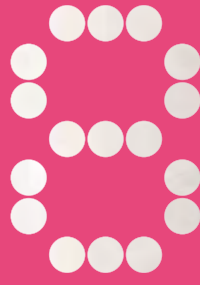


M4

MATHS



AUSTRALIAN CURRICULUM WESTERN AUSTRALIA

AUTHORS: Jennifer Nolan / Melanie Koetsveld / Sonja Stambulic / Robert Bell SERIES CONSULTANT: JAN HONNENS

SAMPLE

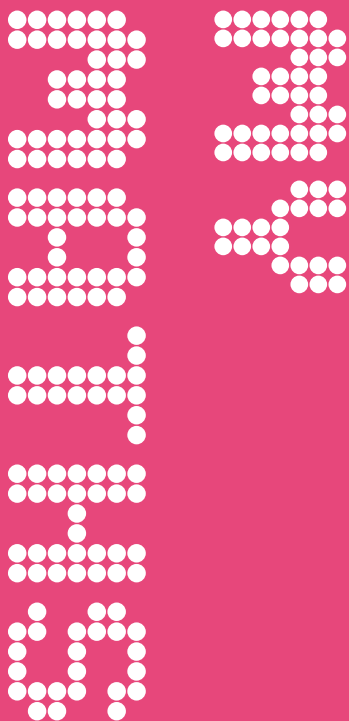
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OXFORD

## NUMBER AND ALGEBRA

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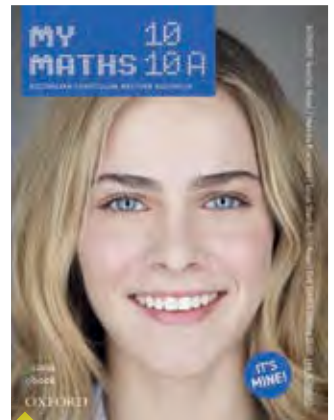
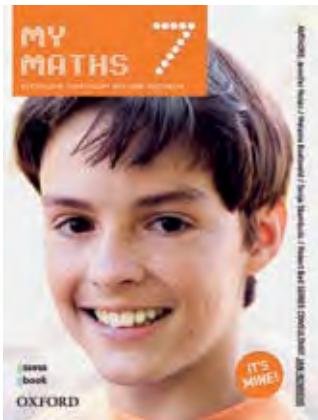
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# OXFORD MYMATHS FOR WESTERN AUSTRALIA



*Oxford MyMaths for Western Australia* has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

## STUDENT BOOK + BOOK/ASSESS

- ▶ Finely levelled exercises to ensure smooth progress
- ▶ Integrated worked examples – right where your students need them
- ▶ Learning organised around the ‘big ideas’ of mathematics
- ▶ Discovery, practice, thinking and problem-solving activities promote deep understanding
- ▶ A wealth of revision material to consolidate and prove learning
- ▶ Rich tasks to apply understanding
- ▶ Highly accessible and easy to navigate
- ▶ Comprehensive digital tutorials and guided examples to support independent progress

3H CONVERTING BETWEEN FRACTIONS, DECIMALS AND PERCENTAGES 167

### EXERCISE 3H Converting between fractions, decimals and percentages

**EXAMPLE 3H-1** Writing a percentage as a decimal

Write 37% as a decimal.

**THINK**

- Write 37% as a fraction.
- Divide the numerator (37) by the denominator (100).
- Write your answer. Show a digit before the decimal point. There are two ones, so write 0.

**WRITE**

$$\frac{37}{100} = \frac{37}{100} = 0.37$$

1 Write each percentage as a decimal.

a 46%	b 13%	c 99%
d 25%	e 20%	f 50%
g 5%	h 8%	i 1%

**EXAMPLE 3H-2** Writing a decimal percentage as a decimal

Write 6.25% as a decimal.

**THINK**

- Write 6.25% as a fraction.
- Divide the numerator (6.25) by the denominator (100). A shortcut is to divide by 100 to ‘move’ the decimal point two places to the left.
- Insert a placeholder zero in the ‘empty’ space (tens place).
- Write your answer. Show a digit before the decimal point.

**WRITE**

$$\frac{6.25}{100} = \frac{6.25}{100} = 0.0625$$

2 Write each percentage as a decimal.

a 23.84%	b 19.65%	c 46.7%
d 3.99%	e 567.4%	f 0.467%
g 12.895%	h 73.28%	i 200.5%
j 10.92%	k 404.04%	l 0.0101%

3H CONVERTING BETWEEN FRACTIONS, DECIMALS AND PERCENTAGES 167

5 Write each fraction as a percentage by first converting to a decimal.

6 Write each fraction as a percentage correct to two decimal places.

7 Check your answers to questions 5 and 6 with a calculator.

8 Eclectus parrots are found in north-eastern Australia. The male is green and the female is red and blue.

9 Write the number of male parrots pictured as a fraction of the total number of parrots.

10 Lachlan scored 18 out of 25 on his first test and 23 out of 30 for his next test.

11 Create your own incomplete table like the one in question 10 with fraction, decimal and percentage equivalents of given amounts. Swap with your classmate.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.25	75%
$\frac{1}{4}$		62.5%
$\frac{1}{5}$	0.4	60%

The student book accurately and carefully delivers the course.

Every question matched to the Australian Curriculum proficiencies.

Worked examples are clearly laid out and located where students need them most.

8G UNDERSTANDING MASS 365

### EXERCISE 8G Understanding mass

1 List these animals in order from lightest to heaviest.

2 For each animal in question 1, which unit would you use to measure mass: milligrams, grams, kilograms, or tonnes?

**EXAMPLE 8G-1** Converting mass units in one step

Convert:

a 820 g into kg	b 12.4 g into mg
-----------------	------------------

**THINK**

- To convert to a larger unit, divide by the conversion factor of 1000. (1000 g = 1 kg)
- To convert to a smaller unit, multiply by the conversion factor of 1000. (1000 mg = 1 g)

**WRITE**

$$820 \text{ g} = (820 \div 1000) \text{ kg} = 0.82 \text{ kg}$$

$$12.4 \text{ g} = (12.4 \times 1000) \text{ mg} = 12\,400 \text{ mg}$$

3 Convert these mass units.

a 1.2 kg into grams	d 1 g into milligrams
e 72 kg into tonnes	e 3.5 t into kilograms
f 450 g into kilograms	f 9.8 g into milligrams
g 750 mg into grams	g 2045 g into kilograms
h 8.13 kg into grams	h 145 kg into tonnes
i 0.93 kg into grams	

8QA CHAPTER 7: SHAPES

### CONNECTION

#### Lamp design

You are to design a lamp up to 20 shapes and 3D objects, at least two different 3D objects must be designed with two different shapes.

**Your task**

- Decide what 2D shapes and 3D objects will make up your lamp.
- Choose an appropriate tessellation that is colour and attractive to cover the base or lampshade.
- Draw a diagram of your lamp using graph or isometric dot paper.
- Draw a set of plans for the lamp.
- Construct a model of the lamp using a series of nets.

# 24/7 LEARNING AND SUPPORT

E-tutors scaffold understanding of key concepts and build confidence.

Self-discovery opportunities for students through guided exploration.

Finely levelled content enables students to progress with ease

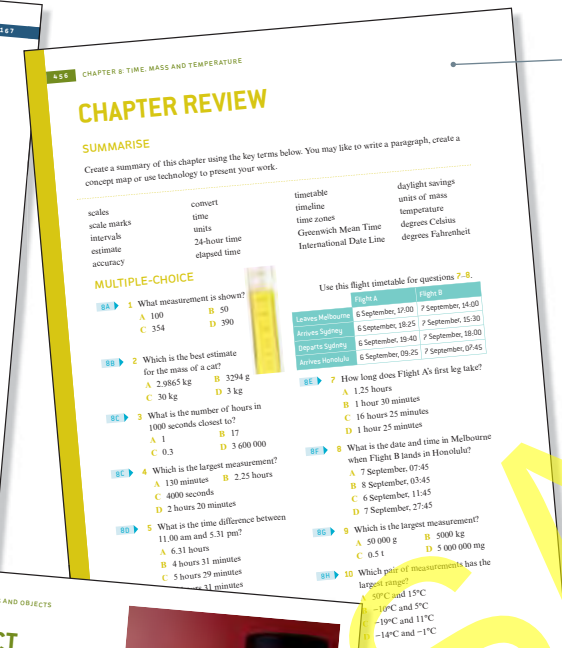


Guided examples support practice and fluency



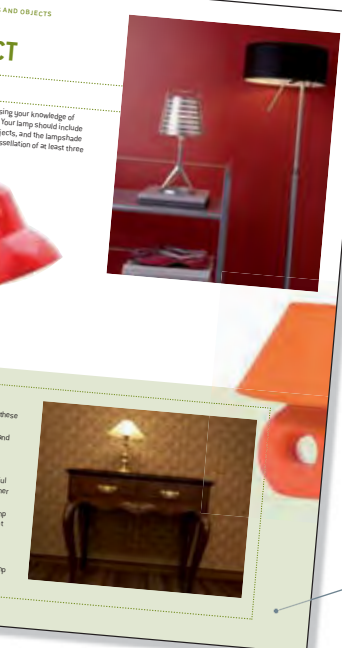
Students receive feedback for incorrect responses

Ample revision to consolidate understanding and prove that learning has happened



Optimise understanding and performance.

