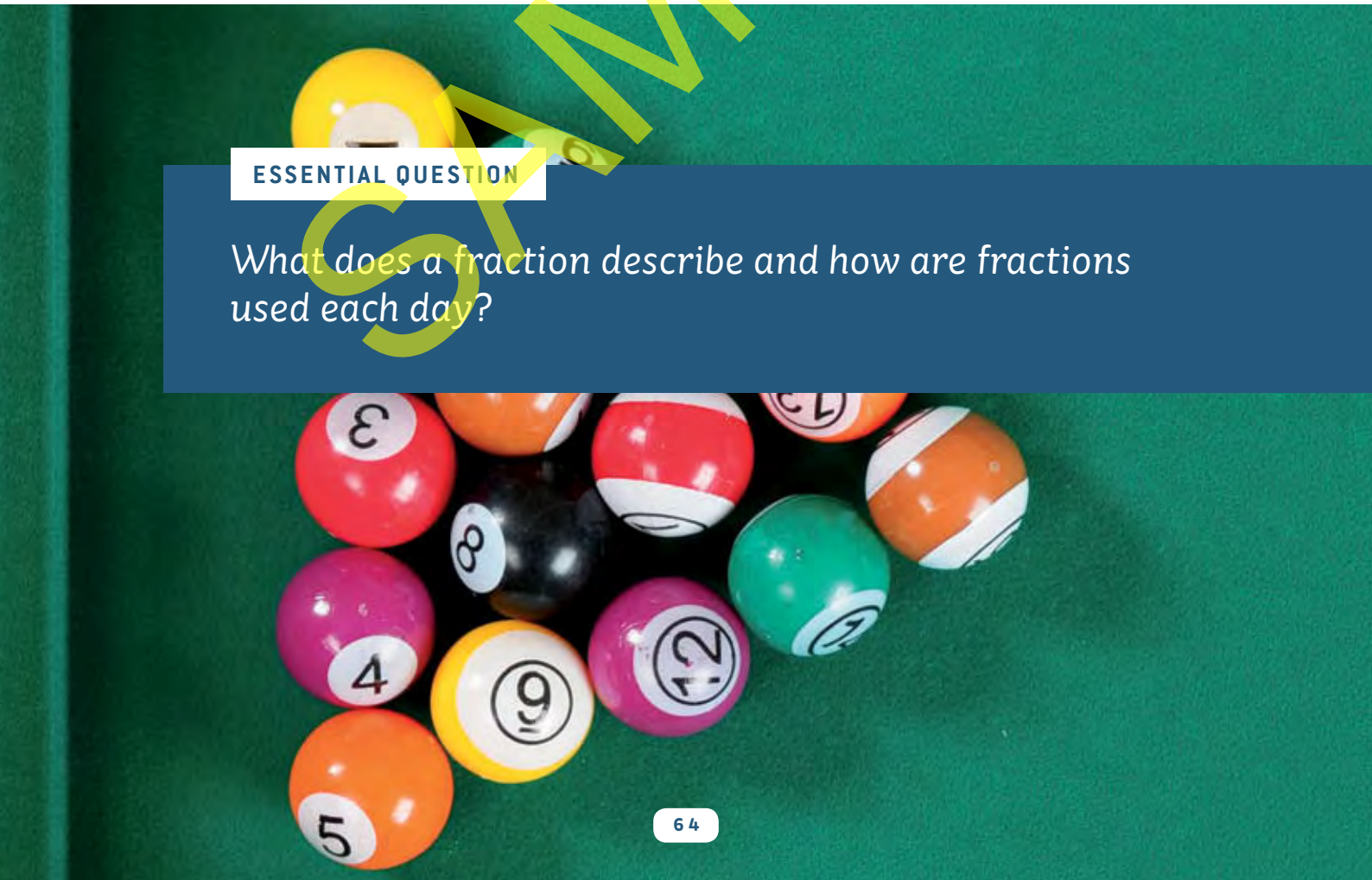
Rich tasks where students can demonstrate understanding

TEACHER OBOOK/ASSESS

Practical classroom resources and tools:

- Manage student differentiation
- Correct common misconceptions
- Assign work
- Set tests
- Monitor results
- Any device, anytime, anywhere.

2 FRACTIONS AND RATIOS

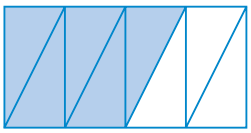


ESSENTIAL QUESTION

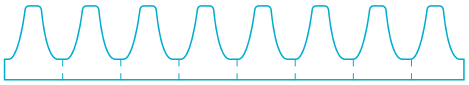
What does a fraction describe and how are fractions used each day?

2A **1** What fraction of each shape has been shaded?

a 

b 

2A **2** Copy this diagram. Shade $\frac{1}{2}$ of the chocolate bar.



2A **3** Describe each number as a proper fraction, an improper fraction or a mixed number.

a $\frac{5}{7}$ **b** $3\frac{2}{3}$ **c** $\frac{5}{18}$

d $\frac{9}{28}$ **e** $\frac{28}{9}$ **f** $2\frac{9}{11}$

2B **4** What is the highest common factor of 48 and 64?

A 4 **B** 8 **C** 16 **D** 24

2B **5** Which fraction is equivalent to $\frac{1}{3}$?

A $\frac{5}{16}$ **B** $\frac{9}{18}$ **C** $\frac{2}{15}$ **D** $\frac{4}{12}$

2B **6** Which fraction is equivalent to $\frac{5}{2}$?

A $\frac{35}{14}$ **B** $\frac{10}{6}$ **C** $\frac{14}{35}$ **D** $\frac{10}{10}$

2B **7** **a** Write this list of numbers in order from smallest to largest.

$\frac{5}{11}, \frac{13}{11}, 1, \frac{7}{11}, \frac{2}{11}$

b Write this list of numbers in order from largest to smallest.

$1, \frac{8}{17}, \frac{3}{17}, \frac{19}{17}, \frac{2}{17}, \frac{21}{17}$

2C **8** Calculate:

a $\frac{2}{15} + \frac{11}{15}$ **b** $\frac{13}{23} + \frac{4}{23}$

c $\frac{7}{9} - \frac{5}{9}$ **d** $\frac{19}{21} - \frac{8}{21}$

2C **9** What is the lowest common multiple of 3, 4 and 5?

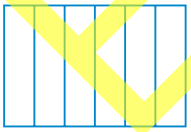
A 120 **B** 60 **C** 20 **D** 12

2D **10** Use this diagram to answer the question.

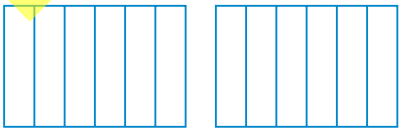
a What is $\frac{1}{2}$ of 16?

b What is $\frac{1}{4}$ of 16?

2E **11** **a** How many lots of $\frac{1}{6}$ are in the rectangle below?



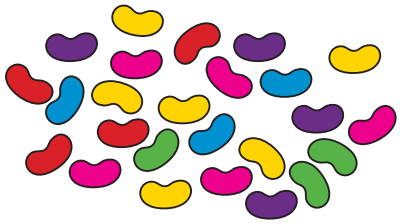
b How many lots of $\frac{1}{6}$ are in the two rectangles below?



2F **12** Calculate:

A 7^2 **B** 5^3 **C** 2^4 **D** $\sqrt{36}$

2G **13** Use this diagram to answer the questions.



a Describe the number of red jellybeans compared to the number of blue jellybeans.

A 5 to 3 **B** 4 to 3

C 3 to 5 **D** 3 to 4

b Describe the number of green jellybeans compared to the number of yellow and pink jellybeans.

A 3 to 5 **B** 3 to 6

C 11 to 3 **D** 3 to 11

2A Understanding fractions

Start thinking!

There are different types of fractions. An example of a **proper fraction** is $\frac{3}{4}$ and an example of an **improper fraction** is $\frac{7}{4}$.

1 What is the difference between a proper and an improper fraction?

An improper fraction can also be written as a **mixed number**.
The improper fraction $\frac{7}{4}$ can be written as the mixed number $1\frac{3}{4}$.

2 What is the difference between an improper fraction and a mixed number?

3 What do an improper fraction and a mixed number have in common?

Mary, David and Xavier are to share three rectangular pizzas. Each pizza is cut into eight pieces.

4 Each slice is $\frac{1}{8}$ of a pizza. If David eats seven slices, what fraction of a pizza has he eaten?

5 Is your answer to question **4** a proper fraction, an improper fraction or a mixed number?

6 Xavier eats 11 slices of pizza. What fraction of pizza has Xavier eaten?
Write your answer as an improper fraction and as a mixed number if you can.

KEY IDEAS

- A fraction is a part or portion of a whole. It can be written as one whole number (**numerator**) over another (**denominator**), separated by a horizontal line (**vinculum**).
- Proper fractions have a numerical value less than 1. The numerator in a proper fraction is smaller than the denominator.
  Some examples are: $\frac{3}{5}, \frac{2}{9}, \frac{11}{15}$.

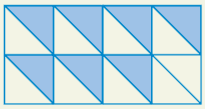
vinculum → $\frac{4}{5}$ ← the **numerator** shows the number of equal parts out of the whole
← the **denominator** shows the total number of parts in the whole

- Improper fractions have a numerical value greater than 1. The numerator in an improper fraction is larger than the denominator. Some examples are: $\frac{7}{2}, \frac{9}{5}, \frac{31}{8}$.
- Mixed numbers also have a numerical value greater than 1. They contain a whole number and proper fraction component. Some examples are: $4\frac{2}{3}, 7\frac{1}{8}, 6\frac{12}{17}$.
- Whole numbers can be written as fractions. For example, $\frac{2}{2} = 1, \frac{9}{3} = 3, \frac{28}{7} = 4, \frac{5}{1} = 5$.

EXERCISE 2A Understanding fractions

EXAMPLE 2A-1 Writing proper fractions

What fraction of the shape is shaded?



THINK

1 Count the number of equal segments in the shape. (16). This is the denominator of the fraction.

2 Count the number of shaded segments in the shape (7). This is the numerator of the fraction.

3 Write the proper fraction.

WRITE

The shape is divided into 16 segments.

Seven of the 16 segments are shaded.

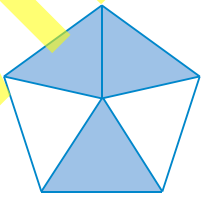
The fraction of the shape that is shaded is $\frac{7}{16}$.

UNDERSTANDING AND FLUENCY

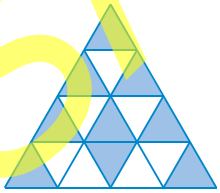
1 What fraction of each shape is shaded?

a

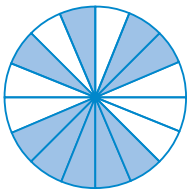
b



c



d

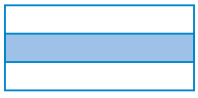


2 What fraction of each shape in question **1** is *not* shaded?

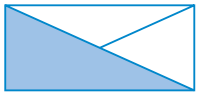
3 Which diagram correctly displays that $\frac{1}{3}$ of the shape has been shaded? Provide a reason for your selection.

A

B



C



EXAMPLE 2A-2 Writing improper fractions and mixed numbers

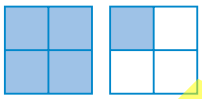
Write the number that is represented by the shading in the diagram as:
a an improper fraction **b** a mixed number.

THINK	WRITE
a 1 Write the total number of quarters that are shaded.	**a** There are seven quarters that are shaded.
2 Write the answer as an improper fraction.	The number represented is $\frac{7}{4}$.
b 1 Look at any shapes in the diagram that are fully shaded. A fully shaded shape represents the whole number of 1.	**b** The whole of the first rectangle is shaded and three quarters of the second rectangle is shaded.
2 Write the answer as a mixed number.	The number represented is $1\frac{3}{4}$.

UNDERSTANDING AND FLUENCY

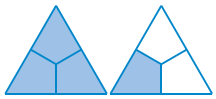
4 Write the number that is represented by the shading in each diagram as:
i an improper fraction **ii** a mixed number.

a



b

c



d

5 Rule up a table with three columns. Title the columns 'Proper fractions', 'Improper fractions' and 'Mixed numbers'. Place each of these fractions into the appropriate column: $\frac{5}{2}$, $2\frac{3}{5}$, $\frac{7}{8}$, $1\frac{5}{9}$, $\frac{17}{2}$, $\frac{16}{31}$, $\frac{29}{15}$, $12\frac{1}{7}$, $\frac{13}{22}$.

EXAMPLE 2A-3 Converting a mixed number to an improper fraction

Convert the mixed number $2\frac{2}{7}$ to an improper fraction.

THINK	WRITE
1 Write the mixed number as the sum of a whole number and a fraction.	$2\frac{2}{7}$
	$= 2 + \frac{2}{7}$
2 Since the denominator is 7, write each whole number as seven sevenths.	$= \frac{7}{7} + \frac{7}{7} + \frac{2}{7}$
3 Add all of the sevenths (7 + 7 + 2 = 16) and write the answer.	$= \frac{16}{7}$

6 Convert each mixed number to an improper fraction.

a $1\frac{2}{9}$ **b** $3\frac{1}{6}$ **c** $5\frac{8}{9}$ **d** $2\frac{6}{11}$ **e** $10\frac{4}{5}$ **f** $8\frac{3}{4}$

g $2\frac{3}{7}$ **h** $12\frac{1}{3}$ **i** $4\frac{5}{8}$ **j** $5\frac{3}{10}$ **k** $7\frac{1}{12}$ **l** $2\frac{4}{15}$

EXAMPLE 2A-4 Converting an improper fraction to a mixed number

Convert the improper fraction $\frac{25}{8}$ to a mixed number.

THINK

1 As 8 eighths make a whole, group 25 eighths into three lots of 8 eighths and 1 eighth left over.

2 Rewrite each lot of 8 eighths as a whole number.

3 Write the total as a whole number and fraction.

WRITE

$\frac{25}{8}$

$= \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

$= 1 + 1 + 1 + \frac{1}{8}$

$= 3\frac{1}{8}$

UNDERSTANDING AND FLUENCY

7 Convert each improper fraction to a mixed number.

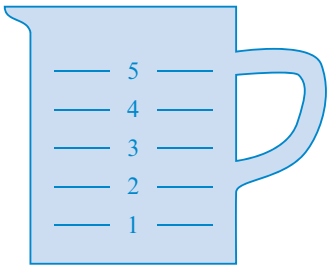
a $\frac{7}{5}$ **b** $\frac{26}{3}$ **c** $\frac{25}{9}$ **d** $\frac{43}{10}$ **e** $\frac{75}{8}$ **f** $\frac{88}{13}$

g $\frac{22}{7}$ **h** $\frac{17}{4}$ **i** $\frac{49}{6}$ **j** $\frac{50}{11}$ **k** $\frac{67}{5}$ **l** $\frac{111}{2}$

8 Copy this diagram of a jug. The scale indicates measurements in litres.

Show the water level for a measurement of $\frac{4}{3}$ L on your jug.

(Hint: first convert the improper fraction to a mixed number.)



9 Repeat the method used in question **8** to show the water level on a jug for each of these measurements.

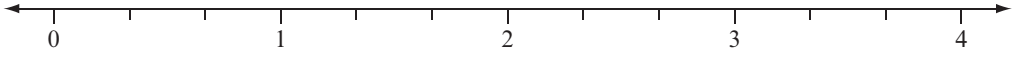
a $\frac{7}{2}$ L **b** $\frac{8}{5}$ L **c** $\frac{17}{4}$ L

10 Convert both fractions to improper fractions and use the symbol < (is less than) or > (is greater than) to make a true statement.

a $2\frac{4}{7}$ $\frac{17}{7}$ **b** $8\frac{2}{3}$ $\frac{28}{3}$

c $\frac{38}{11}$ $2\frac{6}{11}$ **d** $9\frac{4}{5}$ $\frac{51}{5}$

11 The number line below has been divided into four equal intervals from 0 to 4. Each interval has been further subdivided into three equal parts.



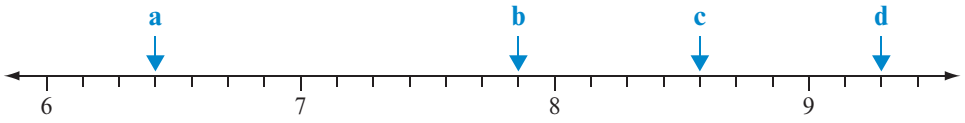
a What fraction of an interval is each part?

b Label each mark on the scale with the value it represents.

12 Draw a number line from 0 to 4 and mark the position of each of these fractions.

a $\frac{2}{5}$ **b** $\frac{5}{5}$ **c** $\frac{8}{5}$ **d** $\frac{13}{5}$ **e** $\frac{17}{5}$

13 Write down the values shown for **a–d** on the number line.



14 The Australian state and territory flags are shown at right.

What fraction of the flags:

a include the Southern Cross in their design?

b include a flower in their design?

