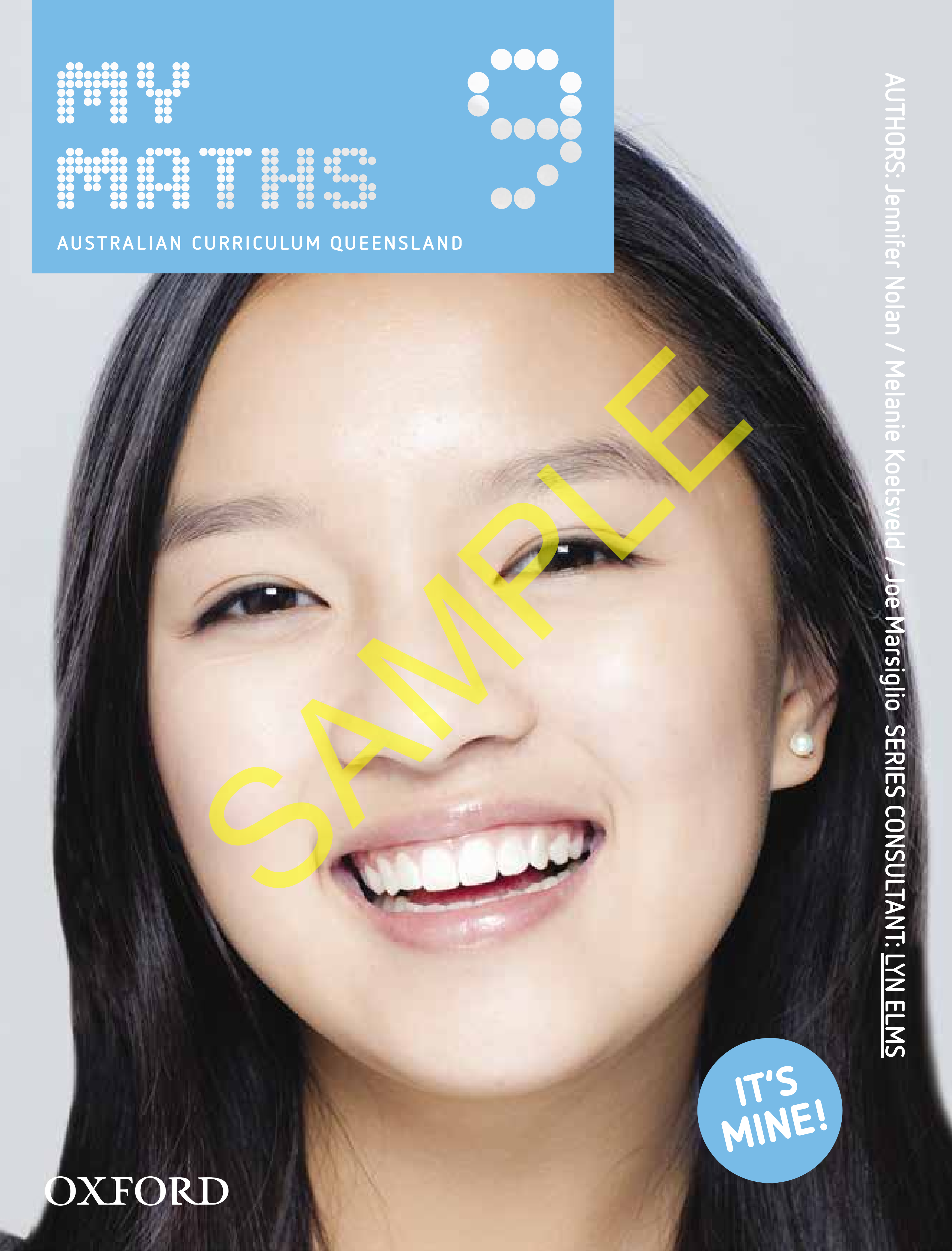
MY MATHS 9

AUSTRALIAN CURRICULUM QUEENSLAND

AUTHORS: Jennifer Nolan / Melanie Koetsveld / Joe Marsiglio SERIES CONSULTANT: LYN ELMS



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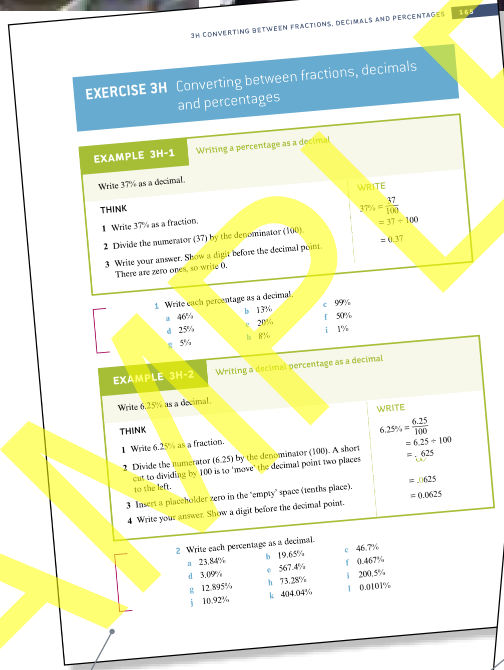
OXFORD MYMATHS FOR QUEENSLAND

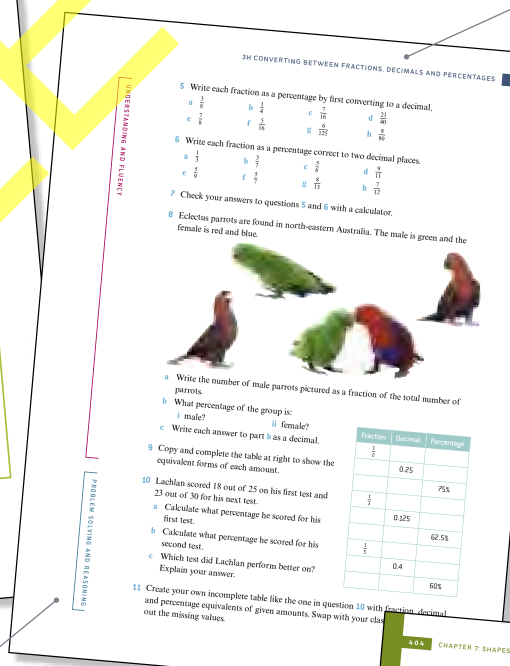
Oxford MyMaths for Queensland has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

STUDENT BOOK + OBOOK/ASSESS

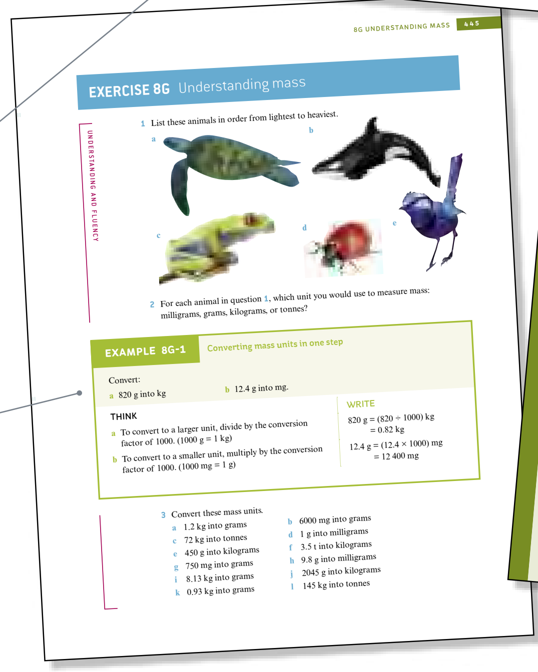
- Finely levelled exercises to ensure smooth progress
- Integrated worked examples – right where your students need them
- Learning organised around the 'big ideas' of mathematics
- Discovery, practice, thinking and problem-solving activities promote deep understanding
- A wealth of revision material to consolidate and prove learning
- Rich tasks to apply understanding
- Highly accessible and easy to navigate
- Comprehensive digital tutorials and guided examples to support independent progress



The student book accurately and carefully delivers the course.



Every question matched to the Australian Curriculum proficiencies.



Worked examples are clearly laid out and located where students need them most.

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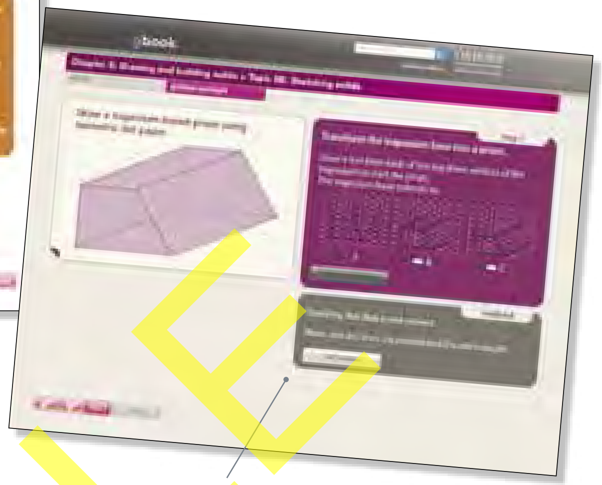
24/7 LEARNING AND SUPPORT

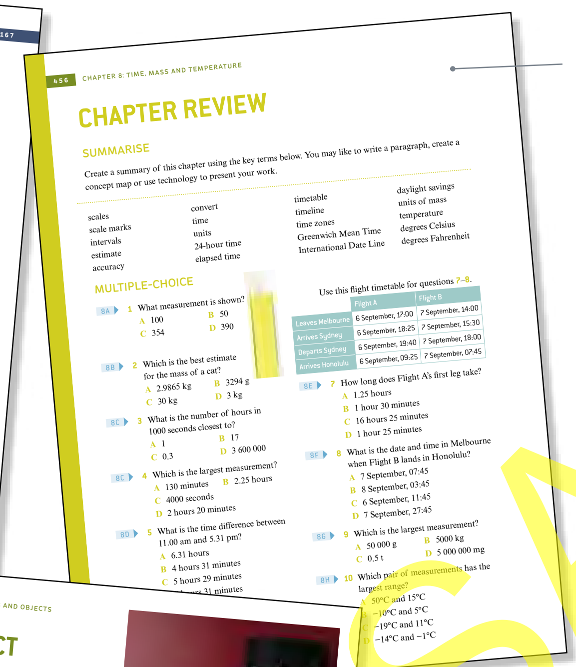
E-tutors scaffold understanding of key concepts and build confidence.

Self-discovery opportunities for students through guided exploration.

Guided examples support practice and fluency



Finely levelled content enables students to progress with ease



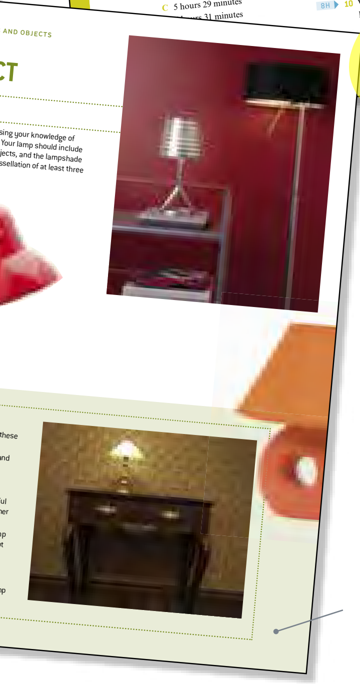
Ample revision to consolidate understanding and prove that learning has happened

Students receive feedback for incorrect responses

Optimise understanding and performance.

Personalised learning: tailor the very best learning experiences for all.

Intervention and extension worksheets supplied for every topic.



Rich tasks where students can demonstrate understanding

TEACHER OBOOK/ASSESS

Practical classroom resources and tools:

- Manage student differentiation
- Correct common misconceptions
- Assign work
- Set tests
- Monitor results
- Any device, anytime, anywhere.

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9 PROBABILITY

ESSENTIAL QUESTION

How do you explain and calculate the probability of an event?

SAMPLE

9A ▶ **1** How would you describe the chance of an event occurring that has a probability of 0.2?

A impossible
B certain
C somewhat likely
D very unlikely

9A ▶ **2** Look at this figure.

a What is the theoretical probability of selecting a yellow jellybean?

A $\frac{1}{4}$ **B** 3 **C** $\frac{3}{11}$ **D** $\frac{3}{10}$

b What is the sample space of this figure?

A 11
B red, green, yellow, blue
C {3 red, 4 green, 3 yellow, 2 blue}
D {red, green, yellow, blue}

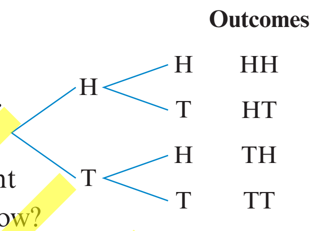
9A ▶ **3** Which of these experiments does not have equally likely outcomes?

A rolling a die
B selecting a letter from the word FLOWER
C drawing a card from a deck and recording its suit
D flipping two coins

9B ▶ **4 a** If you flip a coin 10 times, how many heads would you expect to get?

b If a coin is flipped 20 times and 13 tails are obtained, what is the experimental probability of obtaining a tail?

9C ▶ **5** Look at this tree diagram.



a What experiment does it show?

A rolling a die
B flipping one coin twice
C flipping one coin three times
D flipping four coins

b How many outcomes in total are possible?

c What is the theoretical probability of flipping two tails?

9D ▶ **6** Look at this table.

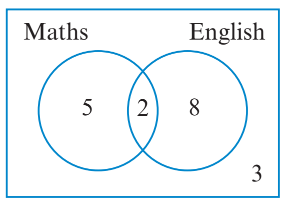
	Male	Female	Total
Dark hair	8	7	15
Fair hair	4	6	10
Total	12	13	25

a How many people were surveyed in total?

b How many females have fair hair?

c How many males were surveyed?

9E ▶ **7** Look at this figure.



a How many people like both Maths and English?

b How many people don't like Maths or English?

c How many people were surveyed in total?

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9A Theoretical probability

Start thinking!

The **probability** of something occurring is how likely it is to happen. To describe probability accurately, you calculate the **theoretical probability** of an **event** occurring. To find theoretical probability, you need to consider all possible outcomes.

Imagine a classmate sells you a raffle ticket for a local club.