Personalised learning: tailor the very best learning experiences for all.



Rich tasks where students can demonstrate understanding

Intervention and extension worksheets supplied for every topic.



## TEACHER QBOOK/ASSESS

Practical classroom resources and tools:

- ▶ Manage student differentiation
- ▶ Correct common misconceptions
- ▶ Assign work
- ▶ Set tests
- ▶ Monitor results
- ▶ Any device, anytime, anywhere.

## 3

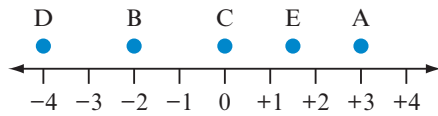
# POSITIVE AND NEGATIVE NUMBERS

- |  |   |
|--|---|
| <b>3A</b> Understanding negative numbers                   | <b>3E</b> Multiplying and dividing integers |
| <b>3B</b> Adding integers                                  | <b>3F</b> Operations with directed numbers  |
| <b>3C</b> Subtracting integers                             | <b>3G</b> Powers of directed numbers        |
| <b>3D</b> Simplifying addition and subtraction of integers | <b>3H</b> The Cartesian plane               |

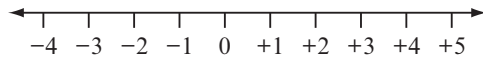
**ESSENTIAL QUESTION**

*This fish lives 1500 m below sea level. Sea level is an example of a reference point: are there any others?*

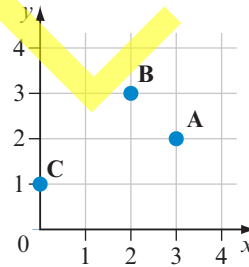
- 3A ▶ 1 List the positions of points A to E.



- 3B ▶ 2 Look at this number line.



- a What number is 1 unit to the right of 3?  
A 2    B 3    C 4    D 5
- b What number is 4 units left of 1?
- c What number is 5 units right of -2?
- 3E ▶ 3 a What is  $30 \times 4$ ?  
A 12    B 34    C 120    D 340
- b What is  $80 \div 4$ ?  
A 20    B 76    C 84    D 320
- 3E ▶ 4 a What is  $163 \times 5$ ?  
b What is  $154 \div 7$ ?
- 3F ▶ 5 a What is  $\frac{2}{3} + \frac{1}{2}$ ?  
A  $\frac{3}{5}$     B  $\frac{7}{6}$     C  $\frac{7}{12}$     D 1
- b What is  $\frac{4}{5} - \frac{3}{4}$ ?
- 3F ▶ 6 a What is  $\frac{15}{28} \times \frac{8}{9}$ ?  
A  $\frac{7}{19}$     B  $\frac{14}{15}$     C  $\frac{23}{37}$     D  $\frac{10}{21}$
- b What is  $1\frac{5}{6} \div 7\frac{1}{3}$ ?
- 3F ▶ 7 What is:  
a  $6.8 + 5.3$     b  $7.4 - 4.5$ ?
- 3F ▶ 8 What is:  
a  $3.6 \times 0.8$     b  $8.4 \div 0.4$ ?
- 3F ▶ 9 What is the average of 5, 7, 8, 10 and 6?
- 3G ▶ 10 a What is the index or power in  $2^7$ ?  
A 2    B 7    C 9    D 14
- b What is the base in  $2^7$ ?  
A 2    B 7    C 9    D 14
- c What is  $4^3$  equivalent to?  
A  $4 \times 3 = 12$   
B  $4 + 4 + 4 = 12$   
C  $4 \times 4 \times 4 = 64$   
D  $3 \times 3 \times 3 \times 3 = 81$
- 3H ▶ 11 Look at this Cartesian plane.



- a What are the coordinates of point A?  
A (1, 0)    B (2, 3)  
C (0, 1)    D (3, 2)
- b What are the coordinates of point B?  
A (1, 0)    B (2, 3)  
C (0, 1)    D (3, 2)
- c What are the coordinates of point C?  
A (1, 0)    B (2, 3)  
C (0, 1)    D (3, 2)



# 3A Understanding negative numbers

## Start thinking!

The average temperature in Beijing is  $-4.5^{\circ}\text{C}$  in January and  $-2^{\circ}\text{C}$  in February. However, the average daily minimum and maximum temperatures (see table) would be more useful if you were visiting Beijing.

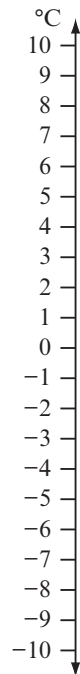


Month	Average daily minimum temperature ( $^{\circ}\text{C}$ )	Average daily maximum temperature ( $^{\circ}\text{C}$ )
January	-9.4	1.6
February	-6.9	4.0

- Which of the six temperatures are:
  - negative numbers?
  - positive numbers?
- Whole numbers that are positive, negative or zero are called **integers**. Which of the six temperatures are integers?
- A vertical number line can represent the scale on a thermometer, showing both positive and negative temperatures.
  - Copy the vertical number line shown and show all six temperatures on it.
  - Which temperature is the highest? Which is the lowest?
  - A number is positive or negative depending on its position from a reference point. What is the reference point on this number line?

Positive and negative numbers are also called **directed numbers**, as they have both size and direction. The number of units away from the reference point is the size of the number, and the sign  $-$  or  $+$  (or no sign) is the direction from the reference point. Directed numbers include positive and negative whole numbers, fractions and decimals.

- Write five examples of directed numbers, including the direction from the reference point.
- Redraw the number line as a horizontal number line and show all six temperatures on it.
- How can you tell whether a number is larger or smaller than another number on:
  - the vertical number line?
  - the horizontal number line?
- List the six temperatures in order from lowest to highest.
- Which month (January or February) is generally colder in Beijing?



## KEY IDEAS

- ▶ Positive and negative numbers are also called directed numbers as they have both size and direction. They include whole numbers, fractions and decimals.
- ▶ Whole numbers that are negative, positive or zero are called integers.
- ▶ Number lines can help you compare the size of directed numbers.
- ▶ **Opposite integers** are located the same number of units away from zero but on opposite sides of a number line.

## EXERCISE 3A Understanding negative numbers

- 1 Look at the number line opposite.
  - a Write three examples of:
    - i a negative integer
    - ii a positive integer.
  - b Which integer on the number line is neither positive nor negative?

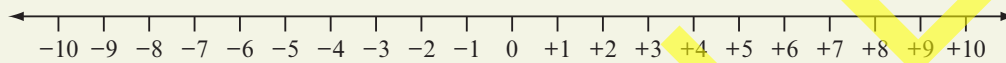
### EXAMPLE 3A-1

#### Identifying the larger of two integers

Which integer is larger in each pair of numbers? Use the number line to help you.

a  $-5$  and  $2$

b  $-4$  and  $-7$



#### THINK

- a Locate the two numbers on the number line and compare their positions.  $2$  is to the right of  $-5$  so  $2$  is larger.
- b Locate the two numbers on the number line and compare their positions.  $-4$  is to the right of  $-7$  so  $-4$  is larger.

#### WRITE

- a  $2$  is larger than  $-5$ .
- b  $-4$  is larger than  $-7$ .

- 2 Which integer is larger in each pair of numbers?
 

a $3$ and $9$	b $0$ and $5$	c $-7$ and $1$
d $5$ and $-8$	e $0$ and $-2$	f $-6$ and $-4$
- 3 Which integer is smaller in each pair of numbers?
 

a $5$ and $6$	b $0$ and $3$	c $-2$ and $7$
d $4$ and $-5$	e $0$ and $-1$	f $-8$ and $-9$
- 4 Which number is larger in each pair of numbers?
 

a $0$ and $5.2$	b $\frac{1}{2}$ and $-1\frac{1}{2}$
c $4.3$ and $-6.2$	d $-\frac{3}{7}$ and $\frac{4}{7}$
e $-1.5$ and $-1.7$	f $-\frac{3}{5}$ and $-\frac{2}{5}$
- 5 Which number is smaller in each pair of numbers?
 

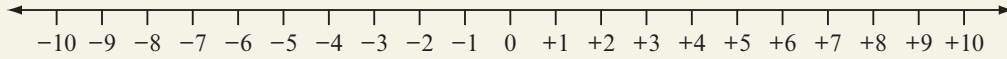
a $0$ and $-4\frac{1}{2}$	b $-2.1$ and $-1.2$
c $5.43$ and $-6.34$	d $-8\frac{5}{11}$ and $-8\frac{6}{11}$
e $-3.54$ and $-3.45$	f $-\frac{2}{3}$ and $-\frac{3}{4}$

**EXAMPLE 3A-2****Comparing directed numbers**

Copy and complete each number statement by writing  $<$  (is less than) or  $>$  (is greater than) in the space provided. Use the number line to help you.