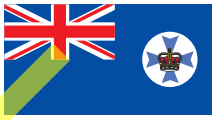
c include blue in their colours?

d do not include animals in their design?


Australian Capital Territory


New South Wales


Northern Territory


Queensland


South Australia


Tasmania


Victoria


Western Australia

15 Write the fraction that is described by each statement.

a 17 correct answers on a quiz containing 25 questions

b the number of months in a year consisting of 31 days

c $23 change from a $50 note

16 There are 60 minutes in 1 hour. What fraction of an hour is:

a 17 minutes? **b** 31 minutes?

c 119 minutes (as an improper fraction)? **d** 311 minutes (as a mixed number)?

17 Georgina, Roisin and Vanessa decide to order large pizzas for a class lunch. Each pizza will be cut into eight equal slices. The girls estimate that each person will eat two slices and there will be a maximum of 22 people.

a Write the number of pizzas required as:

i an improper fraction **ii** a mixed number.

b Will each person be able to receive the estimated two slices of pizza if five large pizzas have been ordered?

c The girls' budget does not allow them to order extra pizzas. What is the minimum number of equal slices each pizza can be cut into, if everyone is to receive two slices?

d What fraction of the pizza will be left over if the pizzas are cut in the way suggested in part **c**?

e On the day of the lunch, three people were away and $4\frac{5}{8}$ of the pizzas were eaten. If each pizza was cut into eight slices, how many slices were eaten?

PROBLEM SOLVING AND REASONING

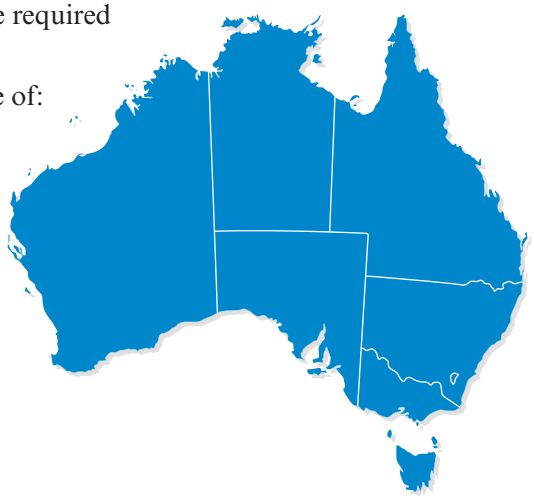
18 Look at the image of the Colosseum shown here.

- **a** By estimation, how many levels high is the tallest point of the Colosseum?
- **b** What fraction of the height of the Colosseum is one level?
- **c** What fraction of the levels contain arches?

19 Estimate what fraction of this ant's whole body length is made up of its head.

CHALLENGE

20 Use this map of Australia to estimate the required fraction in each question.



- **a** What fraction of Australia is the state of:
  - **i** Victoria?
  - **ii** Tasmania?
  - **iii** Queensland?
  - **iv** New South Wales?
  - **v** Western Australia?
- **b** What fraction of Western Australia is the state of New South Wales?
- **c** What fraction of South Australia is the state of Tasmania?

21 Gemma and Cindy disagree on which fraction is larger, $\frac{1}{4}$ or $\frac{1}{5}$. Describe how number lines can be used to compare the fractions and decide which is larger.

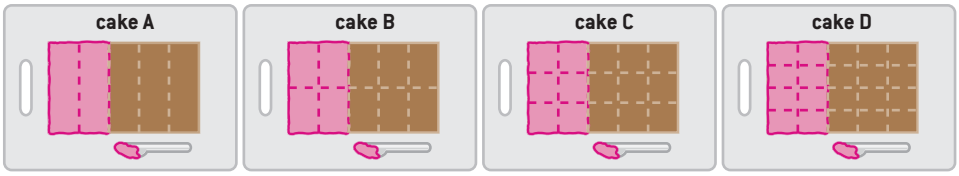
Reflect

Explain how you can compare the size of fractions.

2B Equivalent fractions

Start thinking!

Look at each cake at right. Icing has been spread over some of the top of each cake and the cake has been cut into equal pieces.



1 What do you notice about the amount of cake that has been iced in each case?

2 Copy and complete this table.

Cake	Total number of equal pieces	Number of pieces that have been iced	Fraction of the cake that has been iced
A			
B			
C			
D			

3 Without drawing a diagram, explain how the next cake in the series would look in terms of the total number of pieces, the number of pieces that have been iced and the fraction of the cake that has been iced.

4 The fractions representing the amount of each cake that has been iced are called **equivalent fractions**. Explain why.

KEY IDEAS

- Equivalent fractions have the same numerical value.
- An equivalent fraction can be formed by multiplying both the numerator and the denominator of a fraction by the same value (see Example A).
- Dividing both the numerator and the denominator of a fraction by the same value also produces an equivalent fraction (see Example B). This process is called simplifying or **cancelling**.
- A fraction is in its simplest form once the numerator and the denominator have been divided by the **highest common factor (HCF)**.
- Mixed numbers can be simplified by keeping the whole number the same and simplifying the fraction component.

Example A

$\frac{4}{5} = \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$

Multiplying by $\frac{3}{3}$ is the same as multiplying by 1, so the value of the equivalent fraction is the same.

Example B

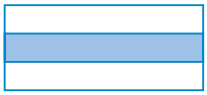
$$\frac{24}{54} \overset{\div 6}{=} \frac{4}{9}$$

EXERCISE 2B Equivalent fractions

UNDERSTANDING AND FLUENCY

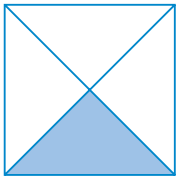
1 What fraction of each shape is shaded?

a



b

c



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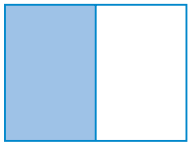
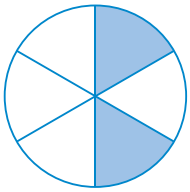
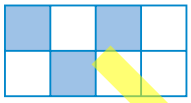


e



f



2 What fraction of each shape in question **1** is *not* shaded?

3 Create two equivalent fractions to represent each fraction found in:

i question **1**

ii question **2**.

EXAMPLE 2B-1 Making equivalent fractions

Fill in the gap to make an equivalent fraction: $\frac{2}{3} = \frac{10}{\square}$.

THINK

1 Since $2 \times 5 = 10$, the numerator of the second fraction is obtained by multiplying the numerator of the first fraction by 5.

$$\frac{2}{3} \overset{\times 5}{=} \frac{10}{\square}$$

2 To find the equivalent fraction, use the same relationship (multiply by 5) to obtain the denominator of the second fraction.

$$\frac{2}{3} \underset{\times 5}{\overset{\times 5}{=}} \frac{10}{\square}$$

WRITE

$\frac{2}{3} = \frac{10}{\square}$

$\frac{2}{3} = \frac{10}{15}$

4 Fill in the gaps to make equivalent fractions. (Hint: change mixed numbers to improper fractions first.)

a $\frac{4}{9} = \frac{}{36}$ **b** $\frac{24}{8} = \frac{12}{}$ **c** $2\frac{5}{11} = \frac{}{33}$

d $\frac{42}{70} = \frac{}{10}$ **e** $\frac{144}{48} = \frac{}{12}$ **f** $3\frac{2}{8} = \frac{}{72}$

g $\frac{2}{3} = \frac{}{9} = \frac{}{90}$ **h** $\frac{5}{6} = \frac{}{18} = \frac{20}{}$ **i** $\frac{2}{5} = \frac{10}{} = \frac{}{15}$

j $5\frac{1}{13} = \frac{}{39} = \frac{330}{}$ **k** $\frac{1}{10} = \frac{}{20} = \frac{}{30} = \frac{8}{}$ **l** $\frac{8}{7} = \frac{24}{} = \frac{}{35} = \frac{}{77}$

m $3\frac{3}{4} = \frac{}{24} = \frac{45}{} = \frac{}{72}$ **n** $\frac{1}{12} = \frac{6}{} = \frac{8}{} = \frac{}{144} = \frac{15}{} = \frac{}{240}$ **o** $\frac{2}{3} = \frac{}{6} = \frac{10}{} = \frac{}{30} = \frac{}{75} = \frac{400}{}$

EXAMPLE 2B-2 Simplifying fractions

Write each fraction in its simplest form.

a $\frac{21}{49}$ **b** $2\frac{45}{65}$

THINK

a 1 Write the fraction and find the highest common factor (HCF) of the numerator and the denominator. HCF = 7.

2 Cancel by dividing both the numerator and the denominator by 7, the HCF.

3 Write the answer.

b 1 Write the mixed number. Find the HCF of the numerator and the denominator of the fraction component. HCF = 5.

2 Leave the whole number as it is and divide both the numerator and the denominator by 5, the HCF.

3 Write the answer.

WRITE

a $\frac{21}{49}$

$= \frac{\cancel{21}^{3}}{\cancel{49}^{7}}$

$= \frac{3}{7}$

b $2\frac{45}{65}$

$= 2\frac{\cancel{45}^{9}}{\cancel{65}^{13}}$

$= 2\frac{9}{13}$

UNDERSTANDING AND FLUENCY

5 Write each fraction in its simplest form.

a $\frac{10}{16}$ **b** $\frac{11}{33}$ **c** $\frac{28}{36}$

d $\frac{30}{48}$ **e** $\frac{63}{56}$ **f** $\frac{116}{94}$

g $2\frac{12}{30}$ **h** $7\frac{20}{45}$ **i** $4\frac{56}{64}$

j $5\frac{33}{36}$ **k** $6\frac{15}{75}$ **l** $3\frac{16}{54}$

6 Larni scored 22 of her team's 34 goals. What fraction of the team's goals did Larni score? Write your answer in simplest form.

7 Cristiano spent 75 minutes of a 90-minute football game on the field. What fraction of time, in simplest form, did he spend off the field?

8 Write four equivalent fractions for $\frac{4}{7}$.

9 Create three proper fractions that are equivalent to $\frac{5}{9}$.

10 Create three improper fractions that are equivalent to $\frac{7}{3}$.

11 Create three mixed numbers that are equivalent to $1\frac{3}{5}$.

12 **a** Write a fraction that is smaller than 1.

b Write a fraction that is greater than 1 with the same denominator as that in part **a**.

c Place these fractions on a number line and comment on their position with respect to 1.

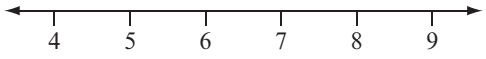
13 **a** Draw a number line and mark the position of each of these fractions.

$\frac{12}{7}, \frac{10}{7}, \frac{15}{7}, \frac{5}{7}, \frac{21}{7}, \frac{32}{7}, \frac{1}{7}, 4\frac{3}{7}$

b Using the number line, list the fractions in ascending order (smallest to largest). What do you notice?

14 Explain how to order fractions that have the same denominator.

15 **a** Copy the number line on the right and divide each interval into three equal parts. Label each of these parts.



b Convert each fraction to a mixed number, simplifying where necessary, and then mark its position on the number line.

i $\frac{23}{3}$ **ii** $\frac{20}{3}$ **iii** $5\frac{2}{6}$ **iv** $7\frac{4}{6}$ **v** $\frac{24}{4}$ **vi** $\frac{12}{3}$

c Compare the position of the largest fraction to the other fractions.

d Compare the position of the smallest fraction to the other fractions.

16 Convert both fractions to mixed numbers and use the < (is less than) or > (is greater than) symbol to make a true statement.

a $\frac{28}{3}$ $\frac{17}{5}$ **b** $\frac{49}{6}$ $\frac{38}{4}$ **c** $\frac{13}{2}$ $\frac{26}{9}$ **d** $\frac{28}{5}$ $\frac{61}{10}$

e $\frac{42}{7}$ $\frac{63}{9}$ **f** $\frac{27}{6}$ $\frac{15}{4}$ **g** $\frac{32}{5}$ $\frac{63}{8}$ **h** $\frac{21}{2}$ $\frac{41}{4}$

17 Write each fraction in simplest form and list those that are equivalent fractions.

$\frac{1}{3}, \frac{5}{10}, \frac{12}{15}, \frac{6}{18}, \frac{17}{25}, \frac{30}{45}, \frac{16}{48}, \frac{18}{54}, \frac{20}{60}, \frac{52}{156}$

EXAMPLE 2B-3 Comparing fractions using the lowest common denominator

Which fraction is larger: $\frac{3}{5}$ or $\frac{4}{7}$?

THINK

1 Find the lowest common denominator (LCD) for the two fractions. It may help to list all the multiples of 5 and 7 and highlight the lowest multiple that is common to both.

2 Write the LCD. Notice that both 5 and 7 are factors of 35.

3 For each fraction, find the equivalent fraction with a denominator of 35.

4 Compare the size of the numerators to decide which fraction is larger. $\frac{21}{35}$ is larger than $\frac{20}{35}$.

WRITE

5, 10, 15, 20, 25, 30, 35…
7, 14, 21, 28, 35, 42…

LCD = 35
$\frac{3 \times 7}{5 \times 7} = \frac{21}{35}$
$\frac{4 \times 5}{7 \times 5} = \frac{20}{35}$
$\frac{3}{5}$ is larger than $\frac{4}{7}$.

UNDERSTANDING AND FLUENCY

18 Which fraction in each given pair is larger?

a $\frac{3}{6}$ or $\frac{4}{9}$ **b** $\frac{7}{10}$ or $\frac{12}{16}$ **c** $\frac{32}{5}$ or $\frac{45}{7}$

d $\frac{5}{6}$ or $\frac{7}{8}$ **e** $1\frac{6}{11}$ or $1\frac{11}{20}$ **f** $\frac{138}{12}$ or $\frac{180}{14}$

g $3\frac{7}{9}$ or $\frac{44}{12}$ **h** $\frac{17}{4}$ or $4\frac{1}{5}$ **i** $\frac{37}{6}$ or $6\frac{1}{12}$

19 **a** Convert these fractions to equivalent fractions using the LCD and then arrange them in ascending order. $\frac{1}{4}, \frac{1}{6}, \frac{1}{3}, \frac{3}{4}, \frac{5}{6}, \frac{2}{3}, \frac{2}{4}$

b Draw a number line and mark the position of the equivalent fractions. What length is represented by the interval between each mark on the scale?

c Write the original fraction below its equivalent fraction on the number line.

20 Write each list of fractions in ascending order (smallest to largest).

a $\frac{7}{5}, \frac{2}{5}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{16}{5}$ **b** $\frac{5}{8}, \frac{11}{4}, \frac{2}{6}, \frac{7}{6}, \frac{5}{4}, \frac{9}{8}$ **c** $\frac{2}{9}, \frac{4}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{3}, \frac{9}{6}$ **d** $\frac{8}{15}, \frac{5}{4}, \frac{6}{3}, \frac{11}{10}, \frac{9}{20}, \frac{10}{6}$

21 Write each list of fractions in descending order (largest to smallest).

a $\frac{3}{2}, \frac{8}{5}, \frac{7}{6}, \frac{10}{6}, \frac{5}{2}, \frac{12}{5}$ **b** $\frac{1}{8}, \frac{11}{12}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}, \frac{2}{6}$ **c** $\frac{10}{7}, \frac{9}{8}, \frac{3}{4}, \frac{3}{2}, \frac{7}{8}, \frac{6}{7}$ **d** $\frac{2}{9}, \frac{1}{3}, \frac{5}{7}, \frac{4}{3}, \frac{11}{7}, \frac{13}{9}$

22 Write three fractions that are:

a smaller than $\frac{1}{4}$ **b** larger than $2\frac{1}{3}$ **c** between $\frac{2}{7}$ and $\frac{2}{5}$

d between $\frac{5}{9}$ and $\frac{4}{3}$ **e** between $2\frac{2}{11}$ and $\frac{7}{2}$ **f** between $\frac{1}{2}$ and $\frac{1}{3}$

23 Philip was comparing two of his test results. He scored $\frac{22}{25}$ on the first test and $\frac{45}{50}$ on the second. On which test did Philip perform better?