1 If there is a total of 100 tickets, what is the probability that you will win the raffle?

2 How can you increase your chances of winning the raffle?

3 Explain why, if you bought seven tickets, you have 7 chances out of 100 to win the raffle.

The number of chances you have to win the raffle can also be called the number of **favourable outcomes**.

4 Explain why you can use the formula

$\Pr(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ to calculate theoretical probability.

5 How many tickets would you have to buy in order to have a 50% or 0.5 chance of winning the raffle?

6 The probability of winning a particular lottery is $\frac{1}{8\,145\,060}$. Explain what this means.

7 How many tickets would you have to buy in order to have a 50% chance of winning this lottery?

8 Why is this so many more than your answer to question 5?

All these calculations have assumed that each ticket has an **equally likely** chance of being drawn.

9 Describe some circumstances where outcomes are not equally likely. Discuss with a classmate.

KEY IDEAS

- The probability of an event occurring can be described using words or numbers in the range 0 (impossible) to 1 (certain).

impossible	highly unlikely	very unlikely	unlikely	somewhat unlikely	even chance	somewhat likely	likely	very likely	highly likely	certain
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

- To find the theoretical probability of an event occurring, use the formula:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- The **sample space** of an **experiment** is a list of all the different outcomes possible and is written within curly brackets. It does not show whether each different outcome is equally likely to occur.
- The complement of an event A is the event where A does not occur. Event A and event ‘not A’ are **complementary events**. Pr(A) + Pr(not A) = 1.

SAMPLE

EXERCISE 9A Theoretical probability

1 Describe the probability of each of these events occurring.

- **a** a random day in winter being cold
- **b** winning the lottery
- **c** a baby being born on a weekday
- **d** the sun rising in the north
- **e** flipping a coin and getting a tail
- **f** selecting a consonant from the word RHYTHM
- **g** selecting a picture card from a deck of cards
- **h** rolling a die and getting a number greater than 2

EXAMPLE 9A-1 Listing sample space

List the sample space for randomly selecting a letter from the word SIMULTANEOUS.

THINK	WRITE
List every different outcome within curly brackets.	{S, I, M, U, L, T, A, N, E, O}

2 List the sample space for each experiment.

- **a** rolling a die
- **b** randomly selecting a letter from the word TECHNOLOGY
- **c** drawing a card from a deck and recording its suit
- **d** spinning this spinner
- **e** randomly selecting a person and recording their birth day of week
- **f** drawing a card from a deck and recording if it is a picture card

EXAMPLE 9A-2 Identifying equally likely outcomes

State whether these experiments have equally likely outcomes.

a flipping a coin **b** selecting a letter at random from the word TELEPHONE

THINK	WRITE
a The possible outcomes are {head, tail}. No outcome is more likely to be selected than any other.	**a** Flipping a coin has two equally likely outcomes.
b The possible outcomes are {T, E, L, P, H, O, N}, where the E is three times as likely to be selected as any other letter.	**b** Selecting a letter at random from the word TELEPHONE does not have equally likely outcomes.

SAMPLE

UNDERSTANDING AND FLUENCY

3 State whether these experiments have equally likely outcomes.

- **a** selecting a marble at random from a bag containing four blue, four yellow, four green and four red marbles
- **b** rolling a die and recording if the number is less than or greater than 3
- **c** selecting a letter at random from the word REGULAR
- **d** selecting a person at random from your class and recording gender
- **e** selecting a person at random and checking if they are left- or right-handed
- **f** selecting a letter at random from the word SUPERB

4 State whether the outcomes in the sample spaces for the experiments in question **2** are equally likely to occur or not.

EXAMPLE 9A-3 Calculating theoretical probability

Find the theoretical probability of rolling a die and obtaining a number greater than 4.

THINK	WRITE
1 Write the formula for theoretical probability.	$\text{Pr(event)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
2 Find the total number of possible outcomes.	total number of outcomes = 6
3 Find the number of favourable outcomes.	Favourable outcomes are rolling a 5 or a 6. number of favourable outcomes = 2.
4 Substitute these values into the formula and write the resulting fraction in simplest form.	$\text{Pr(event)} = \dfrac{2}{6} = \dfrac{1}{3}$

5 Find the theoretical probability of:

- **a** rolling a die and obtaining a 4
- **b** flipping a coin and obtaining a tail
- **c** randomly selecting a C from the word EXCLAIM
- **d** drawing a card from a deck and obtaining an ace
- **e** guessing the correct answer to a multiple-choice question with options A–D
- **f** randomly selecting an R from the word CHARGRILLED.

6 Find the theoretical probability of:

- **a** rolling a die and obtaining a number less than 4
- **b** selecting a picture card from a deck of cards
- **c** spinning green or blue on the spinner shown
- **d** randomly selecting a vowel from the word INTERACTIVE
- **e** rolling a die and obtaining any number except 1
- **f** randomly selecting an N, P or I from the word EMANCIPATION.

SAMPLE

PROBLEM SOLVING AND REASONING

7 Use this spinner to calculate the probability of spinning:

- **a** red or blue
- **b** an odd number
- **c** 1 or 2
- **d** a 5 or green
- **e** yellow and 2
- **f** green but not 4.

8 For each of these spinners:

- **i** write the sample space
- **ii** state which colour you would bet on if the spinner was spun, giving a reason
- **iii** find the theoretical probability of spinning red.

a

b

c

9 Explain the difference between an outcome that has an even chance of occurring and outcomes that are equally likely.

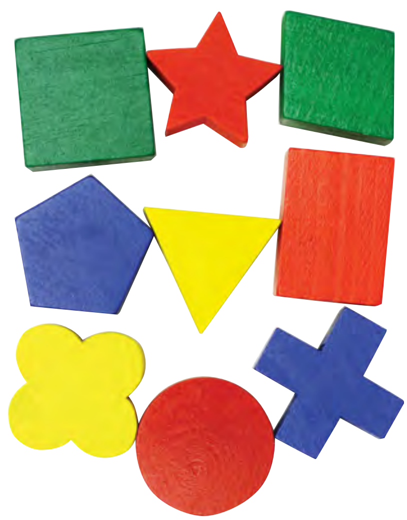
10 Provide an example of an experiment with:

- **a** outcomes that are equally likely, but do not have an even chance of occurring
- **b** outcomes that are equally likely and have an even chance of occurring
- **c** an outcome that has an even chance of occurring but is not equally likely to other outcomes.

11 For each of these experiments:

- **i** list a sample space where the outcomes are equally likely
- **ii** list a sample space where the outcomes are *not* equally likely
- **iii** choose a single outcome from part **i** and calculate its theoretical probability
- **iv** classify the outcome from part **iii** as belonging to the sample space from part **ii** and recalculate its theoretical probability.

- **a** drawing a card from a deck
- **b** selecting a letter from the word COMPUTER
- **c** selecting a coin from Australian currency
- **d** selecting a day of the week
- **e** selecting a shape from this photograph



SAMPLE

12 A number of people made some probability statements that were not quite correct. Explain where each person went wrong and provide a better statement.

- **a** Adele said that the probability of spinning purple on the spinner from question **8a** was $\frac{1}{5}$.
- **b** Thanh flipped a coin five times and got tails each time. He then said that he was very unlikely to get another tail.
- **c** Ethan said that he had an even chance of rolling a 6 on a fair die.
- **d** Bianca said that she was highly likely to spin blue on the spinner from question **8c**.

13 Consider a lucky dip for gift vouchers: 5 are for $50, 10 are for $20 and 15 are for $5.

- **a** List the three different outcomes in this lucky dip.
- **b** Explain why the chance of selecting a $50 gift voucher is not $\frac{1}{3}$.

14 Consider this pile of lollies.

- **a** What is the probability of selecting a red lolly?
- **b** What is the probability of *not* selecting a red lolly?
- **c** What do these two probabilities add to?

Events that are 'opposite' to one another, such as selecting a red lolly and not selecting a red lolly, are complementary events.

- **d** Find the complementary event to:
  - **i** rolling a die and obtaining a 6
  - **ii** selecting a heart card from a deck of cards
  - **iii** flipping a coin and obtaining a tail
  - **iv** rolling a die and obtaining an even number
  - **v** selecting a picture card from a deck of cards
  - **vi** selecting a consonant from this sentence.
- **e** For each event listed in part **d**, find the probability of
  - **i** the event
  - **ii** its complementary event.
- **f** Find the sum of the probability of each event and its complementary event from part **d**. What do you find?
- **g** What can you say about the sum of the probability of complementary events?

SAMPLE

PROBLEM SOLVING AND REASONING

15 In mathematical notation, the probability of an event can be written as Pr(x), where x can be substituted for any letter (usually a capital letter) or even words or a phrase. The probability of a complementary event is denoted using the symbol prime (′); for example the complementary event to A is A′.

a What is Pr(A) + Pr(A′)? (Hint: what do the probabilities of complementary events add to?)

b Find:

i Pr(E) when Pr(E′) = 0.7

ii Pr(W′) when Pr(W) = $\frac{1}{3}$

iii Pr(Y) when Pr(Y′) = $\frac{8}{9}$

iv Pr(M′) when Pr(M) = 0.16

16 Investigate how useful a **probability scale** is in real life.

a Draw a probability scale from 0 to 1, but rather than using decimal numbers (for example, 0.1), use fractions (for example, $\frac{1}{10}$).

b At each of the marks, place a word that describes each probability; for example, impossible, even chance, highly likely etc.

c Place each of these events onto the probability scale you have drawn and hence use a word or phrase to describe the probability of each event occurring.

i a baby being a girl

ii being born on a Monday

iii having at least one day in summer over 25°C

iv not being selected out of a group of three people

vi winning the lottery

d How do you think each event matches to its description and fractional probability? For example, winning the lottery would be placed as close to 0 as possible, which on this scale would give a probability of $\frac{1}{10}$ and a possible description of 'highly unlikely'. Do you think either of these descriptions accurately match the probability of winning the lottery?

e What might you be able to say about events that are close to the middle of the scale versus events that are closer to the ends of the scale?

f How might you improve the probability scale to allow it to better describe the probability of real-life events?

CHALLENGE

17 A spinner with four different colours has a $\frac{1}{5}$ chance of spinning blue, a $\frac{1}{3}$ chance of spinning green and a $\frac{1}{6}$ chance of spinning red.

a What is the probability of spinning the remaining colour (yellow)?

b Draw two examples of spinners that meet this description.

Reflect

Why is it important to establish if outcomes are equally likely before calculating theoretical probability?

SAMPLE

9B Experimental probability and relative frequency

Start thinking!

Experimental probability (or **relative frequency**) is calculated using the results of an experiment rather than using theoretical probability. It is more commonly written as a decimal number rather than a fraction (for example, 0.1 rather than $\frac{1}{10}$) and can be found using the formula

$$\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}.$$

Consider a standard die.

1 List its sample space.

2 List the theoretical probability of each outcome.

3 If it was rolled 60 times, how many times would you expect to obtain a 6?

4 Explain how you got your answer to question **3**.

Before conducting an experiment, it can be useful to calculate the **expected number** of each outcome. This can help to determine if there is any bias in an experiment. To calculate expected number, multiply the theoretical probability by the number of trials in the experiment. This can be written as $E(x) = Pr(x) \times n$.

5 Explain how this is the same as your answer to question **4**.

6 Write down what $E(x)$, $Pr(x)$ and n represent in the expected number formula.

7 Calculate the expected number of each outcome if a die was rolled 30 times.

Imagine that a die was rolled 30 times and the results in the table were obtained.

Outcome	1	2	3	4	5	6
Frequency	2	6	4	5	3	10

8 How do these numbers differ from what you found in question **7**?

9 Copy the table and add two rows. In the first additional row, write the theoretical probability of each outcome as a decimal number. In the second additional row, calculate the relative frequency of each outcome.

KEY IDEAS

- To calculate experimental probability, use the formula $\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}$.
- Before conducting an experiment, calculate the expected number of successful outcomes.
- To calculate expected number, multiply theoretical probability by the number of trials. This can be written as $E(x) = Pr(x) \times n$.
- In an individual experiment, the experimental probability may not match the theoretical probability. However, as the number of trials increases, the experimental probability should get closer to the theoretical probability.

SAMPLE

EXERCISE 9B Experimental probability and relative frequency

EXAMPLE 9B-1 Calculating experimental probability

Find the experimental probability of rolling a 6 if a die is rolled 80 times and 6 is obtained 18 times.

THINK	WRITE
1 Write the experimental probability formula.	$\Pr(\text{success}) = \dfrac{\text{number of successful trials}}{\text{total number of trials}}$
2 Identify the number of successful trials (18) and the total number of trials (80). Substitute into the formula and solve, simplifying if possible.	$= \dfrac{18}{80}$ $= \dfrac{9}{40}$
3 Convert to a decimal number and write your answer.	The experimental probability of obtaining a 6 in this experiment is 0.225.

1 Find the experimental probability for each of these.

a rolling a 4, if a die is rolled 180 times and a 4 is obtained 20 times

b drawing an ace, if a card is drawn from a deck and replaced 100 times and an ace is obtained 5 times

c flipping a tail, if a coin is flipped 36 times and a head is obtained 16 times

d rolling an odd number, if a die is rolled 200 and an odd number is obtained 87 times

e drawing a club, if card is drawn from a deck and replaced 250 times and a club is obtained 92 times

f guessing the correct answer, if 10 answers were guessed correctly out of 50

EXAMPLE 9B-2 Calculating expected number

Find the expected number of 5s or 6s if a die is rolled 90 times.

THINK	WRITE
1 Write the formula for expected number.	$E(x) = \Pr(x) \times n$
2 Identify the theoretical probability of rolling a 5 or a 6 ($\frac{2}{6}$) and the number of trials (90) and substitute into the formula.	$= \dfrac{2}{6} \times 90$ $= 30$

SAMPLE

2 Find the expected number of:
 a heads if a coin is flipped 250 times
 b 1s or 2s if a die is rolled 120 times
 c hearts if a card is drawn from a deck and replaced 100 times
 d 6s if a die is rolled 30 times
 e consonants if a letter is selected randomly from the alphabet 130 times
 f picture cards if a card is drawn from a deck and replaced 260 times.

EXAMPLE 9B-3 Describing long-term probability

The experiment described in Example 9B-2 is performed and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	13	11	12	17	21	16

Find the experimental probability of rolling a 5 or a 6 and describe how you expect this to change in the long term.