**a**  $-4 \underline{\hspace{1cm}} 3$

**b**  $-5.6 \underline{\hspace{1cm}} -8.1$

**THINK**

- a 1** Locate the two numbers on the number line and compare their positions.  $-4$  is to the left of  $3$  so  $-4$  is smaller.
- 2** Complete the statement with the correct symbol. Remember that mathematical statements are read from left to right ( $-4$  is less than  $3$ ).
- b 1** Locate the two numbers on the number line and compare their positions.  $-5.6$  is to the right of  $-8.1$  so  $-5.6$  is larger.
- 2** Complete the statement with the correct symbol. The statement is read as ' $-5.6$  is greater than  $-8.1$ '.

**WRITE**

**a**  $-4$  is smaller than  $3$ .

$$-4 < 3$$

**b**  $-5.6$  is larger than  $-8.1$ .

$$-5.6 > -8.1$$

**6** Copy and complete each number statement by writing  $<$  (is less than) or  $>$  (is greater than) in the space provided. Use the number line above to help you.

**a**  $-3 \underline{\hspace{1cm}} 2$

**b**  $-8 \underline{\hspace{1cm}} -4$

**c**  $0 \underline{\hspace{1cm}} -1$

**d**  $9 \underline{\hspace{1cm}} -9$

**e**  $-3 \underline{\hspace{1cm}} -5$

**f**  $-7 \underline{\hspace{1cm}} -2$

**g**  $1.4 \underline{\hspace{1cm}} -2.6$

**h**  $-1.5 \underline{\hspace{1cm}} -1.2$

**i**  $4.3 \underline{\hspace{1cm}} -4.3$

**j**  $-3.8 \underline{\hspace{1cm}} -5.8$

**k**  $-0.6 \underline{\hspace{1cm}} -0.7$

**l**  $-8.2 \underline{\hspace{1cm}} 0$

**7** Which number is larger in each pair of numbers?

**a**  $35$  and  $-45$

**b**  $-58$  and  $-24$

**c**  $0$  and  $-82$

**d**  $0$  and  $47$

**e**  $-50\frac{1}{4}$  and  $-70\frac{1}{2}$

**f**  $113.2$  and  $-225.7$

**g**  $-94\frac{2}{3}$  and  $-94\frac{1}{3}$

**h**  $-89.7$  and  $-89.3$

**i**  $-2000\frac{1}{6}$  and  $-2000\frac{1}{7}$

**8** Write each list of integers in ascending order (from smallest to largest).

**a**  $14, -20, 10, -7, 8, -11, -12$

**b**  $-33, 42, -19, -41, 0, 6, 29$

**c**  $-8, -88, 68, 8, -28, -18, -48$

**d**  $73, -82, 3, -140, 104, 145, -126$

**9** Write each list of numbers in descending order (from largest to smallest).

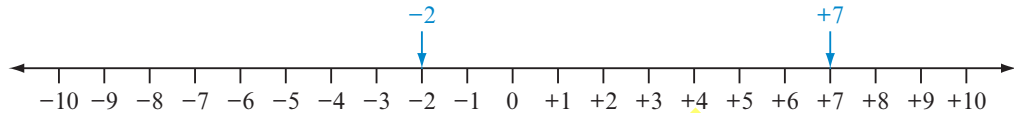
**a**  $-15.4, 71.9, -27.6, -10.8, 4.3, -9.7$

**b**  $-\frac{1}{2}, -2\frac{1}{2}, 5\frac{1}{2}, 10, -11\frac{1}{2}, -5$

**c**  $0, 5.34, -9.04, -14.2, 21.6, -9.4$

**d**  $-1\frac{1}{3}, 5\frac{1}{3}, -7\frac{1}{3}, -6\frac{2}{3}, 7\frac{1}{3}, 1\frac{2}{3}, -7\frac{2}{3}$

- 10** List all the integers between  $-7$  and  $+4$ .
- 11** List five fractions that are between  $-30$  and  $-20$ .
- 12** List five decimals that are between  $-5$  and  $3$ .
- 13** Look at this number line, which shows integers from  $-10$  to  $+10$ . The integer  $-2$  is 2 units to the left of 0 and the integer  $+7$  (or just 7) is 7 units to the right of 0.



- a** What is the reference point for these directed numbers?
- b** Describe the position of each integer from 0 on the number line.
- |                |                |                 |
|----------------|----------------|-----------------|
| <b>i</b> $-4$  | <b>ii</b> $+5$ | <b>iii</b> $+3$ |
| <b>iv</b> $-5$ | <b>v</b> $-3$  | <b>vi</b> $+9$  |
- c** Which pairs of integers in part **b** are the same number of units away from 0 but on opposite sides of the number line? These integers are called opposite integers.
- d** Write the opposite integer to these.
- |                 |                |                 |
|-----------------|----------------|-----------------|
| <b>i</b> $-8$   | <b>ii</b> $+6$ | <b>iii</b> $+1$ |
| <b>iv</b> $-11$ | <b>v</b> $-32$ | <b>vi</b> $17$  |
- e** List three different pairs of opposite integers.

**EXAMPLE 3A-3****Representing a situation with a directed number**

Write a directed number to represent each situation.

- a** The temperature is 15 degrees above zero.    **b** A fish swims 4 m below the surface of the water.  
**c** Your bank account is overdrawn by \$30.

**THINK**

- a** **1** Identify the reference point.  
**2** Decide if the given temperature is above or below the reference point (above).
- b** **1** Identify the reference point.  
**2** Decide if the given distance is above or below the reference point (below).
- c** **1** Identify the reference point.  
**2** Decide if the overdrawn amount is above or below the reference point. If there is money in the account it is 'above', but if money is owed to the account it is 'below'.

**WRITE**

- a** Reference point is  $0^\circ$ .  
 Directed number is  $+15$ .
- b** Reference point is the water surface.  
 Directed number is  $-4$ .
- c** Reference point is  $\$0$ .  
 Directed number is  $-30$ .

- 14 Write a directed number to represent each situation.
- The lift stops at the third floor below the ground floor.
  - You have \$2174.30 in the bank.
  - The temperature inside a freezer is 18 degrees below zero.
  - The shoreline of the Dead Sea, Jordan, is 408 m below sea level.
  - The top of Mt Kilimanjaro, Tanzania, is 5895 m above sea level.
  - A skydiver is 500 m above the ground.
  - Your bank account is overdrawn by \$46.55.
  - A submarine is 40 m below sea level.



- 15 Polar bears live close to the edge of sea ice in the Arctic, where in winter the temperatures range from around  $-45^{\circ}\text{C}$  to  $0^{\circ}\text{C}$ . In summer, the temperatures range from  $-10^{\circ}\text{C}$  to  $10^{\circ}\text{C}$ . The ocean temperature is about  $-1.5^{\circ}\text{C}$ .
- Show  $-45^{\circ}\text{C}$ ,  $0^{\circ}\text{C}$ ,  $-10^{\circ}\text{C}$ ,  $10^{\circ}\text{C}$  and  $-1.5^{\circ}\text{C}$  on a number line.
  - Is  $-10^{\circ}\text{C}$  higher or lower than  $-45^{\circ}\text{C}$ ?
  - Write the five temperatures in order from lowest to highest.
  - A polar bear dips her paw into the water to catch a fish. On a day with an air temperature of  $-18^{\circ}\text{C}$ , does the water feel warmer or colder?



- 16 Bank statements usually show a transaction as a positive number if an amount is added or credited to the account or a negative number for withdrawals or debits.

Use the bank statement to answer these questions.

- What is:
  - the largest amount credited to the account?
  - the smallest amount debited from the account?
- What is:
  - the largest amount credited to the account?
  - the smallest amount debited from the account?
- What does the final balance indicate about the account?

Transaction	Balance
	+\$51.50
+23.65	+\$75.15
-35.00	+\$40.15
-51.30	-\$11.15
+34.45	+\$23.30
+13.40	+\$36.70
-49.95	-\$13.25

- 17** Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.
- Ashwan checks his bank balance and sees he has \$86 in his account. Write this as a positive integer.
  - The bank allows Ashwan's account to be overdrawn. This means that he can withdraw more money than he actually has in his account. If he withdraws \$100 at an ATM, how much does he owe the bank?
  - Write his new bank balance as a negative integer.
- 18** Jasmine needs glasses for reading as she is long sighted (hypermetropic). The prescription for the lenses in her glasses is +2.50. Alex wears contact lenses because he is short sighted (myopic). His lenses have prescriptions of  $-3.50$  for his left eye and  $-4.25$  for his weaker right eye.
- If Justin has a prescription of  $+1.75$ , is he long or short sighted?
  - If Elle has a prescription of  $-4.75$ , is she long or short sighted?
  - How does Elle's prescription compare to Alex's? Who has weaker vision?
  - What do you think the reference of zero means in this situation?