PROBLEM SOLVING AND REASONING

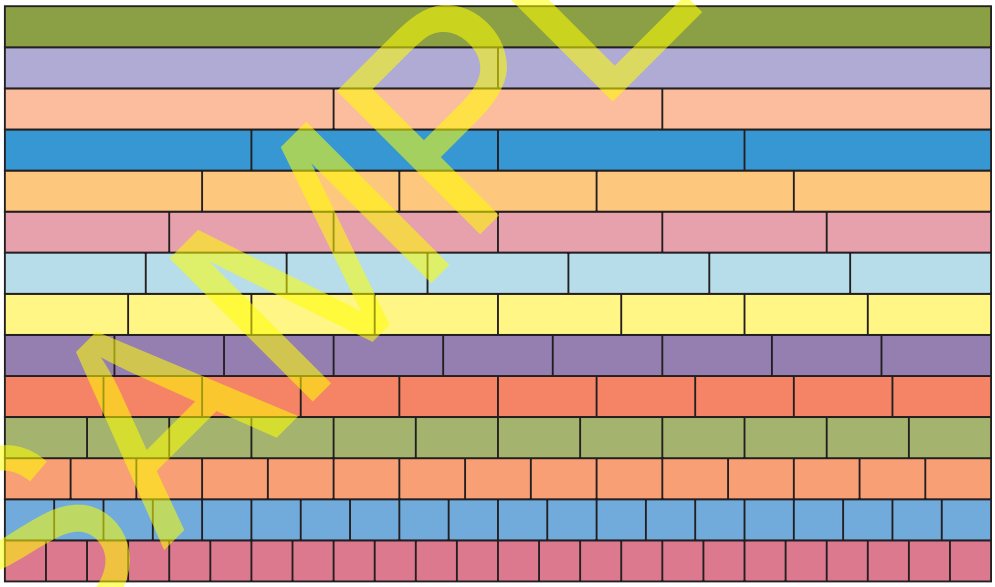
24 Zach and Melanie are comparing their results after an afternoon of archery. From 35 attempts, Zach scored 20 bullseyes. Melanie scored 24 bullseyes from 40 attempts. Who hit the bullseye on the archery target with more accuracy? Justify your answer.

25 The common denominator of three different fractions is 24. What could the denominators of the fractions be?

26 The common denominator of three different fractions is 52. What could the denominators of the fractions be?

CHALLENGE

27 This diagram shows a section of a fraction wall. Fraction walls can be used to obtain equivalent fractions or to compare fractions.



a What do you notice about each horizontal section of the wall?

b What fraction is shown by each segment in the second section down?

c What fraction is shown by each segment in the third section down?

d What do you notice about each segment of a particular horizontal section?

e Use the fraction wall provided to list the set of equivalent fractions for $\frac{1}{4}$.

f Use the fraction wall provided to list four fractions that are larger than $\frac{2}{5}$.

g Use graph or grid paper and follow the teacher's instructions to create your own fraction wall.

h Work in pairs and use the fraction wall to design three questions you could ask your classmates.

Reflect

What calculations could be performed using the fraction wall?

2C Adding and subtracting fractions

Start thinking!

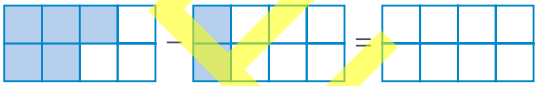
This diagram shows two fractions being added together.

$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

1 What do you notice about the denominator of each fraction?

2 What do you notice about the numerators of the fractions being added and the numerator of the answer?

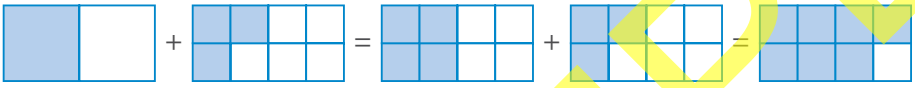
3 Copy and complete this diagram to represent $\frac{5}{8} - \frac{2}{8}$.



$\frac{5}{8} - \frac{2}{8} =$ ____

4 Describe how to add or subtract fractions that have the same denominator.

This diagram shows how to add fractions with different denominators.



$\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$

5 Look carefully at the diagram. Explain what must be done to add fractions with different denominators.

6 Draw a diagram similar to the one in question **3** to represent $\frac{1}{2} - \frac{3}{8}$.

7 Describe how to add or subtract fractions that have different denominators.

KEY IDEAS

- To add (or subtract) fractions that have the same denominator, keep the denominator the same and add (or subtract) the numerators.
- To add (or subtract) fractions that have different denominators, first find the equivalent fractions with the lowest common denominator (LCD) and then add (or subtract) the numerators.
- Before adding or subtracting mixed numbers, convert them to improper fractions and check that they are written with a common denominator.
- If the answer is an improper fraction it should be converted to a mixed number.

EXERCISE 2C Adding and subtracting fractions

EXAMPLE 2C-1 Adding and subtracting fractions with the same denominator

Calculate:

a $\frac{3}{8} + \frac{7}{8}$ b $\frac{9}{14} - \frac{3}{14}$

THINK

a 1 Check if the denominators are the same (yes). Keep the denominator the same and add the numerators.

2 Simplify the fraction if necessary.

3 Convert the improper fraction to a mixed number.

b 1 Check if the denominators are the same (yes). Keep the denominator the same and subtract the numerators.

2 Simplify the fraction if necessary.

WRITE

a $\frac{3}{8} + \frac{7}{8}$
$= \frac{3+7}{8}$
$= \frac{10}{8}$
$= \frac{5}{4}$
$= 1\frac{1}{4}$

b $\frac{9}{14} - \frac{3}{14}$
$= \frac{9-3}{14}$
$= \frac{6}{14}$
$= \frac{3}{7}$

UNDERSTANDING AND FLUENCY

1 Calculate:

a $\frac{1}{5} + \frac{2}{5}$ b $\frac{4}{7} + \frac{2}{7}$

c $\frac{5}{12} + \frac{3}{12}$ d $\frac{5}{18} + \frac{6}{18}$

e $\frac{3}{37} + \frac{16}{37}$ f $\frac{9}{23} + \frac{16}{23}$

g $\frac{15}{11} + \frac{2}{11} + \frac{8}{11}$ h $\frac{2}{13} + \frac{4}{13} + \frac{5}{13}$

i $\frac{23}{100} + \frac{7}{100} + \frac{14}{100}$ j $\frac{11}{7} - \frac{5}{7}$

k $\frac{25}{21} - \frac{8}{21}$ l $\frac{19}{13} - \frac{5}{13}$

m $\frac{13}{15} - \frac{3}{15}$ n $\frac{26}{27} - \frac{25}{27}$

o $\frac{21}{33} - \frac{5}{33}$ p $\frac{48}{17} - \frac{13}{17} - \frac{10}{17}$

q $\frac{55}{3} - \frac{47}{3} - \frac{7}{3}$ r $\frac{21}{8} - \frac{3}{8} - \frac{15}{8}$

EXAMPLE 2C-2

Adding and subtracting mixed numbers with the same denominator

Calculate: a $1\frac{4}{9} + 2\frac{1}{9}$ b $3\frac{1}{4} - 1\frac{3}{4}$

THINK

a **1** Convert each mixed number to an improper fraction. Check if the denominators are the same (yes).

2 Add the numerators. The denominator stays the same.

3 Convert to a mixed number.

NOTE Alternatively, $1\frac{4}{9} + 2\frac{1}{9} = 1 + 2 + \frac{4}{9} + \frac{1}{9} = 3 + \frac{5}{9} = 3\frac{5}{9}$

b **1** Convert each mixed number to an improper fraction. Check if the denominators are the same (yes).

2 Subtract the numerators. The denominator stays the same.

3 Convert to a mixed number and simplify.

WRITE

a $1\frac{4}{9} + 2\frac{1}{9}$
$= \frac{13}{9} + \frac{19}{9}$
$= \frac{32}{9}$
$= 3\frac{5}{9}$

b $3\frac{1}{4} - 1\frac{3}{4}$
$= \frac{13}{4} - \frac{7}{4}$
$= \frac{6}{4}$
$= 1\frac{2}{4}$
$= 1\frac{1}{2}$

2 Calculate:

a $3\frac{1}{7} + 2\frac{2}{7}$	**b** $2\frac{1}{3} - 1\frac{2}{3}$	**c** $3\frac{2}{5} + 4\frac{1}{5}$	**d** $8\frac{1}{9} - 4\frac{4}{9}$
e $2\frac{8}{11} + 1\frac{5}{11}$	**f** $6\frac{4}{5} - 3\frac{2}{5}$	**g** $2\frac{2}{9} + 1\frac{7}{9} + 3\frac{4}{9}$	**h** $4\frac{5}{6} - 2\frac{1}{6} - 1\frac{5}{6}$
i $12\frac{2}{3} - 2\frac{1}{3} - 8\frac{2}{3}$	**j** $3\frac{2}{5} + 4\frac{1}{5} + 1\frac{4}{5}$	**k** $6\frac{5}{12} + 2\frac{1}{12} - 7\frac{11}{12}$	**l** $3\frac{1}{8} + 2\frac{5}{8} - 1\frac{7}{8}$

EXAMPLE 2C-3

Adding and subtracting fractions with different denominators

Calculate: a $\frac{5}{6} + \frac{3}{4}$ b $\frac{2}{3} - \frac{1}{5}$

THINK

a **1** Check if the denominators are the same (no). Identify the LCD by finding the lowest common multiple of the denominators (12). Write each fraction as an equivalent fraction with a denominator of 12.

2 Add the numerators together. The denominator stays the same.

3 Convert the improper fraction to a mixed number.

b **1** Check if the denominators are the same (no). Write each fraction as an equivalent fraction with a denominator of 15.

2 Subtract the numerators. The denominator stays the same.

WRITE

a $\frac{5}{6} + \frac{3}{4}$
$= \frac{5}{6} \times \frac{2}{2} + \frac{3}{4} \times \frac{3}{3}$
$= \frac{10}{12} + \frac{9}{12}$
$= \frac{19}{12}$
$= 1\frac{7}{12}$

b $\frac{2}{3} - \frac{1}{5}$
$= \frac{10}{15} - \frac{3}{15}$
$= \frac{7}{15}$

UNDERSTANDING AND FLUENCY

3 Calculate:

a $\frac{1}{2}+\frac{2}{3}$ **b** $\frac{3}{5}-\frac{1}{4}$ **c** $\frac{3}{8}+\frac{5}{6}$ **d** $\frac{5}{9}-\frac{1}{2}$ **e** $\frac{3}{4}+\frac{2}{3}$ **f** $\frac{1}{3}-\frac{1}{5}$

g $\frac{4}{7}+\frac{1}{2}$ **h** $\frac{5}{6}-\frac{3}{5}$ **i** $\frac{6}{7}-\frac{1}{3}$ **j** $\frac{2}{3}+\frac{7}{10}$ **k** $\frac{2}{11}+\frac{5}{6}$ **l** $\frac{2}{5}-\frac{3}{8}$

m $\frac{3}{4}-\frac{3}{14}$ **n** $\frac{2}{7}+\frac{4}{5}$ **o** $\frac{4}{15}+\frac{7}{10}$ **p** $\frac{5}{8}-\frac{5}{12}$ **q** $\frac{3}{5}+\frac{1}{8}$ **r** $\frac{17}{23}-\frac{1}{4}$

4 For the fractions in each part, write the LCD you would use to work out the result.

a $\frac{8}{9}+\frac{2}{3}$ **b** $\frac{21}{8}+\frac{17}{32}$ **c** $\frac{14}{19}-\frac{5}{38}$

d $\frac{49}{15}-\frac{14}{5}$ **e** $\frac{17}{6}+\frac{7}{12}$ **f** $\frac{17}{10}-\frac{2}{15}$

5 Work out the result to each calculation shown in question **4**.

6 Dean completed a 10 km run in $\frac{5}{7}$ of an hour, while Mark completed the run in $\frac{3}{4}$ of an hour.

a Which runner had the faster time?

b What was the difference between the two times?

7 Calculate:

a $\frac{2}{3}+\frac{11}{12}+\frac{3}{4}$ **b** $\frac{3}{5}+\frac{13}{20}+\frac{12}{15}$ **c** $\frac{7}{8}+\frac{5}{3}-\frac{5}{12}$

d $\frac{5}{6}-\frac{1}{2}+\frac{3}{8}$ **e** $\frac{21}{5}+\frac{8}{3}-\frac{13}{10}$ **f** $\frac{8}{5}-\frac{11}{8}+\frac{1}{4}$

EXAMPLE 2C-4

Adding and subtracting mixed numbers with different denominators

Calculate: **a** $4\frac{1}{2}+5\frac{2}{3}$ **b** $3\frac{7}{10}-2\frac{3}{4}$

THINK

a **1** Convert each mixed number to an improper fraction.

2 Check if the denominators are the same (no). Identify the LCD (6) and write each fraction as an equivalent fraction with a denominator of 6.

3 Add the numerators together. The denominator stays the same.

4 Convert to a mixed number.

b **1** Convert each mixed number to an improper fraction.

2 Check if the denominators are the same (no). Identify the LCD (20) and write equivalent fractions with a denominator of 20.

3 Subtract the numerators. The denominator stays the same.

WRITE

a $4\frac{1}{2}+5\frac{2}{3}$
$=\frac{9}{2}+\frac{17}{3}$
$=\frac{27}{6}+\frac{34}{6}$
$=\frac{61}{6}$
$=10\frac{1}{6}$

b $3\frac{7}{10}-2\frac{3}{4}$
$=\frac{37}{10}-\frac{11}{4}$
$=\frac{74}{20}-\frac{55}{20}$
$=\frac{19}{20}$

UNDERSTANDING AND FLUENCY

8 Calculate:

a $2\frac{2}{3} + 1\frac{1}{2}$ **b** $4\frac{4}{7} - 1\frac{2}{3}$ **c** $3\frac{2}{5} + 2\frac{1}{4}$

d $5\frac{2}{3} - 3\frac{4}{5}$ **e** $1\frac{3}{5} + 7\frac{1}{2}$ **f** $2\frac{1}{3} + 3\frac{3}{4}$

g $6\frac{5}{6} - 4\frac{3}{10}$ **h** $7\frac{1}{4} - 3\frac{5}{6}$ **i** $3\frac{1}{7} + 2\frac{2}{21}$

j $1\frac{5}{6} + 5\frac{1}{2}$ **k** $6\frac{1}{8} - 4\frac{3}{4}$ **l** $4\frac{1}{6} - 2\frac{7}{24}$

9 Calculate:

a $1\frac{2}{3} + \frac{8}{7} + \frac{5}{2}$ **b** $\frac{8}{3} + 5 + 1\frac{1}{8}$ **c** $\frac{9}{2} + 4\frac{3}{7} - 2\frac{1}{6}$

d $\frac{28}{5} - 4\frac{1}{7} + \frac{1}{2}$ **e** $\frac{14}{3} - \frac{16}{9} + 2\frac{4}{5}$ **f** $3\frac{5}{12} - \frac{10}{5} + \frac{17}{6}$

g $3\frac{1}{11} + 2\frac{3}{4} + 1\frac{1}{2}$ **h** $1\frac{1}{7} + 2\frac{1}{2} - 3\frac{5}{14}$ **i** $4\frac{1}{5} + 1\frac{7}{25} - \frac{5}{6}$

PROBLEM SOLVING AND REASONING

10 A factory worker is employed to make t-shirts. On average, each t-shirt takes $3\frac{1}{2}$ minutes for cutting, $2\frac{4}{5}$ minutes for sewing and $4\frac{1}{6}$ minutes for finishing. What is the total time required to make one t-shirt?

11 Ramona has a part-time job in a café. This table displays the hours she worked in one week of the school holidays.

Day	Number of hours and minutes worked	Number of minutes worked	Number of hours worked
Monday	3 h 45 min		
Tuesday	1 h 50 min		
Wednesday		330	
Thursday		420	
Friday	10 h 30 min		
Saturday			$8\frac{1}{6}$

a Copy the table.

b Complete the second and third columns of the table by converting the hours and minutes worked into minutes and vice versa.

c Write the time worked each day in hours in simplest fractional form and complete the last column of the table. (Hint: write the minutes as a fraction of an hour.)

d How many hours (in simplest fractional form) has Ramona spent working at the café in this week?

12 A group of friends organise to catch up for lunch and decide to order pizza. Each person estimates what fraction of a pizza he or she can eat. Luisa can eat $\frac{1}{3}$, Declan $\frac{1}{2}$, Nisa $\frac{3}{8}$, Tomas $\frac{2}{3}$ and Christopher $\frac{3}{2}$ of a pizza.

a How many full pizzas will need to be ordered?

b What fraction of a pizza will be left over?

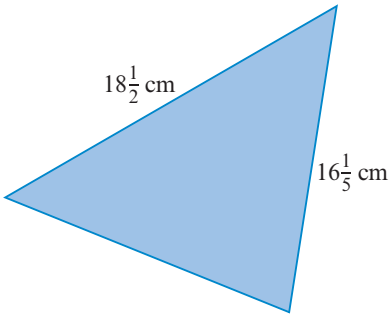
13 Laila is planning to travel overseas at the end of the year and needs to save money so she devises a budget. She decides to allocate $\frac{2}{5}$ of her weekly pay to living expenses, $\frac{1}{3}$ to rent, $\frac{1}{8}$ to going out and the remainder to savings.

a Calculate what fraction of Laila's pay will go towards living expenses, rent and going out.

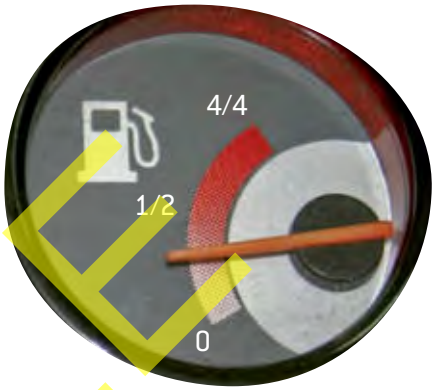
b What fraction of her pay will go towards savings? Explain your answer.