THINK

1 Write the formula for experimental probability.

2 Identify the number of successful trials (21 fives and 16 sixes) out of the total number of trials (90) and substitute into the formula.

3 In the long term, experimental probability should approach theoretical probability ($\frac{2}{6} \approx 0.33$).

WRITE

$$\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

$$\text{Pr(5 or 6)} = \frac{37}{90} \approx 0.41$$

In the long term you would expect the experimental probability of rolling a die and obtaining a 5 or 6 to decrease as it approaches theoretical probability.

SAMPLE

UNDERSTANDING AND FLUENCY

3 A die is rolled 150 times and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	38	32	19	24	21	16

 a Find the experimental probability of rolling a number greater than 2.
 b Describe how you expect the experimental probability of rolling a number greater than 2 to change in the long term.

4 A card is drawn from a deck and replaced, and this is repeated 130 times.
 a What is the expected number of picture cards?
 b If seven picture cards were obtained, find the experimental probability of drawing a picture card.
 c Describe how you expect the experimental probability of drawing a picture card to change in the long term.

PROBLEM SOLVING AND REASONING

5 Three coins are flipped, and this is repeated 200 times. If three tails appear 22 times, describe how you expect the experimental probability of flipping three tails to change in the long term.

6 A magician uses a number of props in his show, but you aren't sure that they are fair. You watch and record his movements over several shows, and results are shown in the tables below. For each experiment:

- **i** find the total number of trials
- **ii** state the theoretical probability of each outcome
- **iii** calculate the expected number of each outcome
- **iv** calculate the relative frequency of each outcome
- **v** state if you think the prop used is fair, biased, or if there are not enough trials to make a firm decision
- **vi** give a reason to support your answer to part **v**.

a

Outcome	Heads	Tails
Frequency	8	2

b

Outcome	1	2	3	4	5	6
Frequency	966	971	1036	994	1031	1002

c

Outcome	Hearts	Diamonds	Clubs	Spades
Frequency	38	32	19	24

7 If you flip three coins (5c, 10c and 20c) at the same time, how often would you expect to get a triple heads or tails?

- **a** Make a list of all the possible outcomes. (Hint: there are eight.) How many of these are 'triples'?
- **b** What is the theoretical probability of flipping a 'triple'?
- **c** How many 'triples' would you expect to get if you performed 40 trials?
- **d** Perform 40 trials of the experiment and record your results.
- **e** Does the relative frequency of a 'triple' match the theoretical probability?
- **f** Describe how you would expect this relative frequency to change if you performed 4000 trials.

8 If you roll two dice (one red and one blue), how often would expect to get a 'double' number? Follow the steps shown in question **7** and perform at least 30 trials of the experiment. Discuss your results. (Hint: there are 36 outcomes.)

SAMPLE

9 A number of experiments were performed and their results recorded below. For each experiment, find the number of times each outcome occurred.

a total number of trials = 40

Outcome	Heads	Tails
Relative requency	0.625	0.375

b total number of trials = 120

Outcome	1	2	3	4	5	6
Relative requency	0.15	0.2	0.175	0.1	0.125	0.25

c total number of trials = 60

Outcome	Hearts	Diamonds	Clubs	Spades
Relative requency	0.2	0.3	0.35	0.15

SAMPLE

10 In real life, it can be difficult to perform a large number of trials in an experiment. **Simulations** can be used to generate results when these experiments are impractical. A simulation makes use of a simple random device, such as a coin, die or spinner, or digital technology that generates random outcomes. When planning to perform a simulation, it is important that all outcomes listed are equally likely and each outcome of a device is matched to each outcome of the experiment.

a List the number of outcomes in these real-life situations.

i a baby's gender at birth

ii guessing the answer to a multiple choice question with answers A, B, C or D

iii selecting a prize from a lucky dip with three different prizes that come in two different colours each

iv selecting your favourite flavour Clinker from a bag (from pink, green and yellow)

b For each situation in part **a**, list:

i a device with the same number of outcomes that could be used to simulate the situation

ii at least two limitations of using the device in order to simulate the situation.

11 Sometimes chocolate companies have promotions where one in six chocolate bars wins a free bar.

a Explain why, even though there are only two outcomes (winning and not winning), you can treat this problem like it has six outcomes.

b Select a device to simulate winning a free chocolate bar and perform as many trials as necessary in order to simulate winning a free bar.

c Repeat part **b** another 19 times and hence state the average number of bars you would have to buy in order to win a free bar.

CHALLENGE

12 A classic probability problem that confuses many people is the Monty Hall problem. The problem is as follows:

Imagine that you are a contestant on a game show. You are shown three doors and told that behind one door is a car, and behind the other two doors are goats.

If you correctly select the right door you win the car. After selecting one of the doors (say door 2), the host opens up another door (say door 1) to show a goat. The host then asks you if you want to stay with door 2 or switch to door 3. Should you switch? Is it an advantage, a disadvantage, or does it not matter if you switch?

a Decide if you would switch or stay.

b What is the probability that you correctly select the car (say, door 2)?

c What is the probability that you select a goat?

d Does showing you what is behind another door (say, door 1) change the probability of your initial selection?

e Explain why there is still a $\frac{1}{3}$ chance that the car is behind door 2 and therefore a $\frac{2}{3}$ chance that the car is not behind door 2.

f Use your answer to part **e** and the fact that there is a goat behind door 1 to explain why there is a $\frac{2}{3}$ chance that the car is behind door 3 and hence it is better to switch.

Many people believe that after seeing that there is a goat behind door 1 that there is now a 50% chance that you selected correctly. Sometimes, some perspective can help. Imagine now that rather being offered to pick one door out of three, you were offered to pick one door out of 1 000 000.

g What is the probability you would correctly select the car now?

Imagine that all doors except the one you picked and one other were opened.

h Would you switch or stay? Why does this seem more obvious than the original problem?

Still, some people can confuse the initial probability of guessing correctly with having only two options left. Use a simulation to explore this.

i With a classmate, obtain materials to simulate this problem. It may be as simple as writing ‘car’, ‘goat’, ‘goat’ on three pieces of paper. Set up the experiment so that one person is the host and the other is the contestant.

j The contestant should decide on a strategy: to switch or stay; and do this consistently for 20 trials.

k Perform 20 trials and record your results. What is the relative frequency of your chosen strategy?

l Switch who is the host and who is the contestant, and now perform 20 trials of the other strategy, recording your results. What is the relative frequency of this other strategy?

m Your results should have roughly given a relative frequency of 0.67 for switching and 0.33 for staying. Did your results reflect this? What do you think would happen to your results if you performed 2000 trials?

Reflect

How is experimental probability related to theoretical probability?

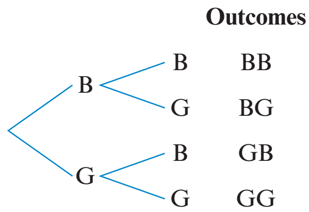
SAMPLE

9C Tree diagrams

Start thinking!

When performing multi-step experiments (or tracking multi-step events), you can use a **tree diagram** to display the possible outcomes.

Consider this tree diagram.



1 What experiment is it showing?

2 How many steps are there in this tree diagram?

3 How many possible outcomes are there? List each one.

4 How many outcomes involve at least one boy?

5 If a family has two children, what is the probability that at least one of the children is a boy?

6 Copy the tree diagram and add another branch to represent the family having a third child.

7 How many outcomes now involve at least one boy?

8 If a family has three children, what is the probability that at least one of the children is a boy?

9 How does drawing a tree diagram help to calculate probabilities in multi-step experiments and events?

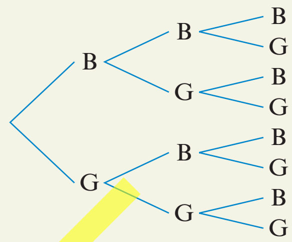
SAMPLE

KEY IDEAS

- Tree diagrams display the outcomes of multi-step experiments.
- The possibilities for each step of the experiment are represented by a number of branches.
- The final outcomes are listed at the end of the branches.
- This list of final outcomes can be used to calculate the probability of an outcome occurring.

EXERCISE 9C Tree diagrams

EXAMPLE 9C-1 Understanding tree diagrams



Use this tree diagram to find:

a the total number of outcomes
b the number of outcomes containing at least one girl
c the probability of a family of three children containing at least one girl.

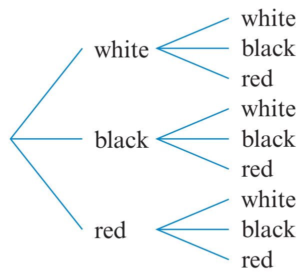
THINK	WRITE
a Count the number of final outcomes at the right end of the tree diagram.	a There are eight possible outcomes.
b Trace the branches carefully and count the number that contain a girl.	b Seven outcomes contain a girl.
c Consider the number of favourable outcomes (7) out of the total number of possible outcomes (8).	c Pr(at least one girl) = $\frac{7}{8}$

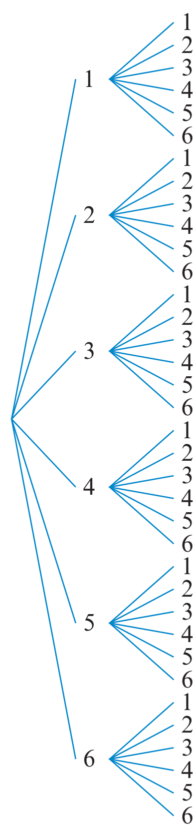
SAMPLE

UNDERSTANDING AND FLUENCY

1 Consider the tree diagram at far right.
   a How many possible outcomes are there?
   b How many of these outcomes contain a 6?
   c How many of these outcomes contain a double number?

2 Consider this tree diagram.
   a How many possible outcomes are there?
   b How many of these outcomes contain at least one red?
   c What is the probability of drawing at least one red?

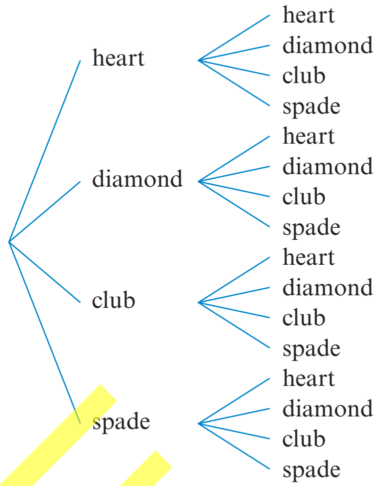




UNDERSTANDING AND FLUENCY

3 Use this tree diagram to find the probability of:

- **a** drawing at least one diamond
- **b** drawing a club and a heart
- **c** drawing two spades
- **d** not drawing a heart
- **e** drawing a diamond or a spade
- **f** drawing a spade then a club.



4 Use the tree diagram from question **1** to find the probability of:

- **a** rolling a double 6
- **b** rolling at least one even number
- **c** rolling two odd numbers
- **d** rolling at least one 4
- **e** rolling a double
- **f** rolling a total of 6.

SAMPLE

EXAMPLE 9C-2 Calculating probability using a tree diagram

Use a tree diagram to calculate the probability of flipping at least three tails when flipping a coin four times.

THINK

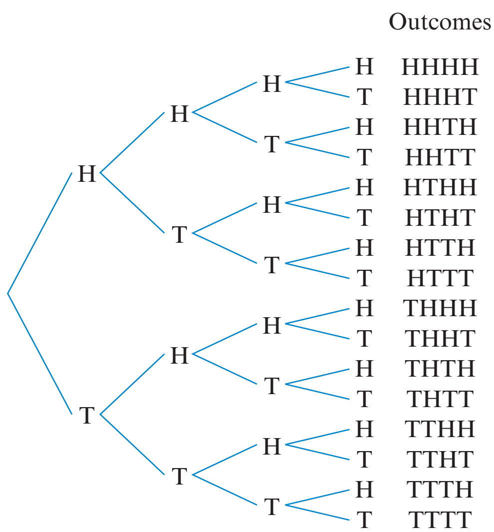
1 Draw the first two branches to represent the first coin flip. Label the end of these branches with H and T to represent the two different outcomes.

2 From each branch, draw another two branches to represent the next coin flip and label them appropriately. Repeat this twice more so that you are representing the four coin flips.

3 At the end of each of the 16 branches, write the final outcome to complete the tree diagram.

4 There are five outcomes that contain at least three tails (three tails or four tails) out of a possible 16 outcomes.

WRITE



$\text{Pr(at least three tails)} = \frac{5}{16} = 0.3125$

UNDERSTANDING AND FLUENCY

5 Use a tree diagram to find the probability of:

a exactly three tails when flipping a coin four times

b no more than one head when flipping a coin four times

c flipping a coin four times and getting the same outcome each time.

6 A coin was flipped three times. Use a tree diagram to find the probability of:

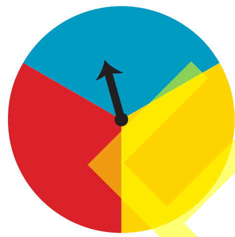
a at least two heads

b no tails

c exactly two tails

d only one head.

7 This spinner was spun three times.
Use a tree diagram to find the probability of:



a three different results

b spinning blue at least once

c spinning the same colour each time

d spinning red each time.

8 A coin was flipped five times. Use a tree diagram to find the probability of:

a no heads

b at least one tail

c exactly three tails

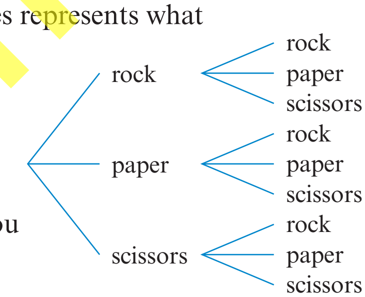
d at least three tails

e more than one head

f less than two tails.

PROBLEM SOLVING AND REASONING

9 This tree diagram represents a single round of a game of rock, paper, scissors. The first set of branches represents what you choose and the second set of branches represents what your opponent chooses.



a List the outcomes that result in you

i winning

ii losing

iii drawing the game.

b Hence calculate the probability that you win a game of rock, paper, scissors.

10 Tree diagrams aren't limited to repeated trials of the same experiment. They can also be used to display unrelated events. Imagine that you flip a coin and then roll a die.

a Draw a tree diagram to represent this multi-step experiment.

b How many outcomes are there?

c What is the probability that:

i you flip a tail and roll a six?

ii you flip a tail and roll a number less than four?

iii you flip a tail or roll a six?

iv you flip a tail or roll a number less than four?

d Explain why the answer to part **c iii** is not $\frac{8}{12}$.