- 19** One way to represent integers is with coloured counters. Use one blue counter to represent  $+1$  and one red counter to represent  $-1$  (or any two colours will do). Collect 10 blue counters and 10 red counters.

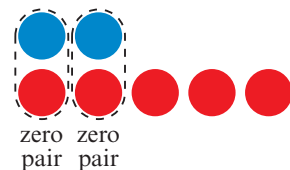


- How many red counters would you use to represent  $-3$ ?
- Use the counters to show each integer listed below. Describe each arrangement.

**i**  $+2$       **ii**  $-4$       **iii**  $+6$

**iv**  $-5$       **v**  $-1$       **vi**  $+8$

- Think about how you could use the counters to represent 0. Why does having one blue counter and one red counter together represent 0? This is called a **zero pair**. (Hint: use the idea of opposite integers.)



- How does this arrangement of counters show  $-3$ ?
- What integer does each arrangement show? Remember that a zero pair can be made by matching one blue and one red counter together.



- You can show any integer in many ways. For example,  $-5$  can be shown with five red counters, or six red counters and one blue counter, or nine red counters and four blue counters and so on. Describe three different ways each integer can be shown.

**i**  $+2$       **ii**  $-4$       **iii**  $-7$

**iv**  $+3$       **v**  $0$       **vi**  $-2$

### Reflect

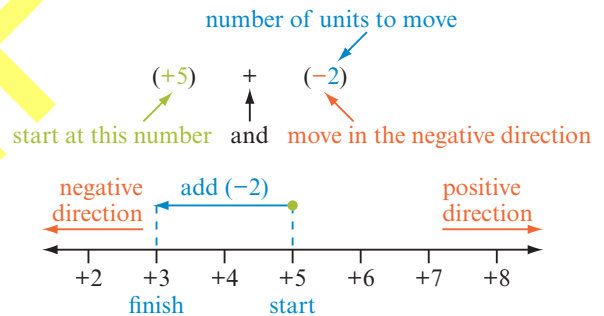
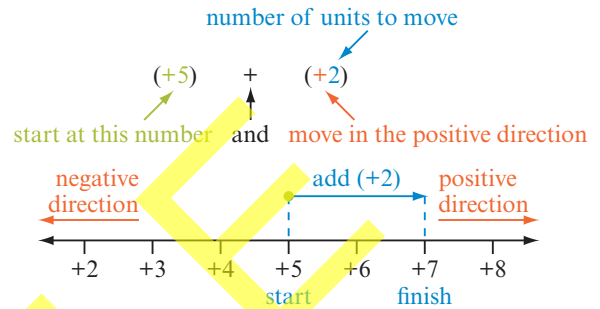
How are negative numbers useful?

# 3B Adding integers

## Start thinking!

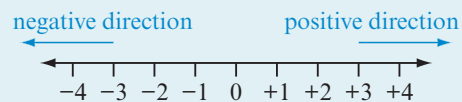
One way to add integers is with a number line.

- What is  $5 + 2$ ?
  - Another way of writing  $5 + 2$  is  $(+5) + (+2)$ .  
To show this on a number line, start at  $+5$  and move  $+2$  units. That is, move 2 units in the positive direction. Do you move to the left or right of  $+5$  to add  $+2$ ?
  - What number do you finish at? Does this match your answer to part **a**?
- Draw a number line for  $(-7) + (+3)$ . Start at  $-7$  and move  $+3$  units.
  - What is  $(-7) + (+3)$ ?
  - Do you move to the left or right of  $-7$  to add  $+3$ ?
- To show  $(+5) + (-2)$  on a number line, start at  $+5$  and move  $-2$  units. That is, move 2 units in the negative direction. Do you move to the left or right of  $+5$  to add  $-2$ ?
  - What is  $(+5) + (-2)$ ?
- Draw a number line for  $(-7) + (-3)$ . Start at  $-7$  and move  $-3$  units.
  - What is  $(-7) + (-3)$ ?
  - Do you move to the left or right of  $-7$  to add  $-3$ ?
- Explain how to use a number line to:
  - add a positive integer
  - add a negative integer.



## KEY IDEAS

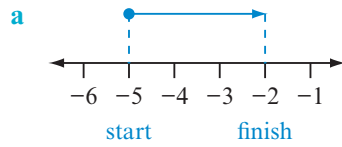
- ▶ On a horizontal number line, moving right is moving in the positive direction and moving left is moving in the negative direction.
- ▶ To add two integers using a number line, start at the position of the first integer and then move left or right the number of units shown by the second integer. The final position is the result.
- ▶ To add a positive number, move to the right (or in a positive direction).
- ▶ To add a negative number, move to the left (or in a negative direction).



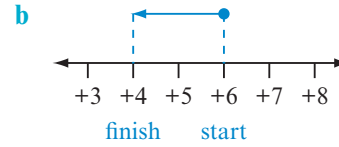
## EXERCISE 3B Adding integers

UNDERSTANDING AND FLUENCY

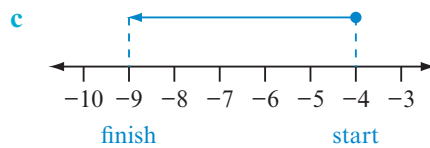
1 Copy and complete each number sentence using the diagram provided.



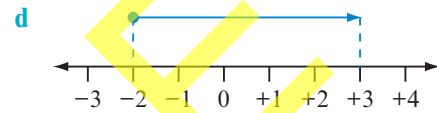
$$(-5) + (+3) = \underline{\quad}$$



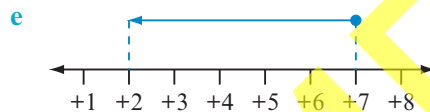
$$(+6) + (-2) = \underline{\quad}$$



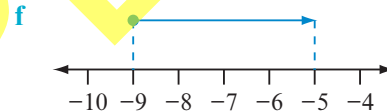
$$(-4) + (-5) = \underline{\quad}$$



$$(-2) + (+\underline{\quad}) = \underline{\quad}$$



$$(\underline{\quad}) + (-5) = \underline{\quad}$$



$$(\underline{\quad}) + (\underline{\quad}) = \underline{\quad}$$

### EXAMPLE 3B-1 Using a number line to add integers

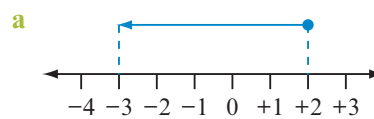
Use a number line to calculate:

**a**  $(+2) + (-5)$       **b**  $(-4) + (+9)$ .

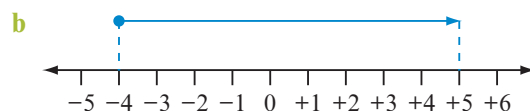
#### THINK

- a** 1 Locate the first number (+2) on the number line. To add  $-5$ , move 5 units to the left (negative direction).
- 2 Locate the finishing point and write the result.
- b** 1 Locate  $-4$  on the number line. To add  $+9$ , move 9 units to the right (positive direction).
- 2 Locate the finishing point and write the result.

#### WRITE



$$(+2) + (-5) = -3$$



$$(-4) + (+9) = +5$$



2 Use a number line to calculate:

- |                        |                        |                        |
|------------------------|------------------------|------------------------|
| <b>a</b> $(-4) + (+3)$ | <b>b</b> $(+1) + (+5)$ | <b>c</b> $(+8) + (-3)$ |
| <b>d</b> $(-2) + (-6)$ | <b>e</b> $(-9) + (+5)$ | <b>f</b> $(+3) + (-6)$ |
| <b>g</b> $(-5) + (-1)$ | <b>h</b> $(+7) + (-2)$ | <b>i</b> $(+2) + (-6)$ |
| <b>j</b> $(-9) + (+9)$ | <b>k</b> $(+4) + (-4)$ | <b>l</b> $(-1) + (+1)$ |

3 Draw a number line showing integers from  $-10$  to  $10$  and use it to calculate:

- |                        |                        |                        |
|------------------------|------------------------|------------------------|
| <b>a</b> $(-4) + (+6)$ | <b>b</b> $(+1) + (+2)$ | <b>c</b> $(-8) + (+3)$ |
| <b>d</b> $(+5) + (-7)$ | <b>e</b> $(+9) + (-4)$ | <b>f</b> $(-3) + (+4)$ |
| <b>g</b> $(+7) + (+2)$ | <b>h</b> $(-6) + (-3)$ | <b>i</b> $(-5) + (+5)$ |
| <b>j</b> $(+8) + (-8)$ | <b>k</b> $(+2) + (-4)$ | <b>l</b> $(-2) + (+2)$ |