PROBLEM SOLVING AND REASONING

14 The total length around the outside of the triangle at right is $49\frac{2}{5}$ cm.
The length measurements of two of the sides are known. What is the length of the unknown side?



15 Jason checked the petrol gauge on his motorbike at the beginning of the day and it showed the tank was $\frac{9}{10}$ full. At the end of the day, the petrol gauge showed the tank was $\frac{1}{2}$ full. What fraction of petrol was used?



16 While on camp, students were required to complete a 21 km bushwalk over three days. They covered $8\frac{2}{5}$ km on the first day and $7\frac{1}{3}$ km on the second day.

a What distance will they need to walk on the third day?

b How much further did they walk on the first day than on the second?

17 **a** Calculate each of these and describe what you see.

i $\frac{1}{2} - \frac{1}{4}$ **ii** $\frac{1}{2} - \frac{1}{4} - \frac{1}{8}$ **iii** $\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}$ **iv** $\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32}$

b Predict the answer for the next two calculations in the pattern. Test your prediction.

18 Complete each of these by filling in the gaps. Explain the pattern you see.

a $\frac{2}{7}, \frac{5}{7}, \frac{8}{7}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_$ **b** $10, 11\frac{2}{5}, 12\frac{4}{5}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_$

c $\frac{19}{3}, \frac{16}{3}, \frac{13}{3}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_$ **d** $12\frac{14}{15}, 10\frac{10}{15}, 8\frac{6}{15}, \_\_\_\_, \_\_\_\_, \_\_\_\_$

CHALLENGE

19 Find the fraction that is halfway between each pair of fractions.

a $\frac{3}{10}$ and $\frac{7}{10}$ **b** $\frac{3}{5}$ and $\frac{4}{5}$ **c** $\frac{4}{11}$ and $\frac{5}{11}$ **d** $\frac{3}{7}$ and $\frac{6}{7}$

e $\frac{1}{6}$ and $\frac{1}{4}$ **f** $\frac{2}{5}$ and $\frac{2}{3}$ **g** $\frac{1}{4}$ and $\frac{1}{3}$ **h** $\frac{1}{10}$ and $\frac{1}{8}$

20 Find a pair of fractions with different denominators that:

a add to give 1 **b** add to give $\frac{5}{8}$ **c** have a difference of $\frac{1}{5}$.

21 The sum of three fractions is $\frac{25}{6}$. Given that one of the fractions is $\frac{10}{3}$, find the other two fractions. (Note: there is more than one answer.)

22 Use a single pair of brackets to make each equation true.

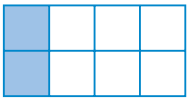
a $\frac{1}{2} - \frac{5}{8} - \frac{2}{5} = \frac{11}{40}$ **b** $\frac{19}{28} - \frac{3}{7} + \frac{3}{14} = \frac{1}{28}$

Reflect

Explain why you cannot add or subtract the numerators of fractions that have different denominators.

2D Multiplying fractions

Start thinking!



1 One quarter of this rectangle has been shaded blue.

a Explain how this diagram shows $\frac{1}{4}$?

b Can you think of a way to show half of one quarter on this diagram?

2 The dark blue shading on this diagram shows half of one quarter.

a What fraction of the rectangle has been shaded dark blue?

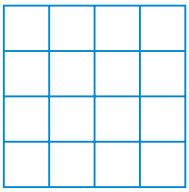
b What is $\frac{1}{2}$ of $\frac{1}{4}$? (Hint: Use your answer to part **a**.)

c Another way to write this is: What is $\frac{1}{2} \times \frac{1}{4}$? Copy and complete: $\frac{1}{2} \times \frac{1}{4} =$ ____.

3 **a** Copy the diagram from question **1** and shade three quarters of the rectangle.

b Highlight half of the shading in a different colour to show half of three quarters.

c What is $\frac{1}{2} \times \frac{3}{4}$?



4 **a** Copy this diagram and shade three quarters of it.

b Highlight a quarter of the shaded area in a different colour to show a quarter of three quarters.

c What is $\frac{1}{4} \times \frac{3}{4}$?

5 Can you see a pattern to follow when multiplying two fractions? Write a description and compare it with a friend's.

KEY IDEAS

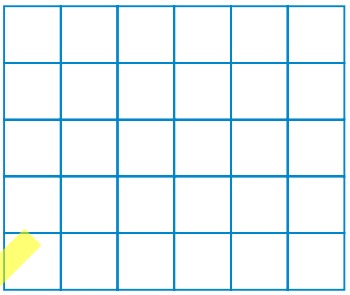
- To multiply fractions, follow these steps.
  1. First look for any factors common to the numerator and denominator.
  2. Cancel a number in the numerator and a number in the denominator by dividing both numbers by the HCF. This will simplify the calculation.

     Cancelling can occur vertically; for example, $\frac{\overset{1}{\cancel{3}}}{\underset{6}{\cancel{30}}} \times \frac{7}{8}$ or diagonally; for example, $\frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{13}$.
  3. Then multiply the numerators together and multiply the denominators together.
- The denominators do not need to be the same when multiplying fractions.
- Mixed numbers must be converted to improper fractions before multiplying.
- Whole numbers can be written as fractions with a denominator of 1. For example, $8 = \frac{8}{1}$.

EXERCISE 2D Multiplying fractions

UNDERSTANDING AND FLUENCY

1 **a** Copy this diagram.



b Shade $\frac{1}{6}$ of the rectangle.

c Using a different colour, shade $\frac{1}{5}$ of $\frac{1}{6}$.

d What fraction of the diagram does $\frac{1}{5}$ of $\frac{1}{6}$ represent?

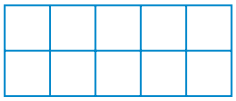
e What is $\frac{1}{5} \times \frac{1}{6}$?

f Repeat parts **a**–**e** for $\frac{2}{5}$ of $\frac{5}{6}$.

g Copy and complete: In mathematics, the word *of* can be replaced by _____.

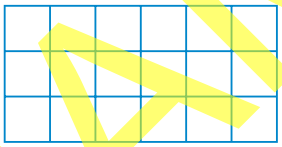
2 Use the diagrams below and the method in question **1** to work out each multiplication problem.

a $\frac{1}{2} \times \frac{3}{5}$

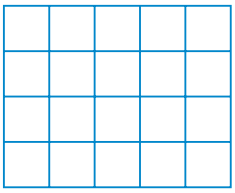


b $\frac{1}{3} \times \frac{1}{4}$

c $\frac{1}{3} \times \frac{5}{6}$



d $\frac{3}{4} \times \frac{2}{5}$



EXAMPLE 2D-1 Multiplying fractions

Calculate $\frac{3}{5} \times \frac{2}{7}$.

THINK

1 Look for any common factors between numerators and denominators (none).

2 Multiply the numerators together ($3 \times 2 = 6$) and then multiply the denominators together ($5 \times 7 = 35$).

WRITE

$\frac{3}{5} \times \frac{2}{7}$

$= \frac{6}{35}$

3 Calculate:

a $\frac{1}{5} \times \frac{1}{2}$ **b** $\frac{2}{3} \times \frac{1}{7}$ **c** $\frac{3}{11} \times \frac{2}{5}$

d $\frac{1}{3} \times \frac{4}{9}$ **e** $\frac{3}{8} \times \frac{1}{2}$ **f** $\frac{11}{13} \times \frac{6}{7}$

g $\frac{2}{5} \times \frac{4}{5}$ **h** $\frac{1}{3} \times \frac{7}{15}$ **i** $\frac{2}{9} \times \frac{5}{9}$

j $\frac{4}{11} \times \frac{5}{9}$ **k** $\frac{3}{10} \times \frac{7}{8}$ **l** $\frac{5}{13} \times \frac{8}{9}$

4 Check your answers to question **2** using the method shown in Example 2D-1.
Calculate:

a $\frac{1}{2} \times \frac{3}{5}$ **b** $\frac{1}{3} \times \frac{1}{4}$

c $\frac{1}{3} \times \frac{5}{6}$ **d** $\frac{3}{4} \times \frac{2}{5}$

EXAMPLE 2D-2 Multiplying fractions with cancelling

Calculate $\frac{8}{5} \times \frac{25}{12}$.

THINK

1 Look for any common factors between numerators and denominators. The numbers 8 and 12 have an HCF of 4. The numbers 25 and 5 have an HCF of 5.

2 Cancel 8 and 12 by dividing both numbers by 4 (shown in green). Then cancel 25 and 5 by dividing both numbers by 5 (shown in blue).

3 Write the result obtained after cancelling.

4 Multiply the numerators together ($2 \times 5 = 10$) and then multiply the denominators together ($1 \times 3 = 3$).

5 Convert the improper fraction to a mixed number.

WRITE

$\frac{8}{5} \times \frac{25}{12}$

$= \frac{{}^{2}\cancel{8}}{{}_{1}\cancel{5}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$

$= \frac{2}{1} \times \frac{5}{3}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

UNDERSTANDING AND FLUENCY

5 Calculate:

a $\frac{8}{9} \times \frac{1}{4}$ **b** $\frac{2}{3} \times \frac{6}{11}$ **c** $\frac{3}{5} \times \frac{10}{13}$

d $\frac{1}{8} \times \frac{4}{17}$ **e** $\frac{7}{9} \times \frac{12}{35}$ **f** $\frac{20}{27} \times \frac{9}{15}$

g $\frac{3}{4} \times \frac{18}{24}$ **h** $\frac{9}{35} \times \frac{20}{21}$ **i** $\frac{14}{28} \times \frac{8}{13}$

j $\frac{8}{12} \times \frac{2}{5}$ **k** $\frac{10}{11} \times \frac{22}{15}$ **l** $\frac{6}{5} \times \frac{4}{12}$

m $\frac{35}{17} \times \frac{6}{42}$ **n** $\frac{21}{25} \times \frac{15}{12}$ **o** $\frac{20}{50} \times \frac{3}{18}$

EXAMPLE 2D-3 Multiplying mixed numbers

Calculate $1\frac{2}{3} \times 2\frac{7}{10}$.

THINK	WRITE
	$1\frac{2}{3} \times 2\frac{7}{10}$
1 Convert each mixed number to an improper fraction.	$= \frac{5}{3} \times \frac{27}{10}$
2 Cancel 5 and 10 by dividing both numbers by 5 (shown in green). Then cancel 3 and 27 by dividing both numbers by 3 (shown in blue).	$= \frac{{}^{1}\cancel{5}}{\cancel{3}_{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}_{2}}$
3 Write the result obtained after cancelling.	$= \frac{1}{1} \times \frac{9}{2}$
4 Multiply the numerators together ($1 \times 9 = 9$) and then multiply the denominators together ($1 \times 2 = 2$).	$= \frac{9}{2}$
5 Convert the improper fraction to a mixed number.	$= 4\frac{1}{2}$

UNDERSTANDING AND FLUENCY

6 Calculate:

a $\frac{3}{5} \times 1\frac{4}{11}$ **b** $\frac{22}{4} \times \frac{6}{7}$ **c** $1\frac{13}{20} \times \frac{7}{11}$ **d** $4\frac{2}{3} \times 3$

e $\frac{7}{9} \times 3\frac{5}{7}$ **f** $5\frac{2}{3} \times 2\frac{2}{5}$ **g** $5\frac{3}{5} \times 3\frac{1}{4}$ **h** $3\frac{1}{3} \times 4\frac{1}{8}$

7 Calculate each of these. (Hint: a whole number can be written as a fraction with a denominator of 1.)

a $\frac{3}{8} \times 24$ **b** $\frac{5}{11} \times 66$ **c** $72 \times \frac{4}{9}$ **d** $\frac{5}{2} \times 16$

e $\frac{7}{6} \times 18$ **f** $3\frac{1}{6} \times 24$ **g** $\frac{12}{15} \times 4$ **h** $5 \times \frac{16}{25}$

i $9 \times \frac{1}{24}$ **j** $6 \times \frac{5}{36}$ **k** $\frac{8}{3} \times 15$ **l** $9 \times \frac{4}{81}$

8 Refer to your answers to question **7**. Copy and complete the following using the words *larger* or *smaller*.

When multiplying a whole number by a proper fraction the answer is always _____ than the whole number.

When multiplying a whole number by an improper fraction or mixed number the answer is always _____ than the whole number.

9 Calculate:

a $\frac{5}{9} \times \frac{9}{10} \times \frac{12}{17}$ **b** $\frac{3}{8} \times \frac{15}{26} \times \frac{8}{9}$ **c** $\frac{28}{5} \times \frac{11}{6} \times \frac{24}{7}$ **d** $1\frac{1}{12} \times \frac{8}{3} \times \frac{15}{4}$

e $\frac{22}{13} \times 10\frac{2}{5} \times 2\frac{1}{11}$ **f** $4\frac{1}{2} \times 5\frac{2}{5} \times \frac{21}{8}$ **g** $\frac{12}{5} \times 7\frac{1}{2} \times 3\frac{1}{4}$ **h** $6\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{2}{7}$

UNDERSTANDING AND FLUENCY

10 Look at these Vespa motorscooters.

a How could you use this diagram to explain that '$\frac{1}{4}$ of the Vespas are red'?

b How many Vespas are there?

c How many Vespas are red?

d What is the relationship between $\frac{1}{4}$ and the answers obtained in parts **b** and **c**?

e Using your answers to parts **b** and **c**, copy and complete this statement: $\frac{1}{4}$ of 12 equals _____.

f Rewrite the statement in part **e** as a mathematical equation. What operation has replaced the word *of*?

11 Calculate:

a $\frac{2}{5}$ of $\frac{1}{8}$ **b** $\frac{3}{4}$ of $\frac{3}{7}$ **c** $\frac{2}{11}$ of $\frac{7}{9}$ **d** $\frac{9}{4}$ of $\frac{5}{11}$ **e** $2\frac{7}{13}$ of 26 **f** $5\frac{1}{2}$ of $3\frac{1}{3}$

PROBLEM SOLVING AND REASONING

12 Hilaire works at the local swimming pool as a lifeguard. Her shift begins at 3.45 pm and ends at 6.15 pm.

a Write the hours Hilaire works in one shift as:

i a mixed number **ii** an improper fraction.

b If Hilaire has the same shift five nights a week, calculate the total number of hours she works as a fraction.

13 Whilst correcting his students' homework, a teacher found three different answers to the question 'Find $2\frac{4}{9} \times 3\frac{3}{8}$ in simplest form'. The answers given were: $6\frac{12}{72}$, $8\frac{1}{4}$ and $6\frac{12}{17}$.

a Which answer is correct?

b Comment on the error that is likely to have been made in each case.

EXAMPLE 2D-4 Multiplying fractions with unit conversions

Calculate $\frac{1}{5}$ of 2 hours. Give your answer in minutes.

THINK	WRITE
1 Convert the number of hours into minutes.	$\frac{1}{5}$ of 2 hours $= \frac{1}{5}$ of 120 minutes
2 Replace 'of' with '×' and cancel any common factors.	$= \frac{1}{5} \times 120$ $= \frac{1}{\cancel{5}_{1}} \times \frac{\cancel{120}^{24}}{1}$
3 Multiply numerators and denominators.	$= \frac{24}{1}$
4 Write your answer.	= 24 minutes

PROBLEM SOLVING AND REASONING

14 Calculate each fraction amount in the units shown in the brackets.

a $\frac{1}{4}$ of 3 hours (minutes)

b $\frac{2}{3}$ of 1 year (months)

c $\frac{4}{5}$ of \$80 (dollars)

d $\frac{6}{13}$ of the year (weeks)

e $\frac{2}{5}$ of 2 minutes (seconds)

f $\frac{3}{4}$ of 100 L (litres)

g $\frac{3}{7}$ of 2 weeks (days)

h $\frac{9}{4}$ of \$200 (dollars)

i $\frac{2}{3}$ of $6\frac{1}{2}$ hours (hours)

j $\frac{7}{5}$ of $11\frac{1}{4}$ km (km)

k $\frac{5}{6}$ of 10 years (months)

l $\frac{4}{7}$ of \$2.80 (cents)

15 Amy baked a batch of 36 cookies and arranged them on a tray in a six by six pattern.

a Draw a diagram of the arrangement and shade $\frac{5}{6}$ of the diagram. How many cookies does this represent?

b Amy decides to take $\frac{2}{3}$ of the cookies to school and share them with her classmates. How many cookies will she take to school?

c When she reached school, Amy realised that $\frac{1}{8}$ of the cookies were broken in her bag. How many cookies were broken?

16 A store takes $\frac{2}{5}$ off the price of its televisions during the stock-take sales. What will an LCD television normally sold for \$6320 now cost?

17 Your friend manages to complete $\frac{3}{8}$ of the $5\frac{1}{2}$ km run before stopping. How far did she run prior to stopping?