11 Use a tree diagram to find the probability in a family of four children that:

a all are girls

b at least one is a boy

c the first child is a boy

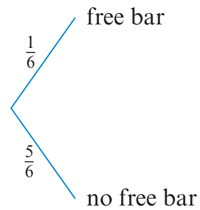
d two are boys

e more than one is a girl

f at least two are girls.

SAMPLE

12 All the tree diagrams you have looked at so far assume that each outcome is equally as likely as any other. However, this is not always the case. Consider the situation mentioned in Exercise 9B question **11** on page 420, where one in every six chocolate bars wins a free bar.



a Explain how this tree diagram represents the probabilities of winning a free chocolate bar when you buy a single chocolate bar.

b Extend this tree diagram so that it represents buying three chocolate bars.

c Write down the final outcomes at the ends of the third set of branches.

To find the probability of each final outcome, you simply multiply together the probabilities of the branches that you move across. For example, the probability of winning a free bar with the first purchase, but then not again (FNN) is $\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{25}{216} \approx 0.12$.

d Find the probability of each of the final outcomes, expressing them both as fractions and as decimals rounded to three decimal places.

e Add together these eight probabilities. What do you find?

f When purchasing three chocolate bars, what is the probability of:

- **i** not winning a free bar?
- **ii** winning a free bar each time?
- **iii** winning a free bar with your second but not first or third purchase?

Each of the parts in part **f** are single final outcomes. To calculate the probability of an event that involves more than one final outcome, you need to add the probabilities of each favourable final outcome.

g Which final outcomes involve winning one free bar? Add together the probabilities of these final outcomes to find the probability of winning one free bar.

h Find the probability of winning:

- **i** at least one free bar
- **ii** winning a free bar with your first purchase
- **iii** two free bars
- **iv** more than one bar.

i How does a tree diagram help to find the probability when outcomes are not equally likely?

13 Consider sitting a quiz consisting of five multiple-choice questions, with answers A–D.

a What is the probability of correctly guessing a problem with four possibilities?

b Draw a tree diagram complete with probabilities to represent guessing the answers to these five questions.

c Use your tree diagram to calculate the probability of correctly guessing:

- **i** all five questions
- **ii** no questions
- **iii** two questions
- **iv** at least one question
- **v** less than four questions
- **vi** at least three questions.

14 A tree diagram is helpful but not necessary when calculating the probabilities of outcomes in multi-step experiments. Rather than drawing a tree diagram, you can just write a list of the outcomes. But how do you know how many outcomes to write down?

SAMPLE

CHALLENGE

Situation	Number of branches at each trial	Number of trials	Number of outcomes
Flipping a coin four times	2	4	
Selecting a card from a deck three times and noting its suit	4		
Rolling a die twice		2	
Recording the gender of three children at birth		3	
Recording two rounds of rock, paper, scissors	3		

a Copy and complete this table.

b Can you see a pattern between the number of outcomes and the other two numbers in the table? (Hint: it involves powers of numbers (repeated multiplication).)

c Explain how the number of outcomes for flipping a coin five times can be calculated using 2^5. How many outcomes is this?

d Describe how to find the total number of outcomes for any experiment with repeated trials.

e When writing down the outcomes for experiments without a tree diagram, what strategy might you use to ensure that you don't leave out any outcomes?

15 Consider a similar situation to question **12**; however, this time one in every five chocolate bars wins. Imagine that you bought four chocolate bars.

a Using the strategy from question **14**, state how many outcomes there could be and list them.

b What is the probability of winning a free bar (F)?

c What is the probability of not winning a free bar (N)?

To find the probability of each outcome, simply multiply together the probabilities like you did in question **12**. For example, the outcome FNNF would have the probability $\frac{1}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} = \frac{16}{625} \approx 0.0256$.

d Calculate the probability of each outcome from your list in part **a**. Write each probability as both a fraction and a decimal (to four decimal places). Check that they all add to 1 to be sure you calculated correctly.

e Use these probabilities to find the probability of:

- **i** not winning a free bar
- **ii** winning four free bars
- **iii** winning a free bar with your first purchase but then not again
- **iv** winning at least one free bar
- **v** winning two free bars
- **vi** winning a free bar with your last purchase.

16 The game of Yahtzee involves five dice. To get a 'Yahtzee', you need to roll the same value on all five dice. Use a tree diagram or the strategy shown in questions **14** and **15** to find the probability of rolling a Yahtzee.

Reflect

How do tree diagrams help you calculate probability in multi-step experiments?

SAMPLE

9D Two-way tables

Start thinking!

Two-way tables can also be used to display outcomes for a two-step experiment.

Consider a family having two children.

	Boy	Girl
Boy	B, B	B, G
Girl	G, B	G, G

1 How does a two-way table display the four possible outcomes?

A two-way table is more commonly used to display the relationship between different sets of data. Consider this two-way table.

	Dark hair	Light hair	Total
Dark eyes	16	4	20
Pale eyes	8	12	20
Total	24	16	40

2 What data is it showing?

3 Of the people surveyed, 24 had dark hair.
How many people with dark hair also had dark eyes?

4 Twelve people had light hair and pale eyes. How many people had pale eyes in total?

5 How many people were surveyed in total?

6 Use your answer to question **5** to help you calculate the probability of a person selected randomly having:

a dark hair **b** light hair and pale eyes
c dark eyes **d** dark hair and dark eyes.

7 Alice said that the probability of a person selected at random having dark hair and dark eyes was $\frac{16}{24}$. Explain where she went wrong.

8 Explain why, even though there are four different outcomes, the probability of each outcome is not $\frac{1}{4}$.

9 How does a two-way table assist in calculating probabilities?

SAMPLE

KEY IDEAS

- A two-way table is another way to display the outcomes of an experiment or survey.
- You can use two-way tables to calculate the probabilities using outcome results and totals.
- **Conditional probability** is the probability of an outcome, given conditions. 'Calculate the probability that a randomly selected card is an ace given that it is a red card' is an example of conditional probability.

EXERCISE 9D Two-way tables

UNDERSTANDING AND FLUENCY

1 Consider this two-way table.

	Male	Female	Total
Prefer sweet food	23	32	55
Prefer savoury food	29	16	45
Total	52	48	100

- **a** What are the four different outcomes?
- **b** How many people were surveyed in total?
- **c** How many males prefer savoury food?
- **d** How many females were surveyed?
- **e** What does the number 32 represent?
- **f** What does the number 45 represent?

EXAMPLE 9D-1 Understanding a two-way table

Consider this two-way table.

	Year 8	Year 9	Total
Jam	28	38	66
Vegemite	25	34	59
Total	53	72	125

- **a** How many students were surveyed in total?
- **b** How many students in Year 9 prefer Vegemite?
- **c** What is the probability that a student chosen randomly from the group in Year 9 prefers Vegemite?

THINK

a Check the bottom right corner cell for the total number of people surveyed.

b Find the cell that is in the 'Year 9' column and the 'Vegemite' row.

c Consider the number of favourable outcomes (34) out of the total number of possible outcomes (125) in this table.

WRITE

a 125 students were surveyed in total.

b 34 students in Year 9 prefer vegemite on their toast.

c Pr(Year 9 student who prefers Vegemite)

$= \frac{34}{125}$

$= 0.272$

SAMPLE

2 Consider this two-way table.

	Primary school	High school	Total
Watch sport	8	7	15
Play sport	22	18	40
Total	30	25	55

- **a** How many students were surveyed in total?
- **b** How many students in high school prefer to watch sport?
- **c** What is the probability of a student chosen randomly from the group being in high school and preferring to watch sport?

3 Consider this two-way table.

	Fair	Dark	Total
Blue/Green	23	11	34
Brown	6	35	41
Total	29	46	75

a How many people were surveyed in total?

b How many people with dark hair have blue eyes?

c What is the probability of a person chosen randomly from the group having dark hair and blue eyes?

EXAMPLE 9D-2 Calculating probability using a two-way table

Use this two-way table to find the probability of a person chosen randomly from the group being a male who does not have a pet.

	Male	Female	Total
Owns pet	11	14	25
Does not own pet	6	4	10
Total	17	18	35

THINK	WRITE
1 Locate the cell that shows the number of favourable outcomes: males who do not own a pet.	Six males do not own a pet.
2 Locate the cell that shows the total number of people surveyed.	35 people were surveyed in total.
3 Write this as a fraction and simplify if possible. You may also like to express your answer as a decimal number.	$\Pr(x) = \frac{6}{35} \approx 0.17$

UNDERSTANDING AND FLUENCY

4 Use this two-way table to find the probability of a person chosen randomly from the group:

	Year 8	Year 9	Total
Alternative	14	23	37
Mainstream	29	19	48
Total	43	42	85

a being a Year 8 student who prefers mainstream music

b preferring alternative music

c being a Year 9 student

d being a Year 9 student who prefers alternative music.

5 Use this two-way table to find the probability that a person chosen randomly from the group:

	Short	Tall	Total
Light hair	16	18	34
Dark hair	21	23	44
Total	37	41	78

a is short with light hair

b is tall

c has dark hair

d is tall with dark hair.

SAMPLE

UNDERSTANDING AND FLUENCY

6 Use this two-way table to find the probability that a person chosen randomly from the group:

	City	Country	Total
Automatic	67	24	91
Manual	15	44	59
Total	82	68	150

a is from the country and drives a manual car
b drives an automatic car
c is from the city
d is from the city and drives an automatic car.

7 Copy and complete the two-way table below using the buttons in this photograph.

	Green	Not green	Total
Two holes			
Four holes			
Total			

8 Use this two-way table to find the probability that a person chosen randomly from this group:

	Year 7	Year 8	Year 9	Total
Tennis	23	26	37	86
Basketball	19	42	34	95
Hockey	31	13	25	69
Total	73	81	96	250

a is in Year 9
b plays basketball
c is in Year 8 and plays tennis
d plays hockey or tennis **e** does not play hockey
f is in Year 7 and doesn't play basketball.

PROBLEM SOLVING AND REASONING

9 **a** Copy and complete this two-way table.

	Cinema	Home	Total
Action	22		
Comedy		14	33
Total			85

b Use it to calculate the probability that a person chosen randomly from the group is someone who on the weekend:
i went to the cinema **ii** watched a comedy film at home
iii watched an action film.

10 Two dice are rolled and the numbers that are uppermost are added together to give the final outcome.
a Create a two-way table that lists all the outcomes. (Hint: List the possibilities for die 1 across the top row and the possibilities for die 2 down the first column.)
b How many different outcomes are there?
c What is the most likely outcome? What is the probability of this occurring?
d What is/are the least likely outcome(s)? What is the probability of this/these occurring?
e State the probability of rolling two dice and obtaining a sum of:
i four **ii** greater than 10 **iii** an odd number **iv** less than seven.

SAMPLE

PROBLEM SOLVING AND REASONING

11 Consider this two-way table.

	Year 8	Year 9	Total
Maths		11	
English	10		
Total	22		50

- **a** Copy and complete the table.
- **b** Use it to find the probability that a person selected randomly from the group:
  - **i** likes Maths
  - **ii** is in Year 9
  - **iii** is in Year 9 and likes Maths.
- **c** A person is selected randomly from the group. You know that they are in Year 9.
  - **i** How many people are in Year 9?
  - **ii** How many people in Year 9 like Maths?
  - **iii** What is the probability that somebody in Year 9 likes Maths?
- **d** What is the difference between parts **b iii** and **c iii**?

The problem represented in part **c iii** is an example of conditional probability. It looks at the probability of an outcome given certain conditions. In part **c iii**, you are looking for the probability that somebody likes Maths given that they are in Year 9. This means that you consider only the limited group of the condition rather than the entire population.

- **e** Use the following steps to calculate the probability of somebody being in Year 8 given that they like English.
  - **i** What is the condition? How many people in this group?
  - **ii** What is the specific group you are after? (Hint: it is not just somebody in Year 8.)
  - **iii** How many people are in this specific group?
  - **iv** Use your answers to parts **i** and **iii** to calculate the probability of selecting somebody being in Year 8 given that they like English.

12 Use the two-way table in question **8** to calculate the probability of randomly selecting a person from the group that:

- **a** plays hockey given that they are in Year 8
- **b** plays tennis given that they are in Year 9
- **c** is in Year 7 given that they play basketball
- **d** is in Year 8 given that they play tennis
- **e** is in Year 9 given that they *don't* play hockey
- **f** is not in Year 7 given that they play basketball.

13 A group of 200 people with a single pet were surveyed on their pets. Of the 113 who owned cats, 29 had a specific breed. This gives a total of 104 pet owners who owned a specific breed. Create a two-way table showing this information and use it to calculate the probability that a person chosen at random is an owner of a cat of a specific breed.

SAMPLE

CHALLENGE

14 Elsa recorded the make and colour of cars that went past her house over a week and recorded her results as shown. Add totals to her results and then find the probability that a car going past her house is:

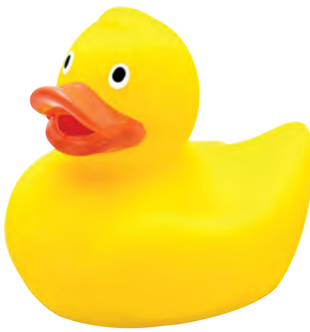
a white
b a Holden
c a white Holden
d white or a Holden
e white given that is a Holden
f a Holden given that it is white
g not a Holden
h not white
i neither white nor a Holden
j white but not a Holden
k a Holden but not white
l not white given that is a Holden.

	Ford	Hyundai	Holden	Mitsubishi	Mazda	Toyota
Silver	11	9	17	8	6	12
White	15	12	16	11	8	10
Red	12	10	8	6	13	14
Blue	8	15	12	7	9	11
Black	13	8	10	10	7	8

15 What is the difference between parts **k** and **l** in question **14**?

16 Use the two-way table shown in question **14** and create four probability questions. Swap these with a classmate and discuss any differences in answers.