4 Copy and complete each number sentence using the diagram provided.

- |   |   |
|---|---|
| <p><b>a</b></p> <p><math>(-11) + (-\underline{\quad}) = \underline{\quad}</math></p>              | <p><b>b</b></p> <p><math>(\underline{\quad}) + (-4) = \underline{\quad}</math></p>                |
| <p><b>c</b></p> <p><math>(\underline{\quad}) + (\underline{\quad}) = \underline{\quad}</math></p> | <p><b>d</b></p> <p><math>(\underline{\quad}) + (\underline{\quad}) = \underline{\quad}</math></p> |

5 Draw a number line showing integers from  $-20$  to  $20$  and use it to calculate:

- |                          |                          |
|--------------------------|--------------------------|
| <b>a</b> $(-9) + (-7)$   | <b>b</b> $(+14) + (-6)$  |
| <b>c</b> $(-11) + (+18)$ | <b>d</b> $(+5) + (+13)$  |
| <b>e</b> $(+16) + (-16)$ | <b>f</b> $(-12) + (+4)$  |
| <b>g</b> $(+8) + (-15)$  | <b>h</b> $(-7) + (-7)$   |
| <b>i</b> $(-13) + (-6)$  | <b>j</b> $(+10) + (+5)$  |
| <b>k</b> $(-15) + (+17)$ | <b>l</b> $(+11) + (-20)$ |
| <b>m</b> $(-19) + (+19)$ | <b>n</b> $(+4) + (-16)$  |
| <b>o</b> $(+10) + (+10)$ | <b>p</b> $(+14) + (-18)$ |
| <b>q</b> $(-17) + (-3)$  | <b>r</b> $(+20) + (-15)$ |

6 Use the words *positive* and *negative* to complete these statements.

- a** Adding two positive integers gives a \_\_\_\_\_ result.
- b** Adding two negative integers gives a \_\_\_\_\_ result.
- c** Adding a positive integer and a negative integer gives a result that is either \_\_\_\_\_ or \_\_\_\_\_ or zero, depending on the numbers involved.

7 Look at your answer to question 6c. How can you tell by looking at the numbers to be added whether the answer will be positive, negative or zero?

**EXAMPLE 3B-2****Identifying positive, negative or zero results**

Decide whether the result of each problem will be positive, negative or zero.

- a**  $(+12) + (+15)$       **b**  $(-7) + (-11)$       **c**  $(-19) + (+19)$   
**d**  $(-13) + (+9)$       **e**  $(-12) + (+17)$

**THINK**

- a** The result of adding two positive numbers is always positive.
- b** The result of adding two negative numbers is always negative.
- c** The result of adding opposite numbers is always zero.
- d** The result of adding a negative number and a positive number could be positive or negative. Since  $-13$  is further from zero than  $+9$  is, the result is negative.
- e** The result of adding a negative number and a positive number could be positive or negative. Since  $+17$  is further from zero than  $-12$  is, the result is positive.

**WRITE**

- a** The result for  $(+12) + (+15)$  is positive.
- b** The result for  $(-7) + (-11)$  is negative.
- c** The result for  $(-19) + (+19)$  is zero.
- d** The result for  $(-13) + (+9)$  is negative.
- e** The result for  $(-12) + (+17)$  is positive.

- 8** Decide whether the result of each problem will be positive, negative or zero.

- a**  $(+10) + (+7)$       **b**  $(-14) + (-5)$   
**c**  $(+15) + (-15)$       **d**  $(+12) + (-19)$   
**e**  $(-8) + (-9)$       **f**  $(-12) + (+16)$   
**g**  $(+18) + (-13)$       **h**  $(+11) + (+30)$   
**i**  $(-25) + (+25)$       **j**  $(-16) + (+11)$   
**k**  $(+22) + (-19)$       **l**  $(+14) + (-17)$

- 9** Without using a number line, calculate each problem in question 8. Explain how you did this.

- 10** Calculate:

- a**  $(+15) + (+13)$       **b**  $(-22) + (-7)$   
**c**  $(+19) + (-11)$       **d**  $(-20) + (+15)$   
**e**  $(+13) + (-13)$       **f**  $(-15) + (-15)$   
**g**  $(+24) + (-17)$       **h**  $(-30) + (+12)$   
**i**  $(+20) + (-24)$       **j**  $(-32) + (-15)$   
**k**  $(+43) + (-31)$       **l**  $(-50) + (+27)$   
**m**  $(+77) + (-94)$       **n**  $(-63) + (+82)$   
**o**  $(+111) + (-49)$       **p**  $(-124) + (-56)$

- 11** An ice sculpture starts to melt. From a starting temperature of  $-6^{\circ}\text{C}$ , the temperature of the surface of the sculpture increases by  $8^{\circ}\text{C}$ .



- a** Which problem would you use to find the temperature of the sculpture's surface?
- A**  $(+6) + (+8)$   
**B**  $(+6) - (+8)$   
**C**  $(-6) + (-8)$   
**D**  $(-6) + (+8)$
- b** Use your answer to part **a** to work out the temperature of the surface.
- 12** Brandon enters a lift three levels below ground level. He selects a floor and travels seven levels upwards.

- a** Which problem would you use to work out the level at which the lift stops?
- A**  $(+3) + (+7)$   
**B**  $(-3) + (-7)$   
**C**  $(-3) + (+7)$   
**D**  $(+3) - (-7)$
- b** Use your answer to part **a** to work out where the lift stops.
- c** Which level did you use as the reference level?



- 13** The air temperature at midnight at the Mawson research base in Antarctica was measured to be  $-21^{\circ}\text{C}$ . At 2 pm, the temperature was  $12^{\circ}\text{C}$  higher.

- Write an addition problem to calculate the temperature at 2 pm.
- What is the temperature at 2 pm?



- 14** A hole 3 m deep is dug for the foot of a pole that is 17 m long.
- Write the depth of the bottom of the hole as an integer if ground level is represented by zero.
  - Write an addition problem to work out the height of the pole above the ground once it is secured.
  - What is the height of the pole above the ground?
- 15** Jess owes her brother \$60.
- Write this amount as an integer.
  - She pays him \$35. Write an addition problem to work out the amount she still owes him.
  - How much does Jess still owe her brother?

- 16** Calculate:

- |                            |                            |
|----------------------------|----------------------------|
| <b>a</b> $(-110) + (-13)$  | <b>b</b> $(+54) + (+126)$  |
| <b>c</b> $(-120) + (+8)$   | <b>d</b> $(+15) + (-165)$  |
| <b>e</b> $(-71) + (+100)$  | <b>f</b> $(+62) + (-69)$   |
| <b>g</b> $(-80) + (-90)$   | <b>h</b> $(+111) + (-98)$  |
| <b>i</b> $(-122) + (+122)$ | <b>j</b> $(+85) + (+95)$   |
| <b>k</b> $(+250) + (-125)$ | <b>l</b> $(-324) + (+300)$ |

- 17** Two samples of frozen nitrogen have a temperature of  $-212^{\circ}\text{C}$ . The temperature of one sample is increased by  $14^{\circ}\text{C}$  and it becomes liquid. The other sample has its temperature increased by  $19^{\circ}\text{C}$  and turns to gas.
- Calculate the temperature of the liquid nitrogen. Show the addition problem you used to work out the result.
  - Calculate the temperature of the nitrogen gas. Show the addition problem you used to work out the result.

### Reflect

How can you predict whether the result of adding two integers will be positive or negative?

# 3C Subtracting integers

## Start thinking!

One way of subtracting integers is to use a number line.

1 a What is  $5 - 2$ ?

b Another way of writing  $5 - 2$  is  $(+5) - (+2)$ .

To show this on a number line, start at  $+5$ . The second number  $(+2)$  indicates to move 2 units in the positive direction. However, the subtraction sign indicates to do the opposite and move in the negative direction.

Do you move to the left or right of  $+5$  to subtract  $+2$ ?

c What number do you finish at? Does this match your answer to part a?

2 a Draw a number line for  $(-7) - (+3)$ . Start at  $-7$  and instead of moving 3 units in the positive direction, do the opposite and move 3 units in the negative direction.

b What is  $(-7) - (+3)$ ?

c Do you move to the left or right of  $-7$  to subtract  $+3$ ?

3 a To show  $(+5) - (-2)$  on a number line, start at  $+5$ .

The number  $-2$  indicates to move 2 units in the negative direction. However, the subtraction sign indicates to do the opposite and move in the positive direction. Do you move to the left or right of  $+5$  to subtract  $-2$ ?

b What is  $(+5) - (-2)$ ?