CHALLENGE

18 Boris spent $\frac{4}{5}$ of an hour training for his upcoming tennis match. Roger spent $3\frac{1}{2}$ times longer than Boris in preparing for his match.

a How long did each boy spend training?

b How much longer than Boris did Roger spend training?

19 When two fractions are multiplied, their product (or answer) is $\frac{5}{7}$. What possible values could the two fractions be? List three pairs of possible fractions.

20 Calculate:

a $3\frac{1}{5} \times \frac{3}{8} - \frac{2}{7}$

b $\frac{2}{7} + 3\frac{1}{2} \times \frac{3}{7}$

c $\left(\frac{3}{2} + 5\frac{1}{3}\right) \times 4\frac{1}{7}$

Reflect

Explain why it is useful to first simplify fractions before multiplying them.

2E Dividing fractions

Start thinking!

1 Look at the rectangles below.

a How many lots of $\frac{1}{4}$ are in one whole?

b How many lots of $\frac{1}{4}$ are in two wholes?

c How many lots of $\frac{1}{4}$ are in three wholes?

2 What do you notice about your answers to question **1**?

3 Repeat questions **1** and **2**, but this time draw rectangles that are divided into fifths. Consider how many lots of $\frac{1}{5}$ are in one whole, two wholes and three wholes.

4 Asking how many lots of $\frac{1}{4}$ are in one whole is the same as asking 'What is 1 is divided by $\frac{1}{4}$?' This can be written as $1 \div \frac{1}{4}$.

Use the diagrams and your results for questions **1** and **3** to answer the following.

a $1 \div \frac{1}{4} =$ **b** $2 \div \frac{1}{4} =$ **c** $3 \div \frac{1}{4} =$ **d** $1 \div \frac{1}{5} =$ **e** $2 \div \frac{1}{5} =$ **f** $3 \div \frac{1}{5} =$

5 Copy and complete these sentences.

a Dividing a number by $\frac{1}{4}$ is the same as _____ that number by 4.

b Dividing a number by $\frac{1}{5}$ is the same as _____ that number by _____.

KEY IDEAS

- To divide fractions, follow these steps.
  1. Change from a division to a multiplication problem by replacing the division sign with a multiplication sign and turning the fraction that follows upside down.
  2. Proceed as for a multiplication problem by first cancelling, if possible, then multiplying the numerators together and the denominators together (see Example A).

  The process of turning a fraction upside down is called taking the **reciprocal** (see Example B).
- To divide mixed numbers, first convert them to improper fractions.
- Where appropriate, convert the answer to a mixed number.

Example A

$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3}$

$= \frac{14}{15}$

Example B

The reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$.

The reciprocal of 4 is $\frac{1}{4}$ (since 4 can be written as $\frac{4}{1}$).

The reciprocal of $2\frac{1}{5}$ is $\frac{5}{11}$ (since $2\frac{1}{5}$ when written as an improper fraction is $\frac{11}{5}$).

EXERCISE 2E Dividing fractions

1 Rewrite each division problem in the form 'How many lots of ____ are there in ____?'

a $2 \div \frac{1}{8}$ **b** $4 \div \frac{1}{3}$ **c** $\frac{1}{2} \div \frac{1}{12}$ **d** $\frac{1}{3} \div \frac{1}{9}$

2 Draw a diagram to help you find the answer to each division problem in question **1**.

3 Write each division problem in question **1** as a multiplication problem.

4 Check your answers to question **2** by working out the results to question **3**. Remember that whole numbers can be written with a denominator of 1.

EXAMPLE 2E-1 Writing reciprocals

Write the reciprocal of each of these.

a $\frac{3}{7}$ **b** $\frac{1}{8}$ **c** 5

THINK

a Turn the fraction 'upside down'. That is, swap the numerator and the denominator.

b Turn the fraction 'upside down'.

c Write 5 as a fraction and then turn the fraction 'upside down'.

WRITE

a The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$.

b The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$ or 8.

c $5 = \frac{5}{1}$. The reciprocal of 5 is $\frac{1}{5}$.

5 Write the reciprocal of each of these.

a $\frac{4}{9}$ **b** $\frac{11}{3}$ **c** $\frac{2}{5}$ **d** $\frac{27}{4}$ **e** $\frac{1}{100}$ **f** 8 **g** $\frac{34}{1}$ **h** $\frac{65}{31}$

EXAMPLE 2E-2 Dividing fractions

Calculate $\frac{1}{7} \div \frac{3}{5}$.

THINK

1 Change the division problem to a multiplication problem. Replace ÷ with × and turn the fraction that follows the division sign upside down.

2 Multiply the numerators together and then multiply the denominators together.

WRITE

$\frac{1}{7} \div \frac{3}{5}$

$= \frac{1}{7} \times \frac{5}{3}$

$= \frac{5}{21}$

UNDERSTANDING AND FLUENCY

6 Calculate:

a $\frac{1}{8} \div \frac{2}{5}$ **b** $\frac{3}{4} \div \frac{3}{10}$ **c** $\frac{1}{12} \div \frac{1}{11}$

d $\frac{1}{9} \div \frac{4}{17}$ **e** $\frac{1}{10} \div \frac{4}{25}$ **f** $\frac{6}{7} \div \frac{8}{21}$

g $\frac{2}{3} \div \frac{8}{9}$ **h** $\frac{9}{14} \div \frac{4}{7}$ **i** $\frac{3}{4} \div \frac{5}{8}$

j $\frac{5}{9} \div \frac{10}{3}$ **k** $\frac{7}{10} \div \frac{5}{4}$ **l** $\frac{4}{11} \div \frac{3}{22}$

m $\frac{6}{7} \div \frac{2}{9}$ **n** $\frac{16}{15} \div \frac{12}{5}$ **o** $\frac{3}{8} \div \frac{9}{20}$

p $\frac{1}{24} \div \frac{13}{18}$ **q** $\frac{14}{3} \div \frac{7}{11}$ **r** $\frac{4}{13} \div \frac{20}{3}$

7 Calculate:

a $6 \div \frac{1}{7}$ **b** $14 \div \frac{7}{6}$ **c** $12 \div \frac{8}{9}$

d $5 \div \frac{10}{13}$ **e** $\frac{2}{9} \div 6$ **f** $\frac{3}{4} \div 9$

g $\frac{5}{2} \div 10$ **h** $\frac{8}{9} \div 2$ **i** $4 \div \frac{7}{5}$

EXAMPLE 2E-3 Dividing mixed numbers

Calculate $4\frac{3}{8} \div 2\frac{6}{7}$.

THINK	WRITE
1 Convert each mixed number to an improper fraction.	$4\frac{3}{8} \div 2\frac{6}{7}$ $= \frac{35}{8} \div \frac{20}{7}$
2 Change to a multiplication problem by replacing ÷ with × and turning the fraction that follows upside down.	$= \frac{35}{8} \times \frac{7}{20}$
3 Cancel 35 and 20 by dividing each by the HCF of 5.	$= \frac{\overset{7}{\cancel{35}}}{8} \times \frac{7}{\underset{4}{\cancel{20}}}$
4 Write the result after cancelling.	$= \frac{7}{8} \times \frac{7}{4}$
5 Multiply the numerators together and then multiply the denominators together.	$= \frac{49}{32}$
6 Change the improper fraction to a mixed number.	$= 1\frac{17}{32}$

8 Calculate:

a $3\frac{1}{2} \div \frac{4}{5}$ **b** $1\frac{4}{7} \div \frac{8}{11}$ **c** $6\frac{1}{3} \div 1\frac{1}{4}$

d $7\frac{1}{5} \div 2\frac{3}{4}$ **e** $1\frac{2}{7} \div \frac{15}{21}$ **f** $2\frac{7}{9} \div \frac{4}{18}$

g $\frac{13}{4} \div 2\frac{6}{10}$ **h** $3\frac{3}{5} \div 1\frac{4}{5}$ **i** $4 \div 1\frac{1}{5}$

j $2\frac{1}{3} \div 5$ **k** $9\frac{1}{11} \div 10$ **l** $7\frac{3}{5} \div 19$

PROBLEM SOLVING AND REASONING

9 Calculate:

a $\frac{15}{27} \div \frac{11}{21}$ **b** $\frac{13}{24} \div \frac{9}{40}$ **c** $2\frac{1}{12} \div 1\frac{5}{6}$

d $1\frac{1}{8} \div 10\frac{1}{2}$ **e** $3\frac{2}{10} \div 1\frac{1}{3}$ **f** $\frac{5}{27} \div 2\frac{7}{9}$

g $12\frac{2}{5} \div 8\frac{4}{15}$ **h** $3\frac{3}{20} \div 4\frac{3}{10}$ **i** $5\frac{7}{9} \div 6\frac{2}{3}$

10 When dividing a mixed number by a proper fraction, do you obtain a smaller or larger number? Explain.

11 The ingredients to make a sticky date pudding for four people are shown below.

$\frac{3}{4}$ cup of chopped pitted dates
1 teaspoon of bicarbonate of soda
60 g butter
$\frac{2}{3}$ cup of sugar
2 eggs
1 cup of self-raising flour

a A triple mixture of sticky date pudding is needed to feed 12 people. How much of each ingredient would be required?

b To feed six people, a one-and-a-half mixture of sticky date pudding is needed. How much of each ingredient is required?

c How much of each ingredient is required for the recipe to feed:

i two people?

ii three people?

12 A batch of a chocolate pudding recipe requires $\frac{1}{6}$ kg of sugar. How many batches of chocolate pudding can be made from this 1 kg packet of sugar?

13 Over the school holidays Brianna worked a total of 44 hours stacking shelves at the supermarket. If she worked $5\frac{1}{2}$ hour shifts, how many shifts did she work?

14 Students in a technology and design class are making costumes for the school production. A total of 35 m of fabric has been purchased. If each garment requires $1\frac{3}{4}$ m of fabric, how many garments can be made?

15 Rhys shares $3\frac{3}{8}$ pizzas equally between himself and eight friends. What fraction of a pizza will each person receive?

16 A new housing estate is to be developed on $43\frac{1}{5}$ hectares of land. If each block measures $\frac{3}{5}$ hectares, how many blocks will there be?

17 Madusha and Cynthia have been training for their school's upcoming walkathon. It takes them $\frac{1}{10}$ of an hour to complete one lap.

a If the time allowed for the walkathon is $7\frac{1}{2}$ hours, what is the maximum number of laps the girls could complete?

b Do you think the girls will complete the maximum number of laps? Explain.

18 Gianluca must pack 15 kg of apples into plastic bags that can hold a maximum of $1\frac{1}{3}$ kg.

a How many $1\frac{1}{3}$ bags can he fill?

b How many kilograms of apples will be left over?

19 After dividing two proper fractions, a mixed number is obtained. Write three possible division problems that show this.

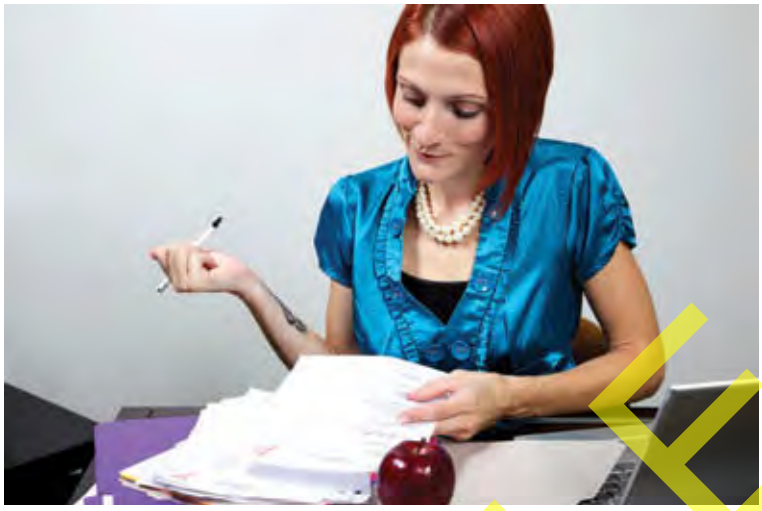
20 After dividing two proper fractions, a whole number is obtained. Write three possible division problems that show this.

21 After dividing two mixed numbers, a proper fraction is obtained. Write three possible division problems that show this.

22 After one fraction is divided by another, the answer is $\frac{5}{8}$. Write three possible division problems that show this.

CHALLENGE

23 While marking test papers, a teacher obtained three different answers to the question 'Divide $4\frac{1}{14}$ into $2\frac{5}{7}$'.



The answers obtained were $\frac{2}{3}$, $\frac{1}{6}$ and $\frac{1083}{98}$.

a Which is the correct answer?

b Explain how the two incorrect answers might have been obtained.

24 Use what you have learnt from question **23** to help you answer the following.

a Divide $\frac{2}{3}$ into $1\frac{5}{12}$.

b Divide $\frac{8}{9}$ into $6\frac{2}{3}$.

c Divide $1\frac{2}{5}$ into $\frac{8}{5}$.

d Divide $12\frac{3}{8}$ into $\frac{11}{8}$.

e Divide 3 into $2\frac{2}{15}$.

f Divide 8 into $5\frac{2}{6}$.

g Divide $\frac{8}{21}$ into $1\frac{1}{7}$.

h Divide $4\frac{1}{4}$ into $5\frac{2}{3}$.

25 Calculate each of these.
Remember to think carefully about which operations are performed first.

a $\frac{5}{3} \div 1\frac{3}{7} + 1\frac{3}{4}$

b $2\frac{1}{3} \div \left(2\frac{2}{4} \div 1\frac{4}{5}\right)$

c $\frac{7}{3} + 2\frac{1}{2} \times \frac{7}{10} \div 1\frac{1}{2}$

d $\left(\frac{4}{5} - \frac{2}{3}\right) \div \left(1\frac{1}{3} + 3\frac{2}{9}\right) \times \left(5 \div \frac{2}{3}\right)$

e $\left(1\frac{5}{7} \times 1\frac{3}{4}\right) + \left(\frac{13}{3} - 3\frac{2}{3}\right) \div \left(2\frac{1}{3} \times \frac{1}{7}\right)$

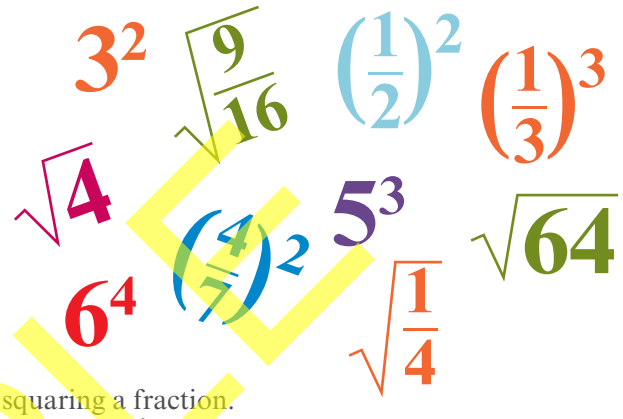
f $\left(1\frac{1}{2} - \frac{11}{12}\right) + \frac{5}{7} \times \left(\frac{1}{4} \div \frac{3}{7}\right)$

Reflect

What would you tell your friend it's important to remember when dividing fractions?

2F Powers and square roots of fractions

Start thinking!



1 Calculate these, then explain how you got your answers.

a 2^2 **b** 5^2 **c** 9^2 **d** 10^2

2 Calculate:

a $\frac{1}{2} \times \frac{1}{2}$ **b** $\frac{1}{5} \times \frac{1}{5}$ **c** $\frac{1}{9} \times \frac{1}{9}$ **d** $\frac{1}{10} \times \frac{1}{10}$

3 Write what you expect the answer to be for each of these.

a $\left(\frac{1}{2}\right)^2$ **b** $\left(\frac{1}{5}\right)^2$ **c** $\left(\frac{1}{9}\right)^2$ **d** $\left(\frac{1}{10}\right)^2$

4 Copy and complete each of these to show two methods of squaring a fraction.

a $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2 = \frac{\square^2}{\square^2}$

$= \_\_\_\_$ $= \frac{1}{4}$

b $\left(\frac{1}{5}\right)^2 = \frac{1}{5} \times \frac{1}{5}$ or $\left(\frac{1}{5}\right)^2 = \frac{\square^2}{\square^2}$

$= \_\_\_\_$ $= \frac{1}{25}$

5 Calculate these, then explain how you got your answers.

a $\sqrt{4}$ **b** $\sqrt{25}$ **c** $\sqrt{81}$ **d** $\sqrt{100}$

6 Use your answers to question **5** to help you calculate each of these.

a $\sqrt{\frac{1}{4}}$ **b** $\sqrt{\frac{1}{25}}$ **c** $\sqrt{\frac{1}{81}}$ **d** $\sqrt{\frac{1}{100}}$

7 Copy and complete the following to show how to calculate the square root of each fraction.

a $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}}$

$= \frac{1}{\square}$

b $\sqrt{\frac{1}{25}} = \frac{\sqrt{1}}{\sqrt{\square}}$

$= \frac{1}{\square}$

KEY IDEAS

- A fraction can be squared in two ways.
  1. Multiply the fraction by itself. For example, $\left(\frac{7}{9}\right)^2 = \frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$.
  2. Square the numerator and then square the denominator. For example, $\left(\frac{7}{9}\right)^2 = \frac{7^2}{9^2} = \frac{49}{81}$.
- To obtain the square root of a fraction, find the square root of the numerator and then find the square root of the denominator. For example, $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$.
- A mixed number must first be converted to an improper fraction before squaring or finding the square root. Where possible, the final answer can then be written as a mixed number.
- Fractions can also be raised to higher powers. For example, $\left(\frac{4}{7}\right)^3 = \frac{4^3}{7^3} = \frac{64}{343}$ and $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$.