17 A group of people were surveyed on their bathing habits. Sixty per cent of women surveyed said they preferred a bath over a shower, whereas 80% of men said they preferred to have a shower rather than a bath. Fifty-five per cent of the group was female.



a Create a two-way table showing these percentages as relative frequencies. Remember that each row and column should add correctly to their totals. (Hint: the statement '60% of women prefer a bath' refers to 60% of the proportion of women, not the total.)

b If a person was randomly selected from a group of 500 people, find the probability that they are:

i a male who prefers to shower
ii someone who prefers a bath
iii a female who prefers to shower.

c Of a group of 500 people, find the number of people who:

i prefer a bath to a shower
ii are female
iii are male and prefer a bath.

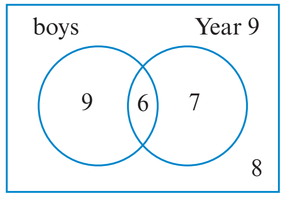
Reflect

How do two-way tables allow you to calculate probabilities when examining the relationship of two different things?

SAMPLE

9E Venn diagrams

Start thinking!

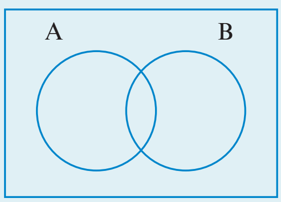


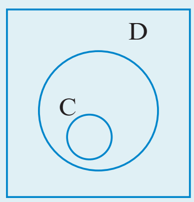
Venn diagrams display the relationship between different sets of data. They consist of a number of circles within a rectangle. Consider this Venn diagram.

1 What are the two sets of data that it is showing?
2 The first circle represents the boys in the group. Which two numbers are in this first circle?
3 Use your answer to question **2** to state how many boys in total are in the group.
4 The second circle represents Year 9 students in the group. Which two numbers are in this second circle?
5 Use your answer to question **4** to state how many Year 9 students in total are in the group.
6 The crossover of the two circles represents people who are in both sets. How many boys are in Year 9?
7 How many girls are in Year 9? (Hint: they belong to the second set but not the first.)
8 The number outside the circles but in the rectangle represents people that don't belong to either set. Describe this set of people.
9 Add all the numbers in the Venn diagram to find the total number of people in the group.
10 Use your answers to calculate the probability of selecting a:
   a boy in Year 9 **b** boy **c** Year 9 student
   d boy not in Year 9 **e** girl in Year 9 **f** girl not in Year 9.
11 The same information can be displayed in a two-way table. Discuss with a classmate the advantages and disadvantages of each display. Which do you prefer?

KEY IDEAS

- A Venn diagram is used to display the relationship between different sets of data. It consists of a number of circles contained within a rectangle.
- Numbers are placed within each section to show how many elements or individuals are in each group and can be used to calculate the probability of elements belonging to different sets.
- In the Venn diagrams shown:





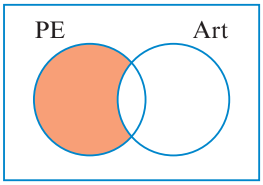
  - $A \cap B$ means A and B, or the **intersection** of sets A and B, and includes the elements in common with both sets.
  - $A \cup B$ means A or B, or the **union** of sets A and B, and includes the elements in either A or B or both.
  - A' means the **complement** of A and includes the elements not in A.
  - If the set C is contained totally in set D, then $C \subset D$ means C is a **subset** of D.

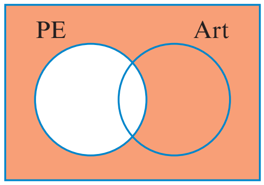
SAMPLE

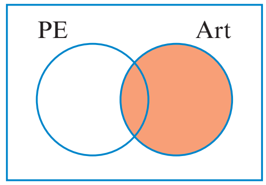
EXERCISE 9E Venn diagrams

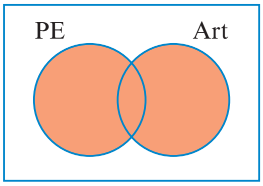
UNDERSTANDING AND FLUENCY

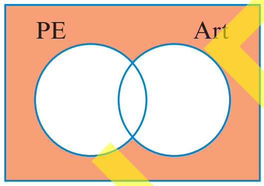
1 These Venn diagrams represent people who like PE and art. Write what the shaded section in each diagram represents.

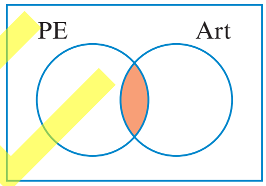
a


b


c


d


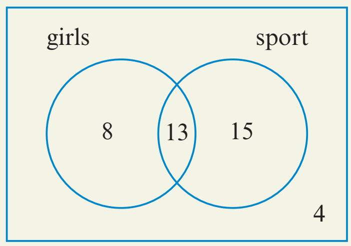
e


f


EXAMPLE 9E-1 Understanding a Venn diagram

This Venn diagram shows people who play sport.

a How many people are girls who play sport?
b How many people play sport?
c How many people are not girls?
d How many people were surveyed in total?



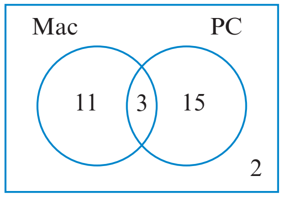
THINK	WRITE
a Look for the section that represents people who are girls and who play sport. This is the middle section where the two circles overlap – the intersection of both sets.	a 13 people are girls who play sport.
b To find the number of people who play sport, add all the numbers that are within the 'sport' circle.	b 13 + 15 = 28 people play sport.
c To find the number of people who are not girls, add all the numbers not inside the 'girls' circle.	c 15 + 4 = 19 people are not girls.
d Add all the numbers in the Venn diagram.	d 8 + 13 + 15 + 4 = 40 people were surveyed in total.

SAMPLE

UNDERSTANDING AND FLUENCY

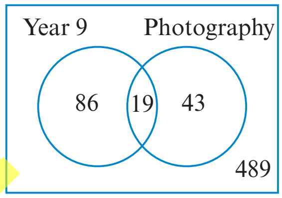
2 Consider the Venn diagram at right.

- **a** How many people own a Mac?
- **b** How many people own a PC but not a Mac?
- **c** How many people don't own either a Mac or a PC?
- **d** How many people were surveyed in total?



3 Consider the Venn diagram at right.

- **a** How many people take Photography?
- **b** How many people are in Year 9 that take Photography?
- **c** How many people are not in Year 9?
- **d** How many people were surveyed in total?



EXAMPLE 9E-2 Calculating probability using a Venn diagram

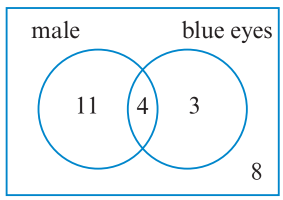
Use the Venn diagram from Example 9E-1 to find the probability that a person chosen randomly from the group plays sport.

THINK	WRITE
1 Find the section(s) that represent people who play sport (all the numbers within the sport circle). Add these.	13 + 15 = 28 28 people play sport.
2 Find the total number of people that were surveyed. Add together all the numbers in the Venn diagram (don't forget the number outside the circles).	8 + 13 + 15 + 4 = 40 40 people were surveyed.
3 Write these two numbers as a probability fraction, simplifying if possible.	$\text{Pr(person who plays sport)} = \frac{28}{40}$ $= \frac{7}{10}$

SAMPLE

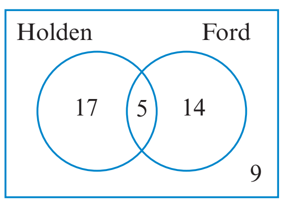
4 Use this Venn diagram to find the probability of a person chosen randomly:

- **a** being male with blue eyes
- **b** having blue eyes
- **c** being male with eyes not blue
- **d** not having blue eyes
- **e** being female with blue eyes
- **f** being female.



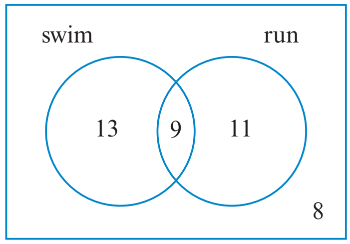
5 Use this Venn diagram to find the probability of a person chosen randomly:

- **a** liking Holden cars only
- **b** liking both Holden and Ford cars
- **c** liking Ford cars
- **d** liking neither
- **e** not liking Holden cars
- **f** liking Holden or Ford (but not both).



PROBLEM SOLVING AND REASONING

6 Use this Venn diagram to find the probability of a person chosen randomly who:



- **a** swims and runs
- **b** does not run
- **c** runs but does not swim
- **d** does not swim or run
- **e** swims
- **f** swims or runs.

7 Construct a Venn diagram for the objects in this photograph, using the categories 'jellybeans' and 'red'.

8 Use the Venn diagram you constructed in question **7** to calculate the probability that a randomly chosen lolly is:

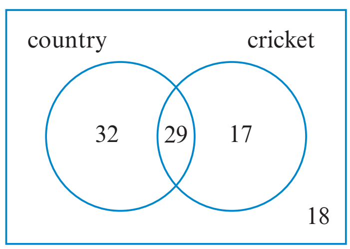
- **a** a jellybean
- **b** a red jellybean
- **c** red but not a jellybean
- **d** not a jellybean
- **e** a jellybean but not red
- **f** neither red nor a jellybean.

9 Consider this statement. In a group of 40 people, 27 have a brother and 29 have a sister. There are four people who do not have either a brother or a sister.

- **a** Explain why there must be some people who have both a brother and a sister. (Hint: What do the last three numbers add to?)
- **b** Draw a Venn diagram with two circles that overlap. Label one 'Brother' and the other 'Sister'.
- **c** Place the number of people who don't have either a brother or a sister in the rectangle outside the circles. How many people are left to fill the circles?
- **d** The entire 'Brother' circle represents people who have a brother. How many people is this?
- **e** Use your answers to parts **c** and **d** to find how many people must have only a sister and write this into the correct section.
- **f** The entire 'Sister' circle must contain 29 people. How many people does this mean must have a brother *and* a sister? Write this in the overlap section.
- **g** Complete the Venn diagram by finding the number of people who have only a brother. Check that all the numbers in your Venn diagram add to 40.

10 In a group of 50 students, 24 are in Year 9, 19 walk to school and 16 are not in Year 9 and do not walk to school. Draw a Venn diagram to represent this situation.

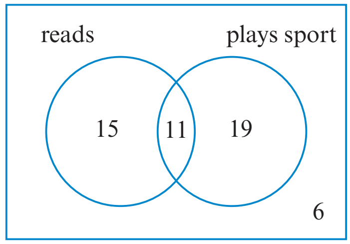
11 Remember that conditional probability examines the probability of an outcome given a condition (page 428). Use this Venn diagram to calculate the probability that a person chosen randomly:



- **a** likes cricket given that they are from the country
- **b** is from the country given that they like cricket
- **c** does not like cricket given that they are from the country
- **d** is not from the country given that they do not like cricket.

SAMPLE

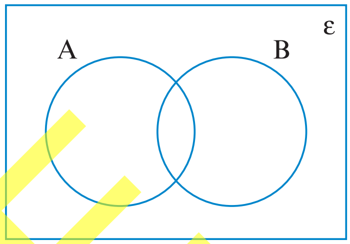
12 Use this Venn diagram to calculate the probability that a person chosen randomly:



- **a** reads given that they play sport
- **b** plays sport given that they read
- **c** does not play sport given that they read
- **d** does not read given that they do not play sport.

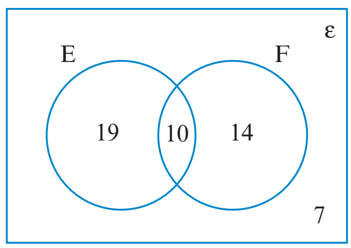
13 When you discuss sets in mathematics, you usually use set notation. Venn diagrams are a useful way to visually display the relationship between sets and understand what set notation represents.

Consider this Venn diagram, showing the relationship between two sets of data, A and B.



- **a** Everything contained within the rectangle is said to belong to the **universal set**. What symbol is used to represent the universal set?
- **b** Copy the Venn diagram into your book five times.
- **c** On your first diagram, shade the overlapping section of sets A and B. This is the intersection of sets A and B and is written as $A \cap B$.
- **d** On your second diagram, shade everything within sets A and B. This is the union of sets A and B and is written as $A \cup B$.
- **e** On your third diagram shade everything that does not belong to set A. This is the complement of set A and is written as A'.
- **f** On your fourth diagram, shade the complement of the union of sets A and B. Label this as $(A \cup B)'$. (Hint: This is related to your second diagram.)
- **g** On your fifth diagram, shade the union of set A and the complement of set B. Label this as $A \cup B'$. (Hint: Remember that union is a combination of sets, not an intersection.)
- **h** Use your diagrams to parts **f** and **g** to explain the difference that brackets can make in set notation.

14 Consider this Venn diagram.



- **a** Given that n(F) means the number of elements in set F, find:

i $n(E)$	**ii** $n(E \cup F)$
iii $n(F')$	**iv** $n(E \cap F)$
v $n(\varepsilon)$	**vi** $n(E' \cap F)$.

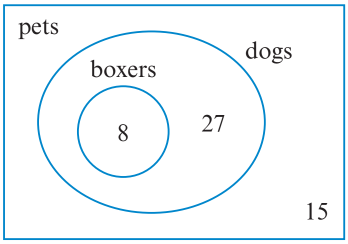
- **b** Given that Pr(E) means the probability of selecting an element from set E, find:

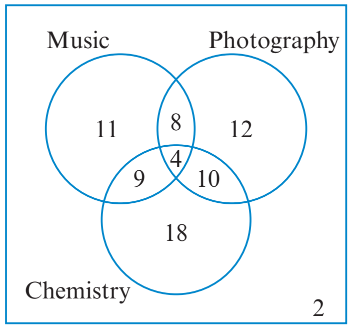
i $Pr(F)$	**ii** $Pr(E')$
iii $Pr(E \cap F)$	**iv** $Pr(E \cup F)'$
v $Pr(E' \cap F')$	**vi** $Pr(F \cup E')$.

SAMPLE

PROBLEM SOLVING AND REASONING

15 Venn diagrams are not restricted to displaying the relationship between two sets of data. They can show the relationship between three or more sets and also include subsets. Consider these two Venn diagrams.





a Which Venn diagram shows a subset of data? Explain the relationship shown in this Venn diagram.

b Copy this Venn diagram three times and on separate copies shade the section that represents:

i pets that are not dogs **ii** dogs that are not boxers

iii all dogs.