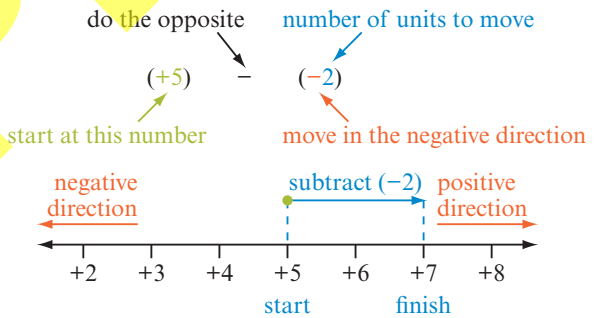
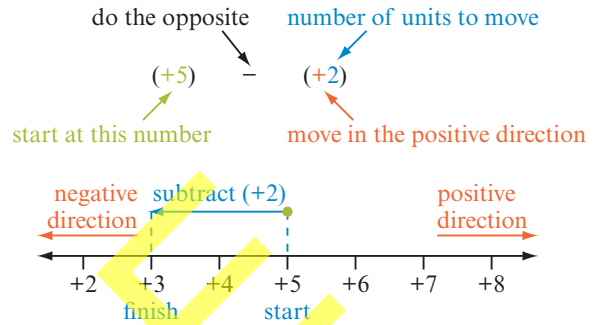
4 a Draw a number line for  $(-7) - (-3)$ . Start at  $-7$  and instead of moving 3 units in the negative direction, do the opposite and move 3 units in the positive direction.

b What is  $(-7) - (-3)$ ?

c Do you move to the left or right of  $-7$  to subtract  $-3$ ?

5 Explain how to use a number line to:

a subtract a positive number      b subtract a negative number.



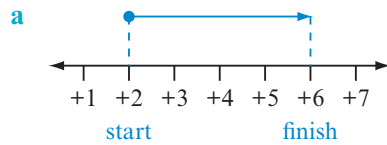
## KEY IDEAS

- ▶ To subtract one integer from another using a number line, start at the first integer and move left or right the number of units indicated by the second. The final position is the result.
- ▶ To subtract a positive number, move to the left (or in a negative direction).
- ▶ To subtract a negative number, move to the right (or in a positive direction).

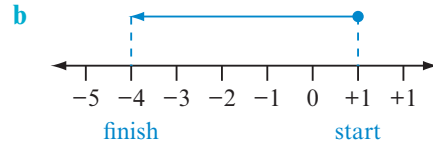
## EXERCISE 3C Subtracting integers

UNDERSTANDING AND FLUENCY

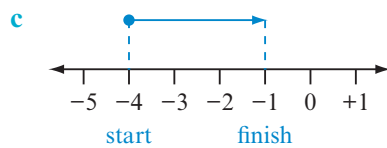
1 Copy and complete each number sentence using the diagram provided.



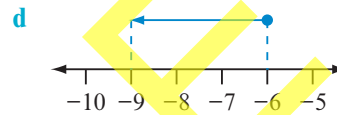
$$(+2) - (-4) = \underline{\quad}$$



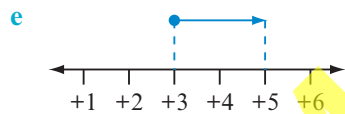
$$(+1) - (+5) = \underline{\quad}$$



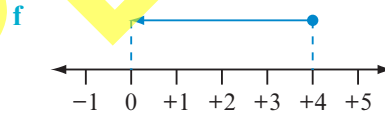
$$(-4) - (-3) = \underline{\quad}$$



$$(-6) - (+\underline{\quad}) = \underline{\quad}$$



$$(\underline{\quad}) - (-2) = \underline{\quad}$$



$$(\underline{\quad}) - (\underline{\quad}) = \underline{\quad}$$

### EXAMPLE 3C-1 Using a number line to subtract integers

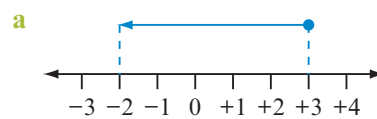
Use a number line to calculate:

**a**  $(+3) - (+5)$      **b**  $(-1) - (-4)$ .

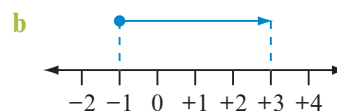
#### THINK

- a** 1 Locate the first number (+3) on the number line. To subtract +5, move 5 units to the left (negative direction).
- 2 Locate the finishing point and write the result.
- b** 1 Locate the first number (-1) on the number line. To subtract -4, move 4 units to the right (positive direction).
- 2 Locate the finishing point and write the result.

#### WRITE



$$(+3) - (+5) = -2$$



$$(-1) - (-4) = +3$$

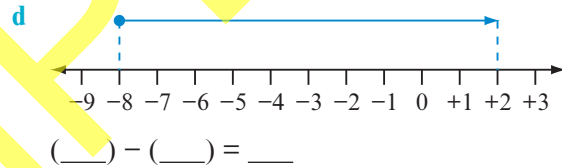
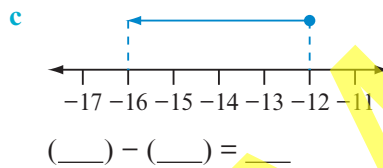
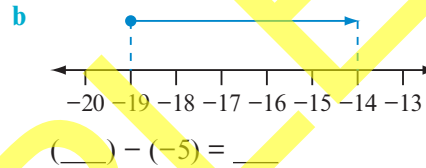
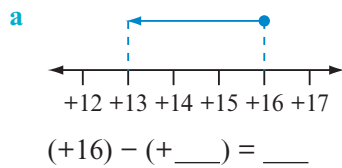
2 Use a number line to calculate:

- |   |               |   |               |   |               |
|---|---------------|---|---------------|---|---------------|
| a | $(+6) - (+4)$ | b | $(-9) - (-2)$ | c | $(+3) - (+3)$ |
| d | $(-5) - (-4)$ | e | $(+7) - (-1)$ | f | $(-4) - (+2)$ |
| g | $(-1) - (+6)$ | h | $(+2) - (-8)$ | i | $(-3) - (-6)$ |
| j | $(+2) - (+7)$ | k | $(+5) - (+8)$ | l | $(-1) - (-9)$ |

3 Draw a number line showing integers from  $-10$  to  $10$  and use it to calculate:

- |   |               |   |               |   |               |
|---|---------------|---|---------------|---|---------------|
| a | $(+3) - (+8)$ | b | $(-6) - (+4)$ | c | $(+5) - (-1)$ |
| d | $(-2) - (-9)$ | e | $(-7) - (+2)$ | f | $(-8) - (-8)$ |
| g | $(+1) - (-3)$ | h | $(-4) - (+5)$ | i | $(+6) - (+5)$ |
| j | $(-3) - (-7)$ | k | $(+2) - (-2)$ | l | $(-5) - (-5)$ |

4 Copy and complete each number sentence using the diagram provided.



5 Draw a number line showing integers from  $-20$  to  $20$  and use it to calculate:

- |   |                 |   |                 |   |                 |
|---|-----------------|---|-----------------|---|-----------------|
| a | $(+13) - (+8)$  | b | $(+18) - (-1)$  | c | $(+10) - (-10)$ |
| d | $(-7) - (+11)$  | e | $(+17) - (+17)$ | f | $(-19) - (+1)$  |
| g | $(+16) - (-2)$  | h | $(-12) - (+8)$  | i | $(-5) - (-12)$  |
| j | $(+16) - (+14)$ | k | $(-15) - (-15)$ | l | $(+6) - (-10)$  |
| m | $(-18) - (-7)$  | n | $(+15) - (+12)$ | o | $(-11) - (-13)$ |
| p | $(+14) - (+20)$ | q | $(-13) - (+7)$  | r | $(-16) - (-12)$ |

6 Use the words *positive* and *negative* to complete these statements.

- a Subtracting a negative integer from a positive integer gives a result that is \_\_\_\_\_.
- b Subtracting a positive integer from a negative integer gives a result that is \_\_\_\_\_.
- c Subtracting a positive integer from a positive integer, or a negative integer from a negative integer, gives a result that is either \_\_\_\_\_ or \_\_\_\_\_ or zero depending on the numbers involved.

7 Look at your answer to question 6c. How can you tell by looking at the numbers to be subtracted whether the answer will be positive or negative or zero?

**EXAMPLE 3C-2****Identifying positive, negative and zero results**

Decide whether the result of each problem will be positive, negative or zero.

**a**  $(+12) - (-15)$

**b**  $(-7) - (+11)$

**c**  $(-19) - (-19)$

**d**  $(-13) - (-9)$

**e**  $(+12) - (+17)$

**THINK**

- a** Subtracting a negative number from a positive number always gives a positive result.
- b** Subtracting a positive number from a negative number always gives a negative result.
- c** Subtracting two identical numbers always gives a result of zero.
- d** Subtracting a negative number from a negative number could give a positive or negative result. Since  $-13$  is further from zero than  $-9$  is, the result is negative.
- e** Subtracting a positive number from a positive number could give a positive or negative result. Since  $+17$  is further from zero than  $+12$  is, the result is negative.

**WRITE**

- a** The result for  $(+12) - (-15)$  is positive.
- b** The result for  $(-7) - (+11)$  is negative.
- c** The result for  $(-19) - (-19)$  is zero.
- d** The result for  $(-13) - (-9)$  is negative.
- e** The result for  $(+12) - (+17)$  is negative.

- 8** Decide whether the result of each problem will be positive, negative or zero.