EXERCISE 2F Powers and square roots of fractions

EXAMPLE 2F-1 Squaring fractions

Calculate $(\frac{3}{5})^2$.

THINK

1 Square the numerator and square the denominator. (Alternatively, write as the multiplication of two fractions.)

2 Calculate the numerator and then the denominator.

WRITE

$(\frac{3}{5})^2$

$= \frac{3^2}{5^2}$ or $\frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

1 Calculate:

a $(\frac{2}{5})^2$ **b** $(\frac{3}{8})^2$ **c** $(\frac{9}{7})^2$ **d** $(\frac{10}{3})^2$

e $(\frac{4}{5})^2$ **f** $(\frac{11}{6})^2$ **g** $(\frac{2}{11})^2$ **h** $(\frac{5}{7})^2$

EXAMPLE 2F-2 Squaring mixed numbers

Calculate $(2\frac{1}{3})^2$.

THINK

1 Convert the mixed number to an improper fraction.

2 Square the numerator and square the denominator. (Alternatively, write as the multiplication of two fractions.)

3 Calculate the numerator and then the denominator.

4 Convert the improper fraction to a mixed number.

WRITE

$(2\frac{1}{3})^2$

$= (\frac{7}{3})^2$ or $= (\frac{7}{3})^2$

$= \frac{7^2}{3^2}$ $= \frac{7}{3} \times \frac{7}{3}$

$= \frac{49}{9}$ $= \frac{49}{9}$

$= 5\frac{4}{9}$ $= 5\frac{4}{9}$

2 Calculate:

a $(2\frac{3}{5})^2$ **b** $(6\frac{1}{4})^2$ **c** $(1\frac{5}{7})^2$ **d** $(3\frac{1}{2})^2$

e $(2\frac{2}{3})^2$ **f** $(1\frac{3}{8})^2$ **g** $(1\frac{3}{7})^2$ **h** $(2\frac{6}{7})^2$

EXAMPLE 2F-3 Finding the square root of fractions

Calculate $\sqrt{\frac{100}{144}}$.

THINK

1 Write as the square root of the numerator divided by the square root of the denominator.

2 Calculate the square root of the numerator and the square root of the denominator.

3 Simplify by cancelling. Divide the numerator and the denominator by the HCF of 2.

4 Write the answer.

WRITE

$\sqrt{\frac{100}{144}}$

$= \frac{\sqrt{100}}{\sqrt{144}}$

$= \frac{10}{12}$

$= \frac{10^{5}}{12_{6}}$

$= \frac{5}{6}$

3 Calculate:

a $\sqrt{\frac{4}{81}}$ b $\sqrt{\frac{25}{64}}$ c $\sqrt{\frac{9}{121}}$ d $\sqrt{\frac{49}{64}}$

e $\sqrt{\frac{64}{144}}$ f $\sqrt{\frac{9}{81}}$ g $\sqrt{\frac{16}{25}}$ h $\sqrt{\frac{36}{100}}$

EXAMPLE 2F-4 Find the square root of mixed numbers

Calculate $\sqrt{11\frac{1}{9}}$.

THINK

1 Convert the mixed number to an improper fraction.

2 Write as the square root of the numerator divided by the square root of the denominator.

3 Calculate the square root of the numerator and the square root of the denominator.

4 Write the answer as a mixed number.

WRITE

$\sqrt{11\frac{1}{9}}$

$= \sqrt{\frac{100}{9}}$

$= \frac{\sqrt{100}}{\sqrt{9}}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

4 Calculate:

a $\sqrt{\frac{144}{25}}$ b $\sqrt{\frac{196}{49}}$ c $\sqrt{5\frac{4}{9}}$ d $\sqrt{7\frac{1}{9}}$

e $\sqrt{3\frac{6}{25}}$ f $\sqrt{3\frac{13}{36}}$ g $\sqrt{7\frac{9}{16}}$ h $\sqrt{1\frac{19}{81}}$

5 First simplify each fraction and then calculate the result.

a $\sqrt{\frac{8}{32}}$ **b** $\sqrt{\frac{50}{72}}$ **c** $\sqrt{\frac{48}{75}}$ **d** $\sqrt{\frac{45}{80}}$

e $\sqrt{\frac{24}{54}}$ **f** $\sqrt{\frac{288}{50}}$ **g** $\sqrt{\frac{72}{162}}$ **h** $\sqrt{\frac{400}{25}}$

6 Explain why it was important to first simplify each fraction in question **5** before finding its square root.

7 Calculate:

a $\frac{3}{5} + \frac{2}{10} + \left(\frac{3}{5}\right)^2$ **b** $\frac{5}{9} - \frac{4}{8} + \left(\frac{4}{3}\right)^2$ **c** $\frac{4}{3} - \frac{2}{5} - \left(\frac{2}{3}\right)^2$

d $\frac{5}{2} - \frac{8}{9} + \left(\frac{5}{6}\right)^2$ **e** $\left(\frac{7}{8}\right)^2 - \frac{3}{4} + \frac{3}{16}$ **f** $\frac{9}{4} - \frac{4}{8} + \sqrt{\frac{16}{25}}$

8 Calculate each of these. Remember to think carefully about which operations are performed first.

a $\frac{4}{9} \times \frac{14}{8} \times \left(\frac{3}{7}\right)^2$ **b** $\frac{2}{3} \div \frac{4}{5} \times \left(\frac{3}{2}\right)^2$ **c** $3\frac{1}{4} \times \frac{4}{5} \div \sqrt{\frac{16}{49}}$

d $\sqrt{\frac{16}{25}} \div \frac{8}{3} + \frac{5}{8} \div 2\frac{1}{2}$ **e** $3\frac{3}{5} \div 2\frac{1}{4} - \sqrt{\frac{27}{12}}$ **f** $\left(\frac{5}{8}\right)^2 - \frac{3}{4} \div 5\frac{1}{3}$

EXAMPLE 2F-5 Raising fractions to the power of three

Calculate $\left(\frac{2}{3}\right)^3$.

THINK

1 Write the numerator to the power of 3 and the denominator to the power of three. (Alternatively, write as three lots of the fraction multiplied together.)

2 Calculate the numerator and then the denominator.

WRITE

$\left(\frac{2}{3}\right)^3$

$= \frac{2^3}{3^3}$ or $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

$= \frac{8}{27}$

9 Calculate:

a 2^3 **b** 4^3 **c** 7^3 **d** $\left(\frac{1}{2}\right)^3$

e $\left(\frac{1}{4}\right)^3$ **f** $\left(\frac{2}{7}\right)^3$ **g** $\left(\frac{3}{11}\right)^3$ **h** $\left(\frac{4}{5}\right)^3$

10 Describe how to calculate the result when a fraction is raised to any power.

11 Calculate:

a $\left(\frac{3}{4}\right)^3$ **b** $\left(\frac{2}{5}\right)^3$ **c** $\left(\frac{4}{3}\right)^3$ **d** $\left(\frac{9}{7}\right)^3$

e $\left(2\frac{1}{4}\right)^3$ **f** $\left(3\frac{1}{2}\right)^3$ **g** $\left(5\frac{2}{3}\right)^3$ **h** $\left(2\frac{5}{8}\right)^3$

UNDERSTANDING AND FLUENCY

12 Calculate:

a $\left(\frac{1}{2}\right)^4$ **b** $\left(\frac{2}{3}\right)^4$ **c** $\left(\frac{1}{4}\right)^5$ **d** $\left(\frac{2}{5}\right)^5$

e $\left(\frac{1}{3}\right)^6$ **f** $\left(\frac{3}{2}\right)^6$ **g** $\left(\frac{3}{4}\right)^4$ **h** $\left(\frac{1}{10}\right)^8$

13 Calculate:

a $\left(2\frac{1}{5}\right)^4$ **b** $\left(\frac{10}{3}\right)^4$ **c** $\left(\frac{5}{2}\right)^5$ **d** $\left(\frac{5}{3}\right)^5$

e $\left(2\frac{1}{3}\right)^6$ **f** $\left(3\frac{1}{4}\right)^6$ **g** $\left(1\frac{2}{5}\right)^4$ **h** $\left(1\frac{1}{8}\right)^4$

14 Carefully study each of these fractions: $\frac{3}{7}$, $\left(\frac{3}{7}\right)^2$, $\left(\frac{3}{7}\right)^3$, $\left(\frac{3}{7}\right)^4$.

a Arrange them in ascending order.

b What did you notice? Is this what you expected?

15 Carefully study each of these fractions: $\frac{6}{5}$, $\left(\frac{6}{5}\right)^2$, $\left(\frac{6}{5}\right)^3$, $\left(\frac{6}{5}\right)^4$.

a Arrange them in ascending order.

b What did you notice?

16 Using your results from questions **14** and **15**, explain what happens when proper fractions and improper fractions are raised to powers of 2 or more.

PROBLEM SOLVING AND REASONING

17 Your grandfather has a number of vinyl LP records in his music collection. The cover of one is a square. Work out the area in square metres by calculating the square of the length or (length × length).

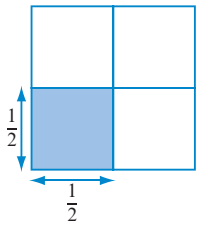
18 In each diagram, a square has been shaded. Use the length shown to answer the following.

a Calculate the value of (length × length) or $(\text{length})^2$ to find the area of the shaded region.

b Write the fraction of the large square that is shaded.

c Compare your answers to parts **a** and **b**. What do you notice?

i



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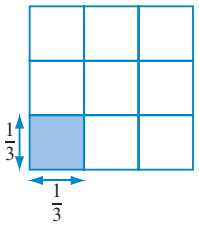

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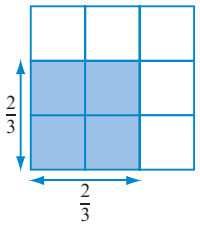

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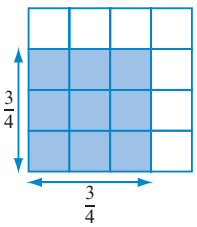


PROBLEM SOLVING AND REASONING

19 Look at each pair of fractions and work out whether they will give the same answer. Explain your reasoning.

a $(\frac{5}{8})^2$ and $\frac{5^2}{8}$ **b** $(\frac{2}{9})^2$ and $\frac{2}{9^2}$

c $(\frac{7}{3})^2$ and $\frac{7^2}{3^2}$ **d** $(\frac{1}{2}+\frac{1}{4})^2$ and $(\frac{1}{2})^2+(\frac{1}{4})^2$

20 Two students produced two different answers when asked to calculate $(2\frac{1}{5})^2$. Look at each student's working and explain which is correct.

Olivia's calculation

$$(2\tfrac{1}{5})^2 = 2^2 + (\tfrac{1}{5})^2$$
$$= 4 + \tfrac{1}{25}$$
$$= 4\tfrac{1}{25}$$

Kate's calculation

$$(2\tfrac{1}{5})^2 = (\tfrac{11}{5})^2$$
$$= \tfrac{121}{25}$$
$$= 4\tfrac{21}{25}$$

21 Jack produced these workings and answer when asked to calculate $\sqrt{9\frac{1}{16}}$. Comment on whether his answer and the process he used are correct.

Jack's workings

$$\sqrt{9\tfrac{1}{16}} = \sqrt{9} + \sqrt{\tfrac{1}{16}}$$
$$= 3 + \tfrac{1}{4}$$
$$= 3\tfrac{1}{4}$$

22 A real estate agent states the size of a square block of land is $2\frac{7}{9}$ square kilometres.

a Explain how the length of the block of land can be worked out if you know that the area of a square is length × length.

b Calculate the length of the block of land for sale.

c How long is the fence that runs along the boundary of the land?

d If the land is for sale at $100 000, calculate the cost per square kilometre.

CHALLENGE

23 Calculate:

a $(2\frac{2}{3})^2 - \sqrt{\frac{25}{81}}$ **b** $\frac{3}{14} \times \sqrt{\frac{169}{36}} \times (1\frac{1}{7})^2$

c $\frac{3}{4} \div \frac{9}{10} + \sqrt{1\frac{19}{81}} - \frac{1}{3}$ **d** $((2\frac{1}{4})^2 - \sqrt{\frac{196}{64}}) \div (1-\frac{1}{4})^2$

e $\sqrt{1\frac{21}{25} - 1\frac{1}{5}} \times 2\frac{2}{5} - \frac{3}{10}$ **f** $\sqrt{\sqrt{\frac{49}{144}} \times 2\frac{1}{3}}$

Reflect

How do you know in which order to perform operations on fractions?

2G Understanding ratios

Start thinking!

1 Use the photo to copy and complete:
   a There are _____ males compared to _____ females.
   b There are _____ adults compared to _____ children.

2 What do you notice about the total in each comparison made in question **1**?

3 Write four more comparisons using the photo.

A comparison of two or more quantities is called a **ratio**. In the photo, there are 2 children compared to 15 adults, so the ratio can be written as 2:15. Notice that a ratio has numbers separated by a colon.

4 Rewrite the comparisons from question **1** as ratios using numbers and a colon.

5 Rewrite the comparisons from question **3** as ratios using numbers and a colon.

To make a particular shade of pink, one part of red paint and four parts of white paint are mixed together.

6 Write this information as a ratio.

7 Describe the colour that would be produced with a ratio of 2:3.

8 Would a mixture with a ratio of 3:2 produce the same shade as a mixture with a ratio of 2:3? Explain.

9 Explain why the order of each amount (part) listed in the ratio is important.

KEY IDEAS

- A ratio is a comparison of two or more quantities of the same kind.
- Before writing a ratio, the numbers must be in the same unit of measurement.
- Ratios do not require units. That is, they are written as whole numbers with no units shown.
- A ratio must be written in the order of the worded description given. For example, one part cordial to five parts water is written as 1:5.
- Ratios do not contain fractions or decimals. They only contain whole numbers.

EXERCISE 2G Understanding ratios

1 Write each comparison as a ratio.

a the number of red jelly beans compared to the number of yellow jelly beans

b the number of orange jelly beans compared to the total number of jelly beans

c the number of pink jelly beans compared to the number of yellow jelly beans compared to the number of blue jelly beans

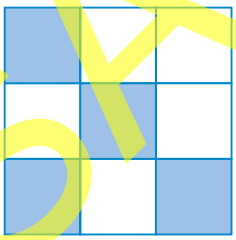
d the number of red and blue jelly beans compared to the number of pink and black jelly beans

2 Write each comparison as a ratio.

i the number of shaded parts to the number of non-shaded parts

ii the number of shaded parts to the total number of parts

a



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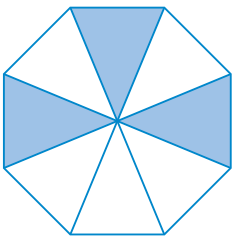

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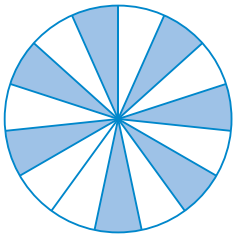

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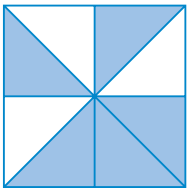


3 Write each comparison as a ratio in the given order.

a 132 cm to 207 cm
b 16 kg to 35 kg
c 18 min to 11 min
d $85 to $121
e 12 weeks to 39 weeks
f 25 L to 17 L

4 Write each comparison as a ratio in the given order.

a Collingwood scored 83 points compared to St Kilda's score of 67 points.
b Kayla spent 5 hours on Facebook and 1 hour doing homework.
c The fastest recorded tennis serve for a male is 247 km/h compared to 209 km/h for a female.
d Australia won three gold, four silver and nine bronze medals in the World Swimming Championships.
e Twenty-eight students tried out for the volleyball team while 23 students tried out for the netball team.
f To make a drink of cordial, use one part cordial to four parts water.

5 Write the number of football players shown to the number of umpires as a ratio.

6 Write each comparison of quantities as a ratio.

a A recipe requires five cups of flour to one cup of fruit and chopped nuts.
b A mixture contains 4 kg of sand, 2 kg of cement and 5 kg of gravel.
c In a group of 100 people, 13 are left-handed, 5 are both left- and right-handed and the rest are right-handed.
d Brittany and Lauren contribute equally to a friend's present.
e There are approximately three male passengers to every five female passengers on the train in the morning.