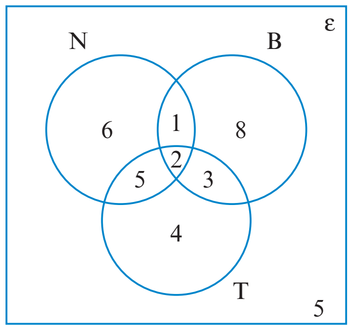
c The other Venn diagram shows the relationship between three different sets. Copy this Venn diagram five times and shade the section that represents students:

i taking all three elective subjects **ii** taking only Photography

iii taking Music or Chemistry **iv** taking Music

v not taking any of these three electives.

16 Consider this Venn diagram. Find the probability that a person chosen randomly from the group:



a plays netball but not tennis

b plays basketball only

c plays all three sports

d doesn't play any of these sports

e plays basketball or tennis

f plays tennis and netball

g plays netball given that they play basketball.

CHALLENGE

17 Draw a Venn diagram that represents the relationship between four different sets of data. There should be a total of 16 sections (including the section outside the sets) with no repeated sections. (Hint: Use ovals rather than circles.)

18 Use the following paragraph to draw a Venn diagram that shows the relationship between three different sets of data.

In a group of 100 people surveyed, 35 liked western films, 45 liked romance films and 46 liked horror films. Nineteen people did not like any of these three types. Fifteen people liked both western and romance, 16 only liked horror and 55 did not like romance films. Five people liked all three types and 18 liked both horror and romance.

Reflect

What mistakes do you think people might make when identifying sections of Venn diagrams?

SAMPLE

9F Experiments with replacement

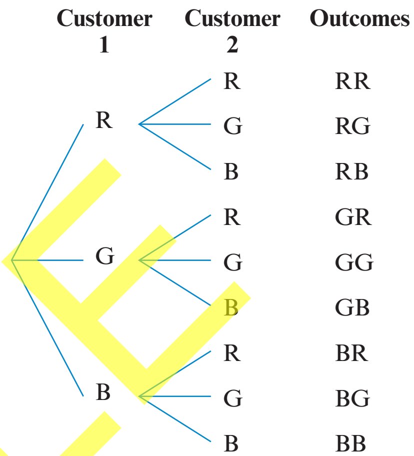
Start thinking!

A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you draw out of a box. If you draw a red ball you get 10% off, if you draw a green ball you get 25% off and if you draw a blue ball you get 50% off. There are 10 balls of each colour in the box.

1 What is the probability of drawing a blue ball?

After you draw a ball for your discount, you must place it back into the box so that the next customer also has an equal chance of drawing a blue ball. This, and other experiments you have seen in this unit, is called experiment with replacement.

Consider the tree diagram on the right.



2 What is it showing?

3 How many outcomes are possible when looking at the results of the first two customers?

4 Use the tree diagram to explain why, when looking at the first two customers, the probability of:

- **a** both customers drawing a blue ball is $\frac{1}{9}$
- **b** both customers drawing a red ball is $\frac{1}{9}$
- **c** both customers drawing a green ball is $\frac{1}{9}$
- **d** the first customer drawing a red ball and the second customer drawing a green ball is $\frac{1}{9}$.

5 Why are these probabilities the same?

When outcomes are simple and equally likely, drawing a tree diagram is not necessary and you may find it easier to just make a list of the outcomes. When outcomes are not equally likely, a tree diagram becomes more helpful. Look back at the tree diagram.

6 What is the probability of any branch in this tree diagram? (Hint: what is the probability of drawing a blue ball? A red ball? A green ball?)

SAMPLE

KEY IDEAS

- Experiments with replacement involve selecting or drawing an item, recording the results, and replacing the item before performing another selection.
- A tree diagram or list of outcomes can help you to find the probabilities of individual outcomes or events involving more than one outcome.
- When outcomes are not equally likely, a tree diagram with probabilities written on the branches is useful in determining the probability of each final outcome.
- The probabilities of the final outcomes will always add to 1.

EXERCISE 9F Experiments with replacement

1 For each experiment, state the theoretical probability of the outcome in brackets in any given trial.

a drawing a card and recording its suit (drawing a club)

b selecting a marble and recording its colour out of a bag containing 10 blue, 5 red, 10 yellow and 5 green marbles (selecting a green marble)

c drawing a card and recording if it is a number or picture card (drawing a picture card)

d rolling a die and recording the number on top (rolling a 5)

EXAMPLE 9F-1 Representing experiments with equally likely outcomes

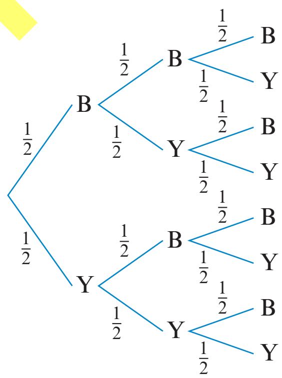
A box contains equal numbers of blue activity cards and yellow activity cards. A card is drawn, its colour recorded, then replaced. This is repeated two more times. Draw a tree diagram to represent this situation, complete with probabilities on each branch and for each final outcome.

THINK

1 Start by drawing a tree diagram for this three-step experiment. To calculate the probability of each final outcome, you need to know the probability of an individual outcome at each step.

2 You have a $\frac{1}{2}$ chance of drawing a blue activity card, and a $\frac{1}{2}$ chance of drawing a yellow activity card. Include these probabilities on the branches and calculate the probability of each final outcome.

WRITE



Outcomes	Probability
BBB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
BBY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
BYB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
BYY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YBB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YBY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YYB	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$
YYY	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

UNDERSTANDING AND FLUENCY

2 Draw a tree diagram with probabilities on the branches for each of these experiments.

a A pencil case contains equal numbers of red and blue pens. A pen is drawn, its ink colour recorded, then replaced. This is repeated one more time.

b A box contains equal numbers of $5, $20 and $75 vouchers. A voucher is drawn, its value recorded, then replaced. This is repeated one more time.

c A box contains 10 cards, numbered 1–10. A card is drawn, it is recorded whether it shows an even or odd number, then it is replaced. This is repeated another two times.

d A ball-pit contains equal numbers of blue, red, yellow and green balls. A ball is drawn, its colour recorded, then replaced. This is repeated one more time.

SAMPLE

EXAMPLE 9F-2 Calculating probability for experiments with equally likely outcomes

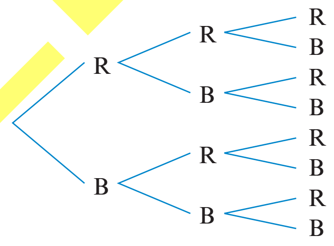
Use the tree diagram from Example 9F-1 to find the probability that:

a three blue activity cards are selected **b** a yellow activity card is selected first

THINK	WRITE
a Locate the outcome(s) where all selections produce a blue activity card (BBB).	**a** Pr(three blue) = Pr(BBB) $= \frac{1}{8}$
b Locate the outcome(s) where a yellow activity card is selected first.	**b** Pr(yellow first) = Pr(YBB, YBY, YYB, YYY) $= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

UNDERSTANDING AND FLUENCY

3 A bag contains six red counters and six black counters. A counter is drawn and replaced three times. This tree diagram shows the possibilities of the three counter draws.



a In any given trial, what is the probability of drawing a black counter?

b How many outcomes are there in total?

c What is the probability of drawing:

i three black counters? **ii** exactly one black counter?

iii at least two red counters? **iv** more than one black counter?

4 A card is drawn from a deck, its suit recorded, then replaced. This is then repeated. What is the probability that:

a both cards are hearts? **b** both cards are spades?

c the first card is a diamond and the second is a club?

d at least one card is a spade?

5 A box contains milk, dark and white chocolates in equal numbers. A chocolate is selected from the box, its flavour recorded, then replaced. This is then repeated. What is the probability that:

a both chocolates are white? **b** at least one chocolate is dark?

c the first chocolate is white and the second chocolate is milk?

d one chocolate is white and one chocolate is milk?

6 A card is drawn from a deck, its suit recorded, then replaced. This is repeated twice more. What is the probability of drawing:

a three hearts? **b** at least two diamonds?

c exactly two clubs? **d** no spades?

e at least one diamond or spade? **f** at least one heart and at least one club?

SAMPLE

EXAMPLE 9F-3 Representing experiments with outcomes that are not equally likely

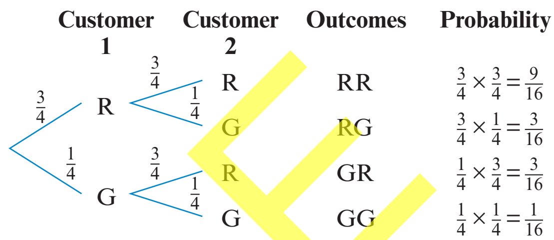
A lucky dip contains five green gift vouchers for \$50 and 15 red gift vouchers for \$5. A gift voucher is drawn, its value recorded and it is then replaced. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment.

THINK

1 Start by drawing a tree diagram for this two-step experiment. To calculate the probability of each final outcome, you need to know the probability of an individual outcome at each step.

2 You have a $\frac{15}{20}$ or $\frac{3}{4}$ chance of drawing a red gift voucher, and a $\frac{5}{20}$ or $\frac{1}{4}$ chance of drawing a green gift voucher. Include these probabilities on the branches and calculate the probability of each final outcome.

WRITE



7 Draw a tree diagram with probabilities on the branches for each experiment.

a A pencil case contains five blue pens and two red pens. A pen is drawn, its ink colour is recorded and then it is replaced. This is repeated one more time.

b A box contains 15 \$5 vouchers, 10 \$20 vouchers and 5 \$75 vouchers. A voucher is drawn, its value recorded is and then it is replaced. This is repeated one more time.

c A box contains 15 cards, numbered 1–15. A card is drawn, it is recorded whether it shows an even or odd number, then replaced. This is repeated another two times.

d A ball-pit contains five blue, four red, three yellow and two green balls. A ball is drawn, its colour recorded, then replaced. This is repeated one more time.

EXAMPLE 9F-4 Calculating probability for experiments with outcomes that are not equally likely

Use the tree diagram from Example 9F-3 to find the probability that:

a a \$50 voucher is selected twice

b a \$50 voucher then a \$5 voucher is selected.

THINK

a Locate the outcome where both customers select a \$50 voucher (GG).

b Locate the outcome where the first customer selects a \$50 voucher and the second customer selects a \$5 voucher (GR).

WRITE

a $\text{Pr(both select \$50)} = \text{Pr(GG)}$
$= \frac{1}{16}$

b $\text{Pr(\$50 then \$5)} = \text{Pr(GR)}$
$= \frac{3}{16}$

SAMPLE

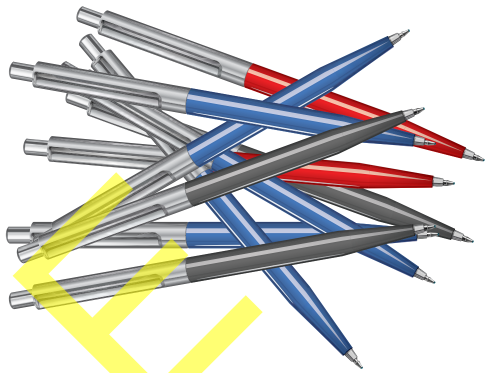
UNDERSTANDING AND FLUENCY

8 A lucky dip contains 10 pink gift vouchers for \$100 and 40 green gift vouchers for \$10. A voucher is drawn, its value recorded and then it is replaced. If this was repeated, find the probability that:

a a \$100 voucher was selected twice **b** a \$100 voucher was not selected at all

c a \$100 voucher was selected first, and a \$10 voucher selected second.

9 The contents of a pencil case are shown here. The owner of the pencil case takes out a pen for each lesson.



a Use the photo to draw a tree diagram to represent the pens drawn for the first two lessons of the day. Remember to include the probabilities along each branch and for the final outcomes.

b Find the probability that the owner draws:

i a blue pen each time **ii** a red pen each time

iii a black pen each time **iv** a blue pen, then a black pen

v a blue pen, then a red pen **vi** a red pen, then a black pen.

PROBLEM SOLVING AND REASONING

10 A card was drawn from a deck of cards; it was recorded if it was a picture card or not, then the card was replaced. This was repeated twice more.

a Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome. It is probably easiest to write these final probabilities as a decimal number rounded to four decimal places.

b Find the probability of drawing:

i exactly one picture card

ii at least one picture card

iii less than two picture cards

iv at least two picture cards

v no picture cards

vi exactly two picture cards.

11 Use your tree diagram from question **9** to find the probability that in the first three lessons of the day, the owner of the pencil case selects:

a at least one blue pen **b** exactly two black pens

c no more than one red pen **d** no blue pens.

12 Consider the pencil case from question **9**. Use a tree diagram or the strategy to find outcomes discussed in Exercise 9C questions **14** and **15** (page 426–7) to find the probability that, over the three double lessons of the day, the owner of the pencil case selects:

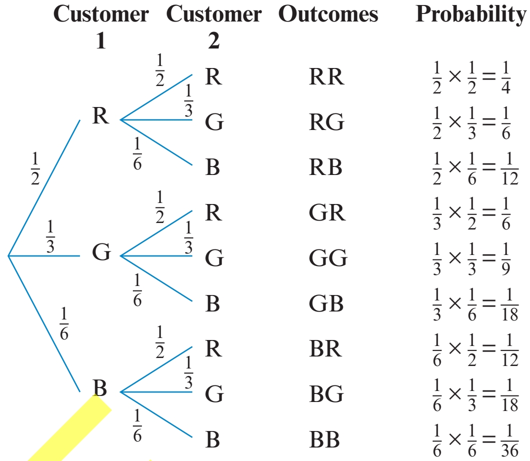
a all black pens **b** a blue pen, then a red pen, then a black pen

c at least one blue pen **d** no red pens **e** at least two black pens.

SAMPLE

PROBLEM SOLVING AND REASONING

13 How do you calculate the probability of a final outcome in a multi-step experiment when the individual outcomes at each step are not equally likely? Consider the experiment from 9F Start thinking! where the box now contains 15 red balls, 10 green balls and 5 blue balls.



a State the theoretical probability of drawing each colour ball.