**a**  $(+11) - (-8)$

**b**  $(-16) - (+4)$

**c**  $(+13) - (+13)$

**d**  $(+15) - (+18)$

**e**  $(-10) - (-7)$

**f**  $(-17) - (+11)$

**g**  $(+8) - (-14)$

**h**  $(+12) - (+19)$

**i**  $(-18) - (-18)$

**j**  $(-14) - (-13)$

**k**  $(+16) - (+21)$

**l**  $(-20) - (-27)$

- 9** Without using a number line, calculate each problem in question 8. Explain how you did this.

- 10** Calculate:

**a**  $(-24) - (+8)$

**b**  $(-3) - (-28)$

**c**  $(+30) - (+16)$

**d**  $(-29) - (-29)$

**e**  $(+38) - (+27)$

**f**  $(-42) - (-22)$

**g**  $(-55) - (+14)$

**h**  $(+71) - (-69)$

**i**  $(+86) - (+90)$

**j**  $(+65) - (-26)$

**k**  $(+45) - (-45)$

**l**  $(-34) - (+52)$

**m**  $(-53) - (+17)$

**n**  $(+68) - (-24)$

**o**  $(-94) - (+72)$

**p**  $(-102) - (-129)$



- 11** An apple pie is moved from the fridge to the freezer. After 30 minutes, the temperature of the pie has decreased from  $4^{\circ}\text{C}$  to  $-6^{\circ}\text{C}$ .

**a** Which problem would you use to work out the difference in the two temperatures?

- A**  $(+6) - (+4)$   
**B**  $(-6) - (+4)$   
**C**  $(-6) - (-4)$   
**D**  $(-6) + (+4)$

**b** Use your answer to part **a** to work out the temperature difference.



- 12** A captain orders his boat's anchor to be pulled up from a depth of 9 m. At a depth of 4 m it is snagged in seaweed, before finally reaching a position 1 m above the water line.

**a** Which problem would you use to work out the vertical distance the anchor has moved before becoming snagged?

- A**  $(-4) - (-9)$   
**B**  $(+1) - (+4)$   
**C**  $(-4) - (+9)$   
**D**  $(+1) - (-4)$

**b** Use your answer to part **a** to work out this vertical distance.

**c** Repeat parts **a** and **b** but this time work out

the vertical distance moved by the anchor after it is freed from seaweed.

**d** What did you use as the reference point for zero in each case?



- 13** What is the difference between a temperature of  $-18^{\circ}\text{C}$  and a temperature of  $33^{\circ}\text{C}$ ? Show the subtraction problem you used to calculate the result.

- 14** A helicopter is flying 28 m above the surface of the ocean. The pilot spots a shark swimming at an estimated depth of 3 m and a stingray at an estimated depth of 11 m below the surface of the water.



- a** If the ocean surface is the reference point for zero, write the position of the helicopter as a positive integer.
- b** Write the positions of the shark and the stingray as negative integers.
- c** Write a subtraction problem to calculate the vertical distance between:
- the helicopter and the shark
  - the helicopter and the stingray
  - the shark and the stingray.
- (Hint: find the difference between the two positions.)
- d** Calculate each subtraction problem.

- 15** Calculate:

- a**  $(-100) - (+24)$     **b**  $(+60) - (+140)$     **c**  $(-150) - (-10)$   
**d**  $(+25) - (-145)$     **e**  $(-168) - (+200)$     **f**  $(+77) - (-58)$   
**g**  $(-93) - (-98)$     **h**  $(+133) - (-87)$     **i**  $(-240) - (+240)$   
**j**  $(+176) - (+84)$     **k**  $(+360) - (-235)$     **l**  $(-415) - (+500)$

- 16** At the start of March, Tyra's bank account shows a balance of  $-\$310$ . At the end of the month, it shows a balance of  $-\$247$ .
- a** Calculate the difference between the two account balances. Show the subtraction problem you used to calculate the result.
- b** Interpret the result. (Hint: what does this result mean?)
- 17** Show three examples of a pair of integers that give a negative result when they are added and when they are subtracted.

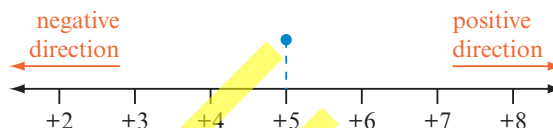
### Reflect

How does a number line help when subtracting integers?

# 3D Simplifying addition and subtraction of integers

## Start thinking!

Is there an easier way to add and subtract integers using number lines?



- a** Use the number line to calculate:

**i**  $(+5) + (+2)$       **ii**  $(+5) - (-2)$ .

**b** Which direction did you move from +5 in each case?

**c** What does this tell you about adding a positive number and subtracting a negative number?

**d**  $(+5) + (+2)$  is the same as  $5 + 2$ . Why can you also write  $(+5) - (-2)$  more simply as  $5 + 2$ ?
- How were you able to make the problems in question 1 easier?
- a** Use the number line to calculate:

**i**  $(+5) - (+2)$       **ii**  $(+5) + (-2)$ .

**b** Which direction did you move from +5 in each case?

**c** What does this tell you about subtracting a positive number and adding a negative number?

**d**  $(+5) - (+2)$  is the same as  $5 - 2$ . Why can you also write  $(+5) + (-2)$  more simply as  $5 - 2$ ?
- How were you able to make the problems in question 3 easier?
- Explain how you can remember which equivalent operation to use when simplifying addition and subtraction problems.

## KEY IDEAS

- ▶ In addition and subtraction problems, you can replace two operation signs that are next to each other with one equivalent sign.
- ▶ For example,  $(-6) - (-2)$  simplifies to  $-6 + 2$ , because  $-(-)$  is equivalent to  $+$ .
- ▶ When there is a  $+$  sign between two numbers, move to the right (positive direction) along the number line.
- ▶ When there is a  $-$  sign between two numbers, move to the left (negative direction) along the number line.

$+$   $(+)$  is equivalent to  $+$   
 $-$   $(-)$  is equivalent to  $+$   
 $+$   $(-)$  is equivalent to  $-$   
 $-$   $(+)$  is equivalent to  $-$

## EXERCISE 3D Simplifying addition and subtraction of integers

- 1 Copy and complete this table to show the equivalent operation that simplifies the problem and which direction to move on a number line.

	Operations	Equivalent operation	Move in positive or negative direction?	Move left or right on a number line?
adding a positive number	+ (+)	+	positive	right
adding a negative number	+ (-)			
subtracting a positive number	- (+)			
subtracting a negative number	- (-)			

### EXAMPLE 3D-1

#### Writing a simpler equivalent problem

Write each problem more simply.

**a**  $(-6) - (+4)$

**b**  $(+5) - (-8)$

#### THINK

**a** Replace  $- (+)$  with  $-$ .

**b** Replace  $- (-)$  with  $+$  and write  $+5$  as  $5$ .

#### WRITE

**a**  $(-6) - (+4)$   
 $= -6 - 4$

**b**  $(+5) - (-8)$   
 $= 5 + 8$

- 2 Copy and complete to write a simpler equivalent problem. Use  $+$  or  $-$  to fill the gap.

**a**  $(-3) - (+7)$

$= -3 \underline{\quad} 7$

**c**  $(-4) + (-5)$

$= -4 \underline{\quad} 5$

**e**  $(+5) - (+9)$

$= 5 \underline{\quad} 9$

**b**  $(+1) + (+6)$

$= 1 \underline{\quad} 6$

**d**  $(+2) - (-4)$

$= 2 \underline{\quad} 4$

**f**  $(-6) - (-8)$

$= -6 \underline{\quad} 8$

- 3 Write each problem more simply.

**a**  $(-1) - (-3)$

**c**  $(-5) + (-2)$

**e**  $(+4) + (+3)$

**g**  $(-5) - (-8)$

**i**  $(-7) + (-3)$

**b**  $(+8) - (+4)$

**d**  $(+6) - (-1)$

**f**  $(+3) - (+9)$

**h**  $(-3) + (+1)$

**j**  $(-6) - (-7)$

**EXAMPLE 3D-2****Adding and subtracting integers using simpler equivalent problems**

Calculate:

**a**  $(-2) - (-3)$

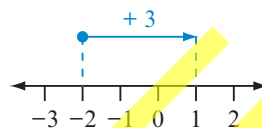
**b**  $(+4) + (-7)$

**THINK**

- a** 1 Write the problem in a simpler equivalent form by replacing  $-(-)$  with  $+$ .
- 2 Locate  $-2$  on the number line. For  $+$ , move 3 units right (positive direction).
- 3 Locate the finishing point and write the result.
- b** 1 Write the problem in a simpler equivalent form by replacing  $+(-)$  with  $-$ .
- 2 Locate 4 on the number line. For  $-$ , move 7 units left (negative direction).
- 3 Locate the finishing point and write the result.