7 List an example for each ratio.
You may like to use a diagram to show your example.

a 5:11 **b** 16:9 **c** 1:1 **d** 7:3

8 A cupcake recipe lists these ingredients:

1 cup of milk
2 cups of sugar
3 cups of flour
4 eggs
5 tablespoons of melted butter
1 teaspoon of vanilla essence

a Write each comparison of quantities as a ratio in the order given.

i flour to sugar

ii milk to flour

iii dry ingredients (flour and sugar) to liquid ingredients (milk)

b Why does it not make sense to write a ratio of eggs to butter for this recipe?

c What would you need to know before you could write a ratio of melted butter to vanilla essence?

9 From the given seven-day forecast, write ratios for:

a the number of windy days to the number of sunny days

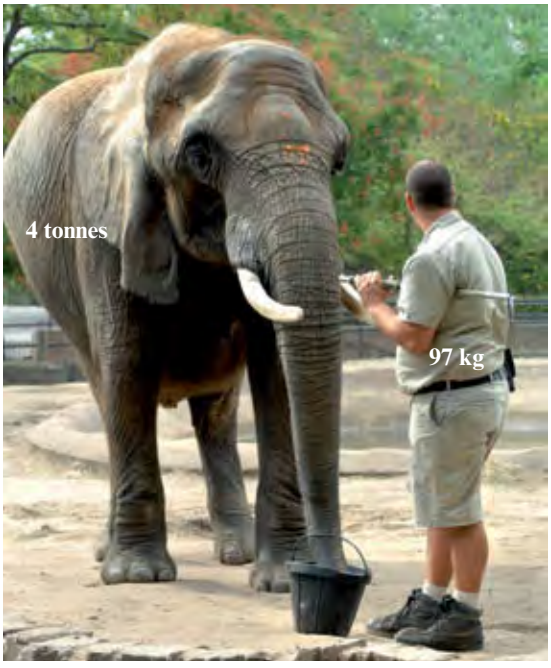
b the number of showery days to the total number of days

c the number of days with a maximum temperature of 13°C to the number of days with a maximum temperature of 15°C to the number of days with a maximum temperature of 18°C.

7 DAY FORECAST

today	max 13	showers
tomorrow	max 15	showers
Monday	max 18	windy
Tuesday	max 13	showers
Wednesday	max 18	sunny
Thursday	max 18	sunny
Friday	max 18	sunny

10 Look at this elephant and its handler. The mass of each is shown.



a Explain why the ratio of the elephant handler's mass to the elephant's mass cannot be written as 97:4.

b Suggest what could be done to one of the measurements so that a ratio comparing their masses can be written.

c Discuss which measurement was selected and why.

d Write the ratio of the elephant handler's mass to the elephant's mass.
(Hint: 1000 kg = 1 tonne.)

11 Can each of these comparisons be written as a ratio? Explain.

- **a** 45 m compared to 67 m
- **b** 375 mL of juice compared to 5 kg of chicken
- **c** $5 compared to 50 cents
- **d** 19 km compared to 3 hours

12 Convert the quantities to the same unit and then write the ratio for each comparison.

- **a** 10 days to 7 weeks
- **b** 23 minutes to 4 hours
- **c** 19 cm to 3.6 m
- **d** 25 cents to $1.99
- **e** 157 kg to 5.2 tonne
- **f** 239 mL to 1.7 L
- **g** 51 months to 5 years
- **h** 11 bananas to 3 dozen bananas
- **i** 49 mL to 1 L
- **j** 5 hours to 47 minutes
- **k** 13.5 km to 8729 m
- **l** 23 seconds to 1.5 hours

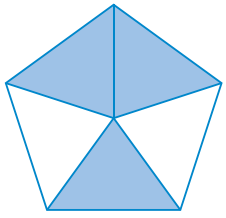
13 In a survey, 173 out of 200 people preferred Dazzle toothpaste to their regular brand. The remainder preferred their regular brand.
Write a ratio for each of these.

- **a** the number of people that prefer Dazzle toothpaste to the total number of people surveyed
- **b** the number of people that prefer their regular brand of toothpaste to the total number of people surveyed
- **c** the number of people that prefer Dazzle toothpaste to the number of people who prefer their regular brand

14 Max scored $\frac{87}{100}$ (or 87 out of 100) for his Geography test.
Write a ratio for:

- **a** the marks received compared to the total number of marks
- **b** the marks lost compared to the total number of marks
- **c** the marks received compared to the marks lost.

15 Look at the pentagon shown.



- **a** What is the ratio of shaded sections to non-shaded sections?
- **b** What is the ratio of non-shaded sections to the total number of sections?
- **c** What is the ratio of shaded sections to the total number of sections?
- **d** Can any of the ratios in parts **a**–**c** be written as fractions? Explain your reasoning.
- **e** Can any of the ratios in parts **a**–**c** *not* be written as fractions? Explain your reasoning.
- **f** Copy and complete: Ratios can be written as fractions when comparing the ratio of a part to its _____.

PROBLEM SOLVING AND REASONING

16 Geoff mixes 4 L of blue paint with 5 L of yellow paint to create a shade of green for his studio.

- **a** Write the ratio of the amount of blue paint to the amount of green paint.
- **b** What fraction of the new shade is blue paint?
- **c** What fraction of the new shade is yellow paint?

17 Elio and Gaetano work at the local supermarket and are called in after school to cover some shifts. They both start at 4 pm. Elio is able to work till 9 pm while Gaetano must leave at 7 pm.

- **a** What is the ratio of the number of hours worked by Elio compared to those worked by Gaetano?
- **b** What is the total amount of time worked by both boys?
- **c** What fraction of the total amount of time did Elio work?

CHALLENGE

18 Barry could not understand why his response to an Aussie rules question was marked incorrect. The question asked whether '10 goals and 3 behinds compared to 158 points' could be written as a ratio, to which Barry replied 'No, since the quantities involved different units'. Explain to Barry why his response was incorrect.

19 Make up four comparisons and ask your friends whether a ratio can be written for each, stating reasons for their answers.

Reflect

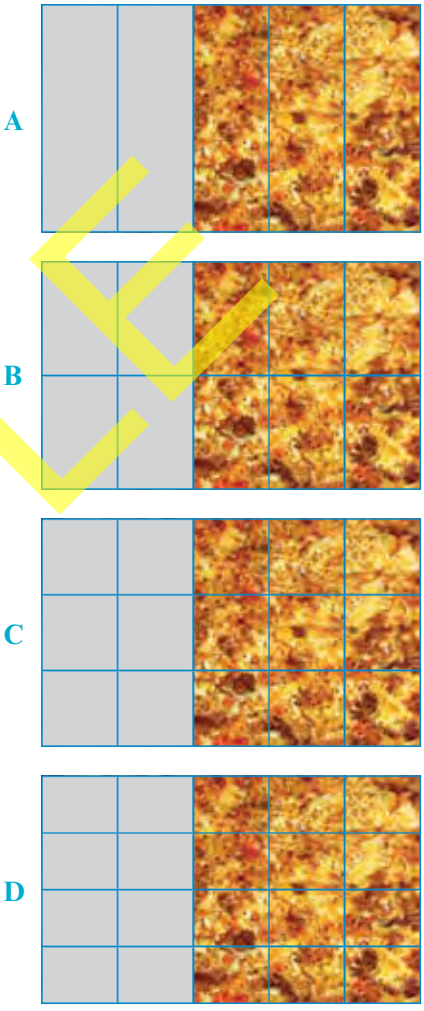
What type of comparisons can be written as a ratio?

2H Working with ratios

Start thinking!

For a family lunch, four trays of lasagne were baked.

The lasagne in the first tray was cut into 5 equal rectangular slices, the second into 10 equal rectangular slices, the third into 15 equal rectangular slices and the fourth into 20 equal rectangular slices. After the lunch, it was noticed that the same amount of lasagne had been eaten from each tray.



1 For each tray of lasagne, write as a ratio the number of slices eaten compared to the original number of slices. For example, the ratio for Tray A is 2:5.

2 Look for a pattern in your answers to question **1**. Write the next three ratios you would expect.

3 Use your answers to question **1** to write each ratio as a fraction. For example, the fraction for Tray A is $\frac{2}{5}$.

4 Explain what each fraction represents.

5 Compare the fractions you have obtained. What is special about them?

6 Copy and complete:
Numbers in a ratio can be _____ or _____ by the same value to create an equivalent ratio.

KEY IDEAS

- Numbers in a ratio can be multiplied or divided by the same value to create an equivalent ratio.
- Ratios are written with whole numbers only.
- A ratio may be written in simplest form by dividing the numbers in the ratio by the HCF.

EXERCISE 2H Working with ratios

EXAMPLE 2H-1 Simplifying ratios

Write the ratio 35:56 in simplest form.

THINK	WRITE
1 Look for the HCF of each number in the ratio. HCF = 7.	35:56
2 Divide each number in the ratio by the HCF of 7. (35 ÷ 7 = 5 and 56 ÷ 7 = 8.)	= 5:8

UNDERSTANDING AND FLUENCY

1 Write each ratio in simplest form.

a 21:6	**b** 12:18	**c** 52:64
d 36:96	**e** 88:48	**f** 40:45
g 65:13	**h** 22:132	**i** 20:50:20
j 125:75:25	**k** 27:45:63	**l** 24:56:40

2 Fill in the gaps to complete the equivalent ratios.

a 3:2 = ___:4 = 15:___	**b** 4:5 = ___:25 = 16:___
c ___:210 = 10:70 = 2:___	**d** 144:___ = 24:9 = 8:___
e ___:121 = 150:___ = 15:11	**f** 147:___ = ___:63 = 7:9

EXAMPLE 2H-2 Converting units to write comparisons as ratios

Write the comparison 6 cm to 42 mm as a ratio in simplest form.

THINK	WRITE
1 Write the quantities in the order given.	6 cm to 42 mm
2 Convert the quantities to the same unit. Write them in the smaller unit (mm). 1 cm = 10 mm.	= 60 mm to 42 mm
3 Write the comparison as a ratio now that the quantities are in the same unit.	= 60:42
4 Divide each number in the ratio by the HCF. HCF = 6. (60 ÷ 6 = 10 and 42 ÷ 6 = 7.)	= 10:7

3 Write each comparison as a ratio in simplest form.

a 42 mm to 7 cm
b 2 km to 400 m
c 50c to $3
d 5 weeks to 15 days
e 500 g to 3.6 kg
f 335 m to 0.6 km
g 55 seconds to $3\frac{1}{2}$ minutes
h 2 h 35 min to 45 min
i 4 weeks to 2 years
j 1.05 million to 950 000
k 72 mL to 2.5 L
l 65 c to $11.85

4 A fruit bowl contains apples, oranges and bananas.

Write each comparison as a ratio in simplest form.

a the number of apples to the number of oranges
b the number of apples to the number of bananas
c the number of oranges to the number of bananas to the number of apples.

5 In an upset win in an AFL round of football, Carlton beat Geelong with a score of 96 points to 84 points. Write the ratio of the scores in simplest form.

6 In a class of 28 students, 16 are girls.

a How many students are boys?
b Write the ratio of girls to boys in simplest form.

7 A breakfast cereal mix is made using 8 kg of oats, 4 kg of dried fruit and 12 kg of wheat flakes.

a Write the ratio of oats to dried fruit to wheat flakes in simplest form.
b Write each ingredient as a fraction of the whole mix.

8 Last week, Shannen completed extra chores around home and earned $32. This week, she earned $14 less. Write as a ratio, in simplest form, the amount of last week's pocket money to this week's pocket money.

9 The table shows the attendance at each performance of the local school production.

Performance	Number of people attending
Thursday evening	486
Friday evening	549
Saturday matinee	390
Saturday evening	520

a Find the total attendance to the school production over the three days.

b Write as a ratio in simplest form:

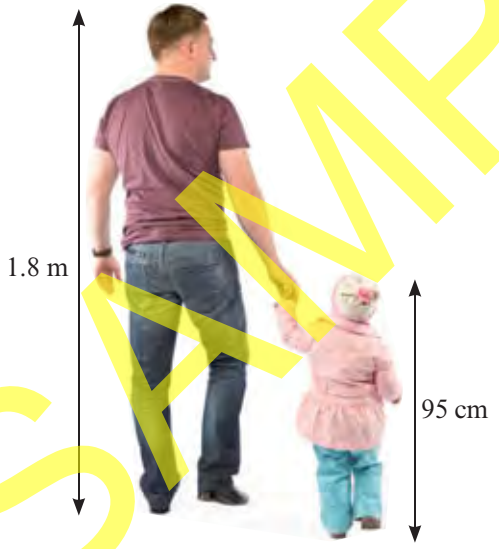
i the number attending Thursday's performance to the number attending Friday's performance

ii the number attending Saturday's matinee performance to the number attending Saturday's evening performance.

c If the venue can seat a maximum of 550 people, write as a ratio in simplest form the total attendance to the total possible attendance over the three days.

10 Use the photos to write each ratio in simplest form.

a The ratio of the height of the taller person to the height of the shorter person.



b The ratio of the amount of liquid in the orange juice carton to the amount of liquid in the milk carton.

11 Write three ratios that simplify to 6:7.

12 Write three ratios that are equivalent to 48:60.

13 Brett is unsure if the ratios 20:32 and 45:72 are equivalent. Explain how he can determine whether they are equivalent.

EXAMPLE 2H-3 Identifying equivalent ratios

Is each pair of ratios equivalent?

a 3:16 and 9:48 **b** 18:12 and 36:28

THINK

a **1** Write each ratio in simplest form. Divide the second ratio by 3.

2 Since the simplest form of each ratio is the same, they are equivalent.

b **1** Write each ratio in simplest form. Divide the first ratio by 6 and the second by 4.

2 Since the simplest form of each ratio is not the same, they are not equivalent.

WRITE

a 3:16 is in simplest form
9:48 = 3:16

The ratios 3:16 and 9:48 are equivalent.

b 18:12 = 3:2
36:28 = 9:7

The ratios 18:12 and 36:28 are not equivalent.

14 Is each pair of ratios equivalent? Give a reason for your answer.

a 7:8 and 21:24 **b** 20:32 and 5:8

c 63:18 and 9:2 **d** 30:45 and 90:60

15 Create equivalent ratios by finding the missing numbers.

a 5:2 = 25:_____ **b** 280:350 = 4:_____

c 1800:2400 = _____:8 **d** _____:7 = 45 000:105 000

e 378:126 = 6:_____ **f** 44:_____ = 220:40

16 The ratio of cats to dogs at the animal shelter is 7:4. If there are 21 cats, how many dogs are at the animal shelter?

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

17 The ratio of boys to girls in a school orchestra is 2:3, and there are 15 girls in it.

a How many boys are in the orchestra?

b How many students are in the school orchestra?

18 Ratios are useful for caterers because a recipe can be scaled up for a large group or scaled down for just a few people.

To make enough dough for two pizzas, a chef uses these ingredients.

1 cup of water
2 cups of plain flour
1 teaspoon of yeast
1 teaspoon of salt

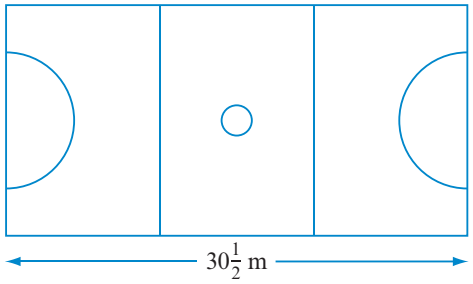
How much of each ingredient would the chef need to make:

a four pizzas? **b** twelve pizzas? **c** one pizza?

19 The ratio of apple juice to soda water for a fruit punch recipe is 5:3. If the recipe requires 750 mL of apple juice, how much soda water is required?

CHALLENGE

20 The ratio of the length of a netball court to its width is 2:1. What is the width of the court?



21 The ratio of the number of supporters for the visiting team to the number of supporters for the home team at Saturday's game was 10:11. If there were 32 500 supporters for the visiting team, how many supporters were there for the home team?

22 Provide three different real-world examples of ratios that are equivalent to 3:4. You may like to use diagrams to illustrate your ratios.

Reflect

How can equivalent ratios be used to solve problems?

CHAPTER REVIEW

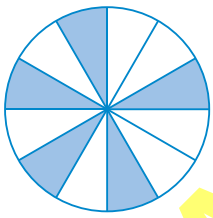
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

fraction
numerator
denominator
vinculum
mixed number
improper fraction
proper fraction
equivalent fractions
simplifying
cancelling
highest common factor (HCF)
lowest common denominator (LCD)
reciprocal
power
square
square root
ratio
equivalent ratios

MULTIPLE-CHOICE

2A **1** What fraction of the shape is coloured blue?



A 5 B $\frac{5}{10}$
C $\frac{7}{12}$ D $\frac{5}{12}$

2B **2** What is an equivalent fraction to $\frac{2}{5}$?