To calculate the probability of a particular event that involves more than one final outcome, you need to add the probabilities of each favourable final outcome.

b Find the probability of exactly one customer drawing a blue ball by:

- **i** listing the final outcomes where exactly one customer draws a blue ball
- **ii** adding together these probabilities.

c Explain why the probability in part **b** is $\frac{10}{36}$ and not $\frac{5}{9}$.

d Calculate the probability of drawing:

- **i** at least one green ball
- **ii** at least one red ball
- **iii** at least one blue ball
- **iv** exactly one red ball
- **v** exactly one green ball
- **vi** a blue ball and a green ball.

14 Explain why you can use the process described in question **13** (adding together the probabilities of each favourable final outcome) to calculate the probability of final outcomes when individual outcomes are equally likely, but it is not necessary to do so.

15 Imagine that you are a customer in the experiment with equally likely outcomes described in 9F Start thinking! on page 440. The sales assistant is in a good mood and will give you a second chance if you don't draw a blue ball out of the box first up, as long as you put the first ball that you draw back into the box.

Use a tree diagram or other means to show that you have a $\frac{5}{9}$ chance of drawing a blue ball from the box.

CHALLENGE

16 A sock drawer contains 10 socks; some are black and some are white. You need to figure out how many of each colour are in the drawer, but you can only select one sock at a time and place it back.

a If you selected with replacement 10 times and selected 3 black socks and 7 white socks, does this mean that there are 3 black and 7 white socks in the drawer? Explain.

b If you selected with replacement 50 times, selecting 21 black socks and 29 white socks, how many socks of each colour would you estimate are in the drawer?

c If you selected with replacement 80 times, selecting 34 black socks and 46 white socks, does this support your previous estimate?

d State how many socks of each colour you believe to be in the drawer.

Reflect

Why is it important to consider all the possible outcomes when calculating probability?

SAMPLE

9G Experiments without replacement

Start thinking!

In real life, games and experiments rarely involve an item being replaced. Rather, once an item is selected, it is usually kept. This affects the probability of every other item remaining. Consider the experiment described in the 9F Start thinking! on page 440.

A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you draw out of a box. If you draw a red ball you get 10% off, if you draw a green ball you get 25% off and if you draw a blue ball you get 50% off. There are 10 balls of each colour in the box.

1 What is the probability of drawing a blue ball?

2 Draw the tree diagram that represents two customers selecting a ball from the box. Do not include any probabilities at this stage, but write the final outcomes at the end of the branches.

Imagine that the first customer draws a blue ball out of the box and keeps it.

3 How many balls are left in total in the box?

4 How many of these balls are:

a blue? **b** green? **c** red?

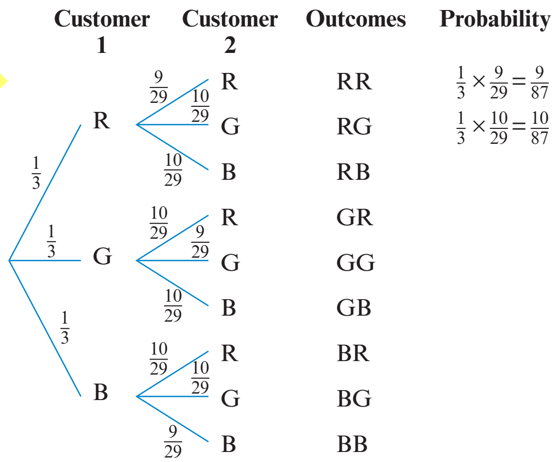
5 Imagine that you are the second customer to come along. What is the probability of you selecting a ball that is:

a blue? **b** green? **c** red?

6 Explain why you have a different probability of drawing a blue ball from the first customer.

These probabilities have now been added to the branches of the tree diagram, as well as the probabilities that result if the first customer drew a green or red ball.

7 Copy and complete the tree diagram.



SAMPLE

KEY IDEAS

- Experiments without replacement involve selecting or drawing an item, recording the results, and not replacing the item before performing another selection.
- Because each item that is selected is not replaced, this changes the probability of selection of the remaining items.
- A tree diagram or list of outcomes can help to find the probabilities of individual outcomes or events involving more than one outcome.

EXERCISE 9G Experiments without replacement

UNDERSTANDING AND FLUENCY

1 A bag contains five red counters and five black counters. A red counter is drawn.
   a How many of the counters that remain are:
      i red? ii black?
   b What is the probability that the next counter will be:
      i red? ii black?

2 A bag contains eight blue counters and eight green counters. A blue counter is drawn.
   a How many of the counters that remain are:
      i blue? ii green?
   b What is the probability that the next counter will be:
      i blue? ii green?

SAMPLE

EXAMPLE 9G-1 Representing experiments without replacement

A lucky dip contains five red gift vouchers for \$50 and five green gift vouchers for \$5. A gift voucher is drawn and the customer keeps it. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment.

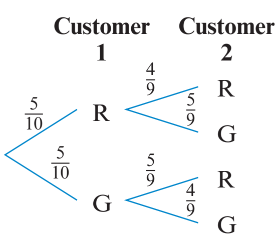
THINK

1 Start by drawing a tree diagram for this two-step experiment. To calculate the probability of each final outcome, you need to know the probability of an individual outcome at each step.

2 Initially you have a $\frac{5}{10}$ chance of drawing a red gift voucher, and a $\frac{5}{10}$ chance of drawing a green gift voucher. Write these on the first branches.

3 If you draw a red gift voucher, that leaves four red and five green vouchers in the box, but if you draw a green gift voucher, that leaves five red and four green gift vouchers in the box. Write these probabilities on the second lot of branches and calculate the probability of each final outcome.

WRITE



Outcomes	Probability
RR	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} \approx 0.22$
RG	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} \approx 0.28$
GR	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} \approx 0.28$
GG	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} \approx 0.22$

UNDERSTANDING AND FLUENCY

3 Draw a tree diagram with probabilities on the branches for each of these experiments.

- **a** A drawer contains five black socks and five white socks. A sock is drawn and its colour recorded. This is repeated two more times.
- **b** An esky contains six cans of Coke and six cans of Pepsi. A can is drawn and its type recorded. This is repeated two more times.
- **c** A box contains five 16 GB SD cards, five 32 GB SD cards and five 64 GB SD cards. A card is drawn and its capacity recorded. This is repeated one more time.
- **d** A bowl contains 10 Smarties and 10 M&Ms. A chocolate is drawn and its type recorded. This is repeated two more times.
- **e** A small ball-pit contains 10 blue, 10 red, 10 yellow and 10 green balls. A ball is drawn and its colour recorded. This is repeated one more time.
- **f** A box contains 10 names from 9D, 10 names from 9E and 10 names from 9F. A name is drawn and it is recorded which class the name is from. This is repeated two more times.

EXAMPLE 9G-2 Calculating probability for experiments without replacement

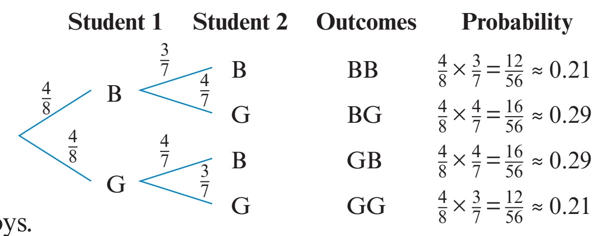
Use the tree diagram from Example 9G-1 to find the probability that:

- **a** both customers select a $50 voucher
- **b** the first customer draws a $50 voucher and the second customer draws a $5 voucher.

THINK	WRITE
a Locate the outcome where both customers select a $50 voucher (GG).	**a** Pr(both select $50) = Pr(GG) $= \frac{20}{90} \approx 0.22$
b Locate the outcome where the first customer selects a $50 voucher and the second customer selects a $5 voucher (GR).	**b** Pr($50 then $5) = Pr(GR) $= \frac{25}{90} \approx 0.28$

4 This tree diagram represents selecting two students from a group of four boys and four girls. Find the probability of selecting:

- **a** two boys
- **b** a boy then a girl
- **c** no boys.



5 A lucky dip contains four purple gift vouchers for $100 and four yellow gift vouchers for $10. A gift voucher is drawn and the customer keeps it. If this was repeated for a second customer, find the probability that:

- **a** both customers select a $100 voucher
- **b** the first customer selects a $100 voucher and the second customer selects a $10 voucher.

SAMPLE

UNDERSTANDING AND FLUENCY

6 A lucky dip contains five purple gift vouchers for $100 and five yellow gift vouchers for $10. A gift voucher is drawn and the customer keeps it. If this was repeated for a second customer, find the probability that:

- **a** both customers select a $100 voucher
- **b** the first customer selects a $100 voucher and the second customer selects a $10 voucher.

7 Each bonbon in a pack of 12 contains one toy, and there are three different kinds of toy: a whistle, a yo-yo and a bouncy ball. In total, the pack contains four of each kind of toy. You and two friends select a bonbon from the pack.

- **a** Draw a tree diagram to represent this situation. Be sure to include probabilities on each branch and the final probabilities for each outcome.
- **b** Find the probability that:
  - **i** all three of you select a bonbon with a whistle
  - **ii** the first two bonbons have a yo-yo and the third has a bouncy ball
  - **iii** the first bonbon has a whistle, the second a bouncy ball and the third a yo-yo.

PROBLEM SOLVING AND REASONING

8 When calculating the probability of an event involving more than one final outcome in a multi-step experiment, remember that you need to add together the probabilities of each favourable final outcome. Consider the situation from question **7**.

- **a** Find the probability of at least one person selecting a bonbon with a whistle by:
  - **i** listing the outcomes where at least one person selects a bonbon with a whistle
  - **ii** adding together these probabilities.
- **b** Find the probability that:
  - **i** the three of you select a bonbon with a different toy
  - **ii** two of you select a bonbon with a yo-yo
  - **iii** at least one of you selects a bonbon with a bouncy ball
  - **iv** a bonbon with a whistle is not drawn.

9 Consider a situation where a bag contains six red counters and six black counters.

- **a** Draw a tree diagram to represent selecting three counters without replacement.
- **b** Use the tree diagram to find the probability of drawing:
  - **i** three black counters
  - **ii** exactly one black counter
  - **iii** at least two red counters
  - **iv** more than one black counter
  - **v** less than two red counters.
- **c** Compare these answers to the experiment with replacement (Exercise 9F question **3**, page 442). What do you notice?

SAMPLE

PROBLEM SOLVING AND REASONING

SAMPLE

10 A card is drawn from a deck, and its suit is recorded. This is then repeated twice more.

a Draw a tree diagram to help you. Remember that there are 52 cards in a pack.

b What is the probability of drawing:

i three hearts?
ii at least two diamonds?
iii exactly two clubs?
iv no spades?
v at least one heart and at least one club?
vi at least one diamond or spade?

c Compare these answers to the experiment with replacement (9F question **6**, page 442). What do you notice?

11 A drawer contains two pink socks, two purple socks and two green socks. Use a tree diagram or other means to calculate the probability that you draw out a pair when you select two socks from the drawer.

12 Imagine instead that the drawer from question **11** contained six of each colour sock. How does this change the probability of drawing a pair when you select two socks from the drawer?

13 A sports team needs to select a captain and a vice-captain. Five people have put their names forward: Adrian, Chantelle, Katie, Ben and Sam.

a Draw a tree diagram to represent the selection (start with 'captain' branches).

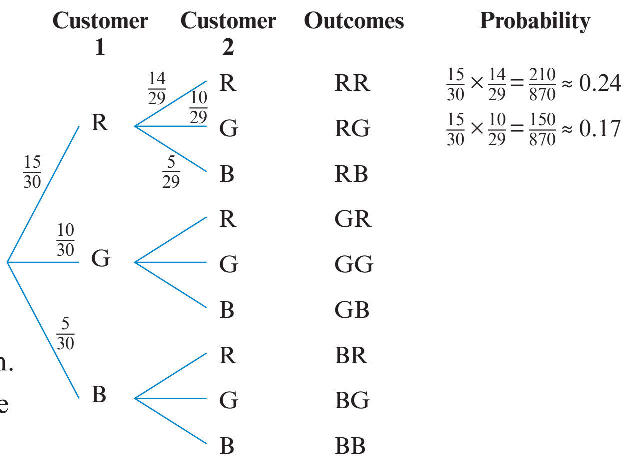
b How does this tree diagram differ from the other ones you have done beforehand? (Hint: Does the second set of branches contain the same number as the first set of branches?)

c How many different combinations of captains and vice-captains are there? Remember that order is important!

d Find the probability that:

i Katie is selected captain
ii Sam is selected either captain or vice-captain
iii Adrian is captain and Chantelle is vice-captain
iv Ben does not get a position
v Katie and Sam both get a position.

14 Experiments without replacement can also start with unequally likely outcomes. Consider the situation in the 9F question **13** (page 445), where the lucky dip contains 15 red balls, 10 green and 5 blue balls, but each ball is not replaced after it has been drawn.



a Copy and complete the tree diagram.

b Use the tree diagram to calculate the probability of:

i both customers drawing a blue ball
ii both customers drawing a green ball

PROBLEM SOLVING AND REASONING

iii both customers drawing a red ball
iv the first customer drawing a red ball and the second customer drawing a green ball.

15 Use the tree diagram from question **14** to calculate the probability of selecting:

a at least one green ball
b at least one red ball
c at least one blue ball
d exactly one red ball
e exactly one green ball
f a blue ball and a green ball.

16 If a drawer contained five red socks, four black socks and three white socks, find the probability that the first two socks selected from the draw form a pair.

17 A box of chocolates contains four milk chocolates, three white chocolates and two dark chocolates. Three chocolates are selected from the box. Find the probability of selecting:

a all three white chocolates
b no dark chocolates
c one of each chocolate type
d at least one white chocolate
e at least one milk chocolate
f all the dark chocolates.

18 A card was drawn from a deck of cards and it was recorded if it was a picture card or not. This was repeated twice more.

a Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome. It is probably easiest to express these final probabilities as a decimal number rounded to four decimal places.
b Find the probability of drawing:
   i exactly one picture card
   ii at least one picture card
   iii less than two picture cards
   iv at least two picture cards
   v no picture cards
   vi exactly two picture cards.
c Compare your answers to that in Exercise 9F question **10** (page 444). How do the probabilities change when the cards are not replaced?

CHALLENGE

19 What is the probability of drawing a 21 in blackjack with the first two cards of the deck?

20 A common lottery consists of 45 numbered balls, of which six winning balls are drawn and two supplementary balls are drawn. To win the first division prize, you must pick all six winning numbers. What is the probability of winning the first division prize in this lottery?