A $\frac{3}{5}$ B $\frac{6}{15}$
C $\frac{2}{10}$ D $\frac{1}{10}$

2C **3** What is $\frac{2}{5} + \frac{1}{3}$ equal to?

A $\frac{3}{8}$ B $\frac{2}{15}$
C $\frac{1}{2}$ D $\frac{11}{15}$

2C **4** What is $1\frac{1}{2} + 2\frac{5}{6} - 1\frac{2}{3}$?

A $2\frac{2}{3}$ B 4
C $2\frac{4}{5}$ D 6

2D **5** What is $\frac{2}{7}$ of $\frac{8}{9}$?

A $\frac{16}{63}$ B $\frac{10}{16}$
C $\frac{6}{2}$ D $\frac{18}{56}$

2D **6** 6 What is $\frac{14}{5} \times \frac{3}{21} \times \frac{25}{30}$ in simplest form?

A 3 B $\frac{5}{3}$
C $\frac{3}{5}$ D $\frac{1}{3}$

2E **7** What is $\frac{7}{20} \div \frac{14}{5}$ equal to?

A $\frac{1}{2}$ B $\frac{1}{8}$
C $\frac{7}{15}$ D $\frac{21}{25}$

2F **8** What is $\left(\frac{2}{3}\right)^3$?

A $\frac{6}{9}$ B $\frac{8}{27}$
C $\frac{5}{6}$ D $\frac{8}{9}$

2F **9** What is $\sqrt{1\frac{7}{9}}$?

A $\frac{16}{9}$ B $\frac{3}{4}$
C $\frac{4}{3}$ D $1\frac{2}{3}$

2G **10** Written as a ratio, what is 20 cm to 1 m?

A 20:1 B 20:10
C 20:100 D 20:1000

2H **11** What is a mixture of 100 mL cordial to 1.4 L of water written as a ratio in its simplest form?

A 100:1.4 B 100:1400
C 1:14 D 101.4

SHORT ANSWER

2A **1** Copy and complete this table.

	Mixed number	Improper fraction
a	$2\frac{3}{8}$	
b		$\frac{11}{2}$
c		$\frac{17}{9}$
d	$5\frac{1}{3}$	

2B **2** Simplify each fraction.

a $\frac{16}{20}$ B $\frac{5}{45}$

C $\frac{50}{100}$ D $\frac{21}{49}$

2B **3** Place these fractions in ascending order.
$\frac{1}{4}, \frac{2}{7}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}, \frac{11}{20}$

2C **4** Calculate:

a $\frac{1}{9} + \frac{4}{9}$ b $\frac{7}{11} - \frac{5}{11}$

c $\frac{2}{3} - \frac{1}{2}$ d $\frac{3}{10} + \frac{2}{5}$

2C **5** Calculate:

a $2\frac{5}{7} + 3\frac{1}{7}$ b $2\frac{1}{5} - 1\frac{2}{5}$

c $1\frac{3}{8} - \frac{1}{4}$ d $1\frac{4}{5} + 2\frac{2}{3}$

2D **6** Calculate:

a $\frac{2}{5} \times \frac{4}{7}$ b $\frac{3}{10} \times \frac{5}{9}$

c $\frac{2}{3} \times \frac{10}{11}$ d $\frac{12}{5} \times \frac{4}{9}$

2D **7** Calculate:

a $\frac{2}{3} \times 1\frac{4}{5}$ b $2\frac{3}{7} \times \frac{1}{5}$

c $2\frac{1}{4} \times 3\frac{2}{3}$ d $1\frac{3}{8} \times 4\frac{1}{3}$

2E **8** Calculate:

a $\frac{3}{4} \div \frac{2}{3}$ b $\frac{25}{12} \div \frac{5}{3}$

c $\frac{49}{24} \div \frac{7}{16}$ d $\frac{1}{4} \div \frac{1}{3}$

2E **9** Calculate:

a $3\frac{4}{7} \div 2\frac{1}{7}$ b $\frac{8}{9} \div 1\frac{1}{3}$

c $3\frac{3}{5} \div \frac{6}{15}$ d $2\frac{3}{5} \div 1\frac{9}{15}$

2F **10** Calculate:

a $(\frac{3}{4})^2$ b $(\frac{5}{6})^2$

c $(\frac{3}{10})^3$ d $(\frac{1}{2})^4$

2F **11** Calculate:

a $\sqrt{\frac{25}{49}}$ b $\sqrt{\frac{144}{81}}$

c $\sqrt{\frac{1}{9}}$ d $\sqrt{1\frac{28}{36}}$

2G **12** Use the photo to write each comparison as a ratio.

a the number of pink Smarties to the number of yellow Smarties

b the number of yellow Smarties to the number of green Smarties

c the number of orange Smarties to the total number of Smarties

d the number of orange and pink Smarties to the number of green Smarties

2H **13** Write as ratios in their simplest form.

a 24:36

b 15:20

c 75c to $1.20

d 8 weeks to 4 days

2H **14** The ratio of snakes to lizards in a reptile zoo is 7:5. If there are 40 lizards, how many snakes are there in the zoo?

NAPLAN-STYLE PRACTICE

1 What is $3\frac{4}{7}$ as an improper fraction?

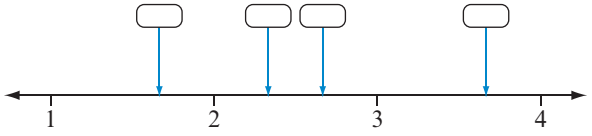
2 Angela needs $\frac{33}{5}$ m of material. How many whole metres should she buy?

33 ○ 7 ○ 6 ○ 5 ○

3 What is $\frac{24}{5}$ as a mixed number?

$2\frac{4}{5}$ ○ $4\frac{4}{5}$ ○ $\frac{5}{24}$ ○ $4\frac{2}{5}$ ○

4 Which arrow is pointing closest to $2\frac{2}{3}$?



5 Which fraction is equivalent to $\frac{5}{12}$?

$\frac{15}{25}$ ○ $\frac{7}{14}$ ○ $\frac{10}{24}$ ○ $\frac{10}{25}$ ○

6 Which list is ordered smallest to largest?

○ $\frac{3}{4}, \frac{7}{2}, 2\frac{1}{4}, \frac{5}{8}, \frac{1}{2}$ ○ $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

○ $1\frac{1}{3}, \frac{12}{5}, 2\frac{4}{5}, \frac{13}{3}, \frac{26}{15}$ ○ $\frac{2}{5}, 1\frac{3}{4}, \frac{9}{4}, 2\frac{7}{10}, \frac{16}{5}$

7 Write $\frac{24}{16}$ in simplest form.

8 Hayley has 18 red and 6 orange icy-poles. What fraction are orange?

$\frac{1}{2}$ ○ $\frac{1}{3}$ ○ $\frac{1}{4}$ ○ $\frac{1}{6}$ ○

9 What is $3\frac{1}{5} - 2\frac{4}{5}$?

10 What is $\frac{64}{5} - \frac{16}{3}$?

$7\frac{3}{5}$ ○ $7\frac{7}{15}$ ○ $3\frac{7}{15}$ ○ $7\frac{1}{3}$ ○

11 What is $\frac{4}{5} \times \frac{2}{3}$?

12 Angelina has a recipe that uses 3 cups of flour. She decides to make a $\frac{3}{4}$ batch. How much flour will Angelina need?

○ 3 cups ○ $\frac{3}{4}$ of a cup

○ 2 cups ○ $2\frac{1}{4}$ cups

13 What is $\frac{8}{5} \div \frac{4}{9}$?

14 What is $\frac{7}{8} \div 3\frac{1}{2}$?

$\frac{7}{48}$ ○ $\frac{49}{16}$ ○ $4\frac{3}{8}$ ○ $\frac{1}{4}$ ○

15 If $4\frac{1}{2}$ L of juice is divided evenly between five friends, how much does each person get?

4 L ○ $\frac{9}{10}$ L ○ $1\frac{1}{8}$ L ○ $\frac{4}{5}$ L ○

16 What fraction is halfway between $\frac{7}{11}$ and $\frac{8}{11}$?

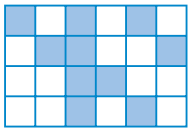
17 What is $(\frac{3}{7})^2$?

18 What is $(2\frac{1}{4})^2$?

$\frac{1}{4}$ ○ $\frac{81}{16}$ ○ $4\frac{1}{16}$ ○ $\frac{16}{81}$ ○

19 What is $\sqrt{\frac{16}{25}}$?

Questions **20** and **21** refer to this diagram.



20 In simplest form, what fraction is *not* shaded?

$\frac{14}{24}$ ○ $\frac{7}{12}$ ○ $\frac{10}{24}$ ○ $\frac{5}{12}$ ○

21 What is the ratio of shaded segments to unshaded segments?

22 Monique is 1.21 m tall. Lachlan is 98 cm tall. What is the ratio of Lachlan's height to Monique's?

- ◯ 98:1.21
- ◯ 98:121
- ◯ 1.21:98
- ◯ 121:98

23 Alex has 16 jellybeans: 7 pink, 5 white and the rest blue. What is the ratio of blue to pink jellybeans?

24 Which ratio is *not* equivalent to the ratio 3:7?

- ◯ 9:21
- ◯ 12:28
- ◯ 6:10
- ◯ 30:70

25 A netball game consists of four quarters and three rest breaks. The ratio of game time to rest time is 15:4. If total rest time is 16 minutes, what is the game's duration?

- ◯ 16 minutes
- ◯ 44 minutes
- ◯ 60 minutes
- ◯ 76 minutes

26 Write the ratio 36:54 in its simplest form.

27 The ratio of girls to boys at camp is 7:6. If there are 54 boys, how many girls are there?

ANALYSIS

1 A survey of 300 families found that 150 have one pet and 100 have two pets.

- **a** What fraction of families have one pet? Write this in simplest form.
- **b** What fraction of families have two pets? Write this in simplest form.
- **c** Write a ratio for the number of families with one pet compared to those with two pets.
- **d** If $\frac{1}{12}$ of families have three or more pets, how many families is this?
- **e** What fraction of families do *not* have pets? How many families is this?
- **f** A news reporter said that 5 in every 10 people have one pet, and 3 in every 10 people have two pets. Are they correct? Use your knowledge of equivalent fractions to support your answer.

2 Brendan wants to buy a new mp3 player worth $50. He receives $10 pocket money every week.

- **a** If he saves $\frac{1}{5}$ of his pocket money each week, how much is this?
- **b** How many weeks would it take to save $50?
- **c** How much would he save in 12 weeks?
- **d** Write your answer to part **c** as a fraction of his weekly pocket money in the form of:
  - **i** an improper fraction
  - **ii** a mixed number.
- **e** If, after 12 weeks, Brendan spends $\frac{2}{3}$ of his savings, how much does he have left?
- **f** Brendan has saved $\frac{42}{5}$ of his weekly pocket money. How much is this and how long would it take at the rate mentioned in part **a**?
- **g** If he instead saved $\frac{3}{5}$ of his pocket money each week, how long would it take to achieve the amount of savings calculated in part **f**? (Hint: divide $\frac{42}{5}$ by $\frac{3}{5}$.)
- **h** Write the initial amount saved each week (part **a**) compared to the second amount saved each week (part **g**) as a ratio in simplest form.

CONNECT

Catering for a birthday party

Imagine you are planning a birthday party. You need to know how many people are attending and to estimate how much food and drink each person may want. How are fractions and ratios related to preparing for a party?

Assume that 24 people (including you) are expected. Of these 24 people, 14 are boys and 10 are girls. The menu will include: pizza, chicken, salad, fruit, drinks and cake. Ingredients required for some of the recipes are given.

Smoky barbecue chicken kebabs

(serves 8)

2 kg skinless and boneless chicken thighs
2 tablespoons olive oil
pita bread
mixed salad leaves

Sauce:

2 brown onions
$\frac{1}{2}$ cup golden syrup
1 cup cider vinegar
$\frac{2}{3}$ cup water
$1\frac{1}{4}$ cups tomato puree
$\frac{1}{3}$ cup Worcestershire sauce
2 tablespoons hot English mustard
$\frac{1}{4}$ cup olive oil
2 teaspoons smoked ground paprika
$\frac{1}{2}$ teaspoon hot chilli powder
$\frac{1}{3}$ cup brown sugar
sea salt and cracked black pepper

Your task

Carry out these steps to prepare for the party.

- Calculate the number of pizzas required for the party.
- Work out the new quantities of ingredients needed to cater for 24 people, given that each recipe serves 8 people.
- Work out the new quantities of ingredients needed to cater for any given number of guests.
- Obtain a recipe for fruit punch and adjust it to cater for the number of guests.
- Calculate the ratio of boys to girls at the party.
- Work out which quantities can and cannot be compared as a ratio.

Chocolate brownie and raspberry trifles

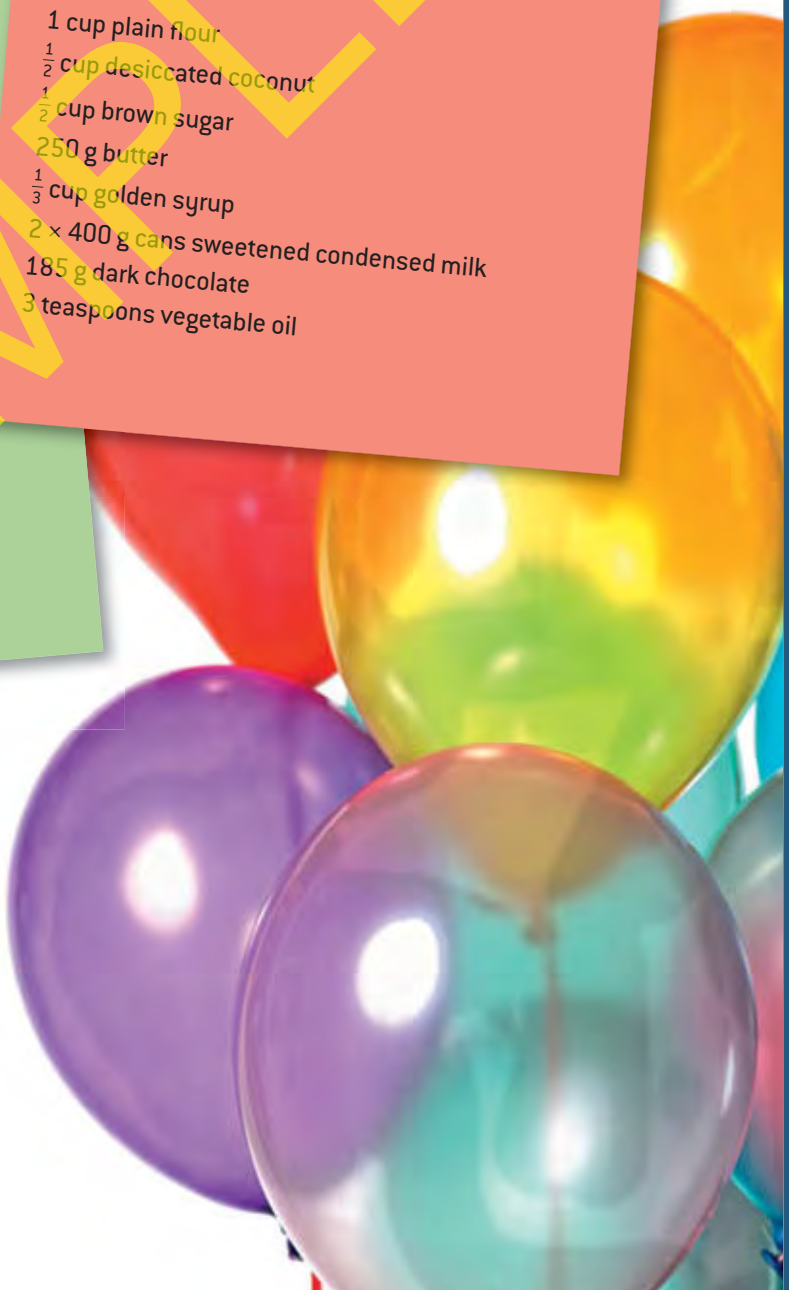
(serves 8)

200 g dark chocolate
250 g unsalted butter
$1\frac{1}{3}$ cups brown sugar
4 eggs
$\frac{1}{3}$ cup cocoa powder
1 cup plain flour
$\frac{1}{4}$ teaspoon baking powder
4 × 120 g punnets raspberries
1 cup crème de cassis
2 cups cream
$\frac{1}{4}$ cup icing sugar

Chocolate caramel slice

(serves 8)

1 cup plain flour
$\frac{1}{2}$ cup desiccated coconut
$\frac{1}{2}$ cup brown sugar
250 g butter
$\frac{1}{3}$ cup golden syrup
2 × 400 g cans sweetened condensed milk
185 g dark chocolate
3 teaspoons vegetable oil



Complete the 2 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a booklet
- a PowerPoint presentation
- a video
- other (check with your teacher).