Reflect

How does experiment without replacement differ from experiment with replacement?

SAMPLE

CHAPTER REVIEW

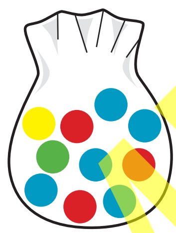
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

certain	favourable outcome	trials	set notation
impossible	equally likely	expected number	universal set
even chance	sample space	tree diagrams	union
probability scale	complementary events	multi-step experiment	elements
event	experimental probability	conditional probability	subsets
outcome	relative frequency	two-way table	replacement
theoretical probability		Venn diagram	without replacement

MULTIPLE-CHOICE

9A **1** The theoretical probability of selecting a blue marble from the bag of marbles shown at right is:



A $\frac{3}{10}$ **B** 5 **C** $\frac{1}{2}$ **D** $\frac{1}{10}$

9B **2** Consider this table. Which of these statements is true?

Number on die	Frequency
1	45
2	35
3	27
4	30
5	32
6	31

A The number of trials performed was 200 rolls.

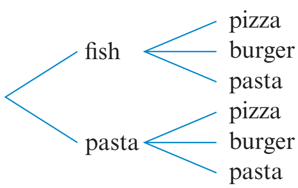
B The experimental probability of rolling an even number is $\frac{13}{25}$.

C The number rolled the least frequently was 1.

D The experimental probability of rolling a 6 is $\frac{1}{6}$.

9C **3** The probability that a person orders pasta for both entrée and main is:

Meal choices at a local café



A $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{1}{6}$ **D** $\frac{1}{2}$

9D **4** The probability of a person randomly selected who prefers Holden is:

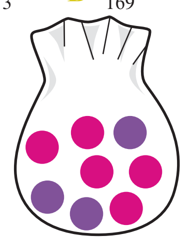
	Male	Female	Total
Prefer Holden	176	98	
Prefer Ford	101	25	126
Total	277		400

A $\frac{137}{200}$ **B** $\frac{49}{200}$ **C** $\frac{11}{25}$ **D** $\frac{101}{400}$

9F **5** A card is selected from a playing deck and an ace is drawn. The card is replaced into the deck. What is the probability of drawing an ace on the second selection?

A $\frac{1}{13}$ **B** $\frac{1}{169}$ **C** $\frac{12}{13}$ **D** $\frac{144}{169}$

Questions **6** and **7** refer to diagram at right.
A marble is selected and not replaced and then a second selection is made.



9G **6** The probability of selecting a pink marble on the second selection, given that the first marble was pink, is:

A $\frac{4}{7}$ **B** $\frac{3}{7}$ **C** $\frac{5}{7}$ **D** $\frac{2}{7}$

9G **7** The probability of selecting two purple marbles is:

A $\frac{3}{8}$ **B** $\frac{2}{7}$ **C** $\frac{5}{14}$ **D** $\frac{3}{28}$

SAMPLE

SHORT ANSWER

9A **1** Consider rolling a 12-sided die. State the:

- i probability of the listed event
- ii probability of the complementary event.

a rolling a 6
b rolling an even number
c rolling a number less than 10

9C **2** A company runs a competition where one out of every four purchases contains the winning bar code for an eBook.

a What is the probability of not winning an eBook?
b Complete a tree diagram showing all probabilities on the branches for three steps.
c Calculate the probability of:
- i winning three times in a row
- ii not winning three times in a row
- iii not winning on the first two tries and then winning on the third try.

9D **3** Refer to this two-way table for Years 8, 9 and 10 girls and their favourite brands of make-up.

	Year 8	Year 9	Year 10	Total
Napoleon Perdis	32	25	45	
Rimmel	22		22	54
Covergirl	17	31	13	
Maybelline		29		
Total	120		95	310

a Copy and complete the table.
b Calculate the probability that a girl chosen at random from the group:
- i prefers Maybelline
- ii is in Year 8 and prefers Rimmel
- iii is in Year 9 and prefers Napoleon Perdis

c Calculate the probability that a girl chosen from the Year 9 girls:
- i prefers Napoleon Perdis
- ii prefers Covergirl or Maybelline.

9E **4** Counters numbered 1 to 15 are placed in a bag. One is drawn and recorded if it fits any of the following events.

Event 1: {numbers ≤5}
Event 2: {multiples of 2}
Event 3: {odd numbers}

a List the sample space for each of the events in this experiment.
b Draw a Venn diagram to represent this experiment.
c Calculate the following:
- i Pr(Event 1)
- ii Pr(Event 2)
- iii Pr(Event 2 or 3)
- iv Pr(neither event 1, 2 nor 3).

d Explain why there is no number placed in the intersection of all three circles.

9F **5** A bag contains six chocolates. Two have orange wrappers, one has a green wrapper and three have pink wrappers. A chocolate is drawn, the colour recorded, the chocolate replaced and another selected.

a Show all possible outcomes on a tree diagram.
b Calculate the probability that:
- i both wrappers are orange
- ii both wrappers are green
- iii both wrappers are pink.

c Calculate the probability that the first wrapper is green.
d Calculate the probability that the second wrapper is pink.

9G **6** Two cards are drawn from a deck of 52 and the suit(s) noted. Assuming selection without replacement, calculate:

a Pr(two spades)
b Pr(heart, then spade).

A third card is also drawn. Calculate:

c Pr(spade, heart, heart)
d Pr(spade, heart, diamond).

SAMPLE

NAPLAN-STYLE PRACTICE

1 The theoretical probability of rolling a number greater than 4 on a standard die is:

$\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{2}$ 0

2 The results of an experiment are recorded in this table.

Option	Frequency
A	400
B	250
C	360
D	375
E	198
F	220
G	315
H	

Which of these statements is *not* correct?

- If 2500 trials were performed, the frequency for outcome H was 382.
- If 2500 trials were performed, the experimental probability for option A is $\frac{4}{25}$.
- A spinner with six equal segments may have been used in this simulation.
- If an additional 2500 trials were performed and the options were equally likely, the experimental probability for each option would theoretically get closer to $\frac{1}{8}$.

3 In a coin-flipping experiment, this result was recorded.

Outcome	Heads	Tails
Relative frequency		0.465

Which of these statements is false?

- The relative frequency of heads for this experiment is 0.535.
- If 3000 trials were performed, 1605 heads were recorded.
- If 5000 trials were performed, in theory 2500 tails would be expected.
- The relative frequency of heads for this experiment is 0.465.

4 Two spinners are spun at the same time. The first spinner has four equally-likely outcomes and the second has five equally-likely outcomes. The total number of possible outcomes to be represented on a tree diagram is:

4 5 10 20

Questions **5** and **6** refer to the two-way table below.

A survey of 700 students was conducted relating to student enjoyment in different subjects at primary, secondary and tertiary levels.

	Primary	Secondary	Tertiary	Total
Maths	100		50	
English		69	122	
Sport	75	96		
Total	250	250		700

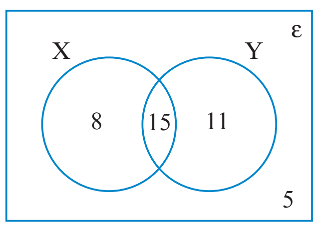
5 The probability that a student chosen at random from the group was a secondary student who enjoyed Maths is:

$\frac{1}{7}$ $\frac{1}{14}$ $\frac{17}{140}$ $\frac{47}{140}$

6 The probability that a student chosen from the primary group of the survey group enjoyed English is:

$\frac{2}{5}$ $\frac{3}{10}$ $\frac{69}{250}$ $\frac{3}{28}$

Questions **7**–**9** refer to this Venn diagram. The number of elements in sets X and Y is shown in the Venn diagram.



7 n(Y) is:

8 11 23 26

8 $n(X \cup Y)$ is:

23 26 34 39

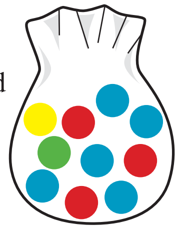
9 $n(X \cap Y)$ is:

8 15 11 5

SAMPLE

Questions **10**–**13** refer to the diagram at right.

A bag contains a number of coloured marbles. A marble is drawn, the colour recorded, the marble replaced and another selected.



10 The probability of selecting two blue marbles is:

$\frac{1}{2}$ ▭ $\frac{1}{4}$ ▭ $\frac{9}{100}$ ▭ $\frac{1}{100}$ ▭

11 The probability of selecting a red marble first is:

$\frac{3}{20}$ ▭ $\frac{3}{100}$ ▭ $\frac{9}{100}$ ▭ $\frac{3}{10}$ ▭

Assume instead that marbles are selected without replacement.

12 The probability of selecting a blue marble on the second try, given that the first marble was not blue, is:

$\frac{5}{9}$ ▭ $\frac{1}{2}$ ▭ $\frac{3}{9}$ ▭ $\frac{1}{9}$ ▭

13 The probability of selecting three red marbles, if a third trial is performed, is:

$\frac{2}{9}$ ▭ $\frac{1}{120}$ ▭ $\frac{2}{72}$ ▭ 0 ▭

ANALYSIS

When moving from Year 9 into Year 10, students undertake core studies such as English and Mathematics, but are able to select from a range of electives to complete their study program.

A student is considering four subject choices: Art, Health and Human Development (HHD), Italian and Food Technology. Three subjects must be selected from this offering. He decides to write each subject onto a card and places them into a hat.

a Would this selection be with or without replacement? Explain your reasoning.

b **i** Draw a tree diagram to represent the possible subject combinations.

ii How many combinations are possible?

c Find the probability of selecting:

i Art, HHD and Food Technology

ii Italian, Art and HHD.

d The year level was surveyed about the Food Technology and Art electives offered. Twelve per cent of girls said that they preferred Art to Food Technology; 28% of boys said that they preferred Food Technology to Art. The year level is 52% boys.

i Construct a two-way table showing these percentages as relative frequencies.

Based on these survey results, if a person was randomly selected from the year level group of 150 students, find the probability that they are:

ii a girl who prefers Food Technology

iii a boy who prefers Art

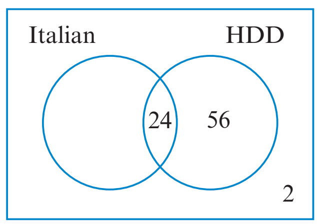
iv a student who prefers Art.

Of these 150 students, find the number of people who:

v are boys and prefer Food Technology

vi prefer Food Technology.

e An analysis of numbers of students studying Italian and/or HHD was also undertaken and the results collated into this Venn diagram.



i How many of the 150 students study both Italian and HHD?

ii How many students study neither subject?

iii How many students study Italian only?

iv If selecting a student at random from the cohort, what is the probability that they study Italian only?

v If selecting a student at random from the cohort, what is the probability that they study HHD only?

SAMPLE

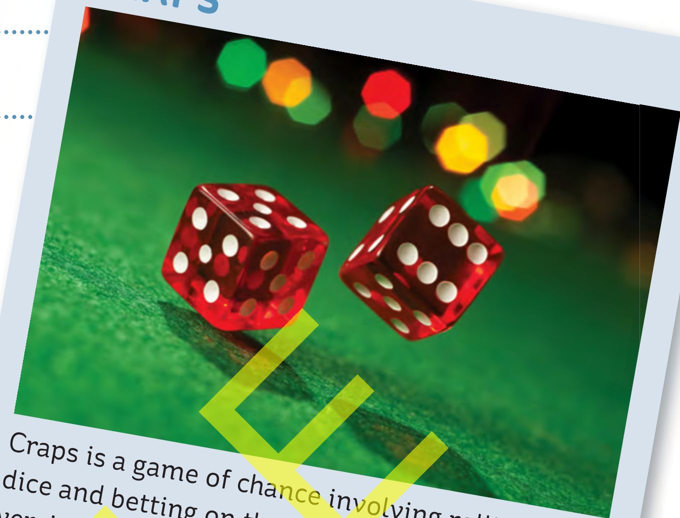
CONNECT

The house always wins

Casinos take advantage of long-term trends in experimental probability to make millions of dollars. Casinos use probabilities and payouts that slightly favour themselves (also known as 'the house'), and the mathematics always ensures that the casinos make money.

You are to investigate the probabilities and payouts of at least two simple games of chance and use this knowledge to construct a lottery-style game that will benefit the casino but that gamblers will still play. Within your investigation you should also address and explain the 'Gambler's fallacy'. Two simple games are listed on this page and you can use the Internet to find many others.

CRAPS



Craps is a game of chance involving rolling two dice and betting on the outcome. In a simplified version of craps, a player bets $1 and if they roll a sum of 2, 3, 4, 10, 11 or 12, they win $2, but if they roll a 5, 6, 7, 8 or 9, they lose.

Your task

For each game that you investigate, you will need to:

- calculate the theoretical probability of a player winning a game
- research and find a common 'payout' for a bet in the game
- calculate the expected winnings/loss for the player for 10 rounds of the game
- perform an experiment or simulation for 10 rounds of the game
- compare your theoretical probability and expected winnings/loss to the experimental probability
- alter the payout so that it pays the player more and recalculate expected winnings/loss and perform another experiment/simulation
- alter the payout so that it pays the player less and recalculate expected winnings/loss and perform another experiment/simulation
- discuss your findings and how the player is disadvantaged in a casino with a 'standard' payout.
- research the 'Gambler's fallacy' and explain how it applies to the game and why it is an incorrect line of thinking.

When constructing the lottery-style game, you will need to:

- decide how many numbered balls will be in the lottery
- decide how many numbered balls will be drawn from the lottery
- decide how many numbers that players must guess
- decide on payouts for the players based on the number of numbers they correctly guess
- perform a simulation to see if your payouts favour the casino
- alter your variables if necessary to ensure that the casino will make money in the long term.

You will need:

- access to the Internet
- access to equipment such as dice and cards.

SAMPLE

ROULETTE

Roulette is a game of chance involving betting on the outcome of a spinning wheel. A standard roulette wheel has 37 numbers: 18 are red, 18 are black, and the number 0 is green. In a simplified version of roulette, a player bets \$1 on red or black and they win \$2 if they guess correctly.

You may like to present your findings as a report. Your report could be in the form of:

- a digital presentation
- a poster
- an information brochure
- other (check with your teacher).

SAMPLE