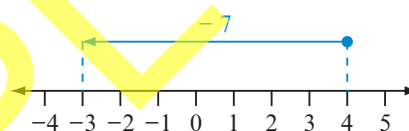
**WRITE**

$$\begin{aligned} \mathbf{a} \quad & (-2) - (-3) \\ & = -2 + 3 \end{aligned}$$



$$(-2) - (-3) = 1$$

$$\begin{aligned} \mathbf{b} \quad & (+4) + (-7) \\ & = 4 - 7 \end{aligned}$$



$$(+4) + (-7) = -3$$

- 4** Simplify each problem and then calculate the result.

**a**  $(-4) - (-7)$

**b**  $(+9) + (-3)$

**c**  $(+2) - (+6)$

**d**  $(-5) - (-5)$

**e**  $(-8) + (+7)$

**f**  $(+6) + (-1)$

**g**  $(-3) - (-4)$

**h**  $(+4) - (+9)$

**i**  $(+2) - (-7)$

- 5** Draw a number line from  $-10$  to  $10$  and use it to calculate:

**a**  $9 - 4$

**b**  $3 - 5$

**c**  $-2 + 8$

**d**  $-6 - 4$

**e**  $7 - 10$

**f**  $-1 - 6$

**g**  $8 - 4$

**h**  $-7 + 9$

**i**  $-10 + 10$

**j**  $0 - 9$

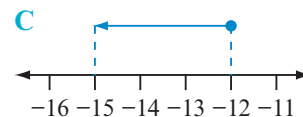
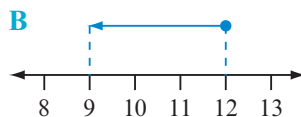
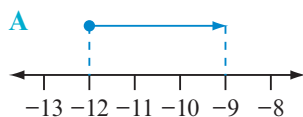
**k**  $-5 - 4$

**l**  $-1 + 8$

- 6** Calculate each result for question 2.

- 7** Calculate each result for question 3.

- 8** Match each diagram (A, B, C) to the appropriate problem (a, b, c).



**a**  $-12 - 3$

**b**  $-12 + 3$

**c**  $12 - 3$

- 9** Calculate each result for question **8**.
- 10** Draw a number line from  $-20$  to  $20$  and use it to calculate:
- a**  $15 - 12$       **b**  $-17 + 11$       **c**  $9 - 13$   
**d**  $-4 + 20$       **e**  $-2 - 17$       **f**  $8 - 18$   
**g**  $-14 - 3$       **h**  $-16 + 14$       **i**  $11 - 24$   
**j**  $-19 + 19$       **k**  $-10 - 10$       **l**  $-18 + 20$
- 11** Decide whether each result will be positive, negative or zero.
- a**  $-3 - 7$       **b**  $16 + 14$       **c**  $8 - 15$   
**d**  $-13 + 13$       **e**  $-11 + 18$       **f**  $19 - 12$   
**g**  $-17 + 4$       **h**  $-14 - 10$       **i**  $17 - 20$
- 12** Without using a number line, calculate each result for question **11**. Explain how you did this.
- 13** Copy and complete each addition table.

**a**

+	-5	-3	0	1	4
-3				-2	
-2					
-1			-1		
3					
6	1				

**b**

+	-10	-14	-7	13	19
11					
14					
-10					
-22					
-18					

- 14** Calculate:
- a**  $-45 + 40$       **b**  $-72 - 27$       **c**  $56 - 88$   
**d**  $-34 + 43$       **e**  $130 - 170$       **f**  $-213 - 62$   
**g**  $-158 + 400$       **h**  $286 - 168$       **i**  $-505 + 505$   
**j**  $1000 - 3000$       **k**  $-2040 - 940$       **l**  $1469 - 1500$   
**m**  $-858 + 885$       **n**  $3400 - 4040$       **o**  $-5006 - 679$

**EXAMPLE 3D-3**

## Adding and subtracting integers

Calculate  $-8 + 3 - 6$ .

**THINK**

- 1 Work from left to right. Calculate  $-8 + 3$ .
- 2 Calculate  $-5 - 6$ .

**WRITE**

$$\begin{aligned} & -8 + 3 - 6 \\ & = -5 - 6 \\ & = -11 \end{aligned}$$

15 Calculate:

a  $-2 + 5 + 7$

b  $3 - 9 + 2$

c  $6 - 1 - 8$

d  $-10 + 4 - 7$

e  $-12 + 19 - 7$

f  $-20 - 11 - 2$

g  $33 + 22 - 65$

h  $70 - 81 - 15$

i  $-24 - 18 + 12$

j  $59 - 93 + 17$

16 Write each as a simpler equivalent problem and then calculate the result.

a  $(-5) - (+7) + (+6)$

b  $(+9) + (-2) - (-8)$

c  $(-4) - (-3) + (+1)$

d  $(+8) + (-6) - (-10)$

e  $(-22) - (+5) + (+13)$

f  $(-14) - (-8) + (-25)$

g  $(+34) + (-19) - (+7)$

h  $(-9) - (+26) - (-37)$

17 A bungee jumper dives from a platform (position A) 50 m above the water. He dips 2 m under the water (position B) before rebounding to a height 18 m below the platform (position C).

- If the reference is the surface of the water, write positions A, B and C as integers.
- Write a problem to find the distance between:
  - A and B
  - B and C
  - A and C.
- Calculate each distance.



18 You enter a lift at the 15th floor and travel down 19 floors. Which level do you finish at? Show how you calculated your answer.

19 Tahini has \$285 in her bank account.

- Write this as an integer.
- She withdraws \$300. Write a subtraction problem to calculate her new account balance.
- What is her new account balance as an integer? Interpret this result. (Hint: what does the integer tell you?)



- 20 A deep sea submersible is lowered into the ocean from a ship's deck 15 m above sea level. The unmanned vessel is on a mission to take photos of deep sea animals.



For each part, write an addition or subtraction problem and then calculate the answer.

- a A deep sea anglerfish is 900 m below sea level. What vertical distance has the submersible travelled from the deck of the ship to reach this fish?
- b A deep sea amphipod is 2500 m below sea level. How much further has the submersible travelled vertically to reach it?
- c What vertical distance does the submersible now need to travel to return to the deck of the ship?
- 21 Sarah enters a lift at the third floor and travels down nine floors and then up five floors. What floor does she finish at? Show the problem you used to get your answer.
- 22 Write two different sets of three integers where the sum of the integers is negative.
- 23 What number should be added to each of these to give 0?
- a -24      b 312      c 5  
d -70      e -1018      f 4218
- 24 What number should be subtracted from each of these to give 0?
- a 100      b -21      c 366  
d -249      e -4000      f 9007
- 25 What number should be subtracted from each of these to give 20?
- a 65      b -10      c 1281  
d -347      e -1000      f -3240
- 26 Find the missing integer to make each number sentence true.
- a  $-54 + 33 + \underline{\quad} = 12$   
b  $121 - \underline{\quad} - 50 = -6$   
c  $-300 + 225 - \underline{\quad} = 50$   
d  $\underline{\quad} - 58 + 73 = -25$

**Reflect**

How can you simplify addition and subtraction problems?

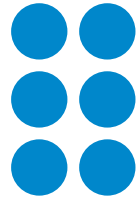


# 3E Multiplying and dividing integers

## Start thinking!

Collect two sets of coloured counters. Choose one colour (say, blue) to represent positive integers and the other colour (say, red) to represent negative integers.

- The product  $3 \times 2$  or  $+3 \times +2$  is the same as three lots of  $(+2)$ .
  - Arrange counters to show  $(+2) + (+2) + (+2)$ . Describe the arrangement.
  - What result does this give for  $+3 \times +2$ ?
- Show  $2 \times 3$  or two lots of  $(+3)$  with your counters. What is the result for  $+2 \times +3$ ?
- Compare your results for questions 1 and 2. Does the order in which you multiply two numbers affect the result?
- When multiplying two positive numbers, is the result positive or negative?
- The product  $3 \times -2$  or  $+3 \times -2$  is the same as three lots of  $(-2)$ .
  - Arrange counters to show  $(-2) + (-2) + (-2)$ . Describe the arrangement.
  - What is the result for  $+3 \times -2$ ?
- When multiplying a positive number by a negative number, is the result positive or negative?
- What would the result for  $-2 \times +3$  be? (Hint: look at question 3.)
- When multiplying a negative number by a positive number, is the result positive or negative?
- What happens when you multiply two negative numbers together?  
To find the result for  $-2 \times -3$  you can first look at  $+2 \times -3$ .  
Since  $-2$  is the opposite of  $+2$ , the result of  $-2 \times -3$  is the opposite of the result to  $+2 \times -3$ .
  - What is the result for  $-2 \times -3$ ?
  - When multiplying two negative numbers, is the result positive or negative?



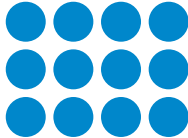
## KEY IDEAS


- ▶ Multiplying or dividing two integers can give a result that is positive or negative (or zero), according to these rules.
- ▶ Remember:
  - ▷ if the numbers have the same sign, the result is positive
  - ▷ if the numbers have different signs, the result is negative.

$+$	$\times$	$+$	$=$	$+$	$+$	$\div$	$+$	$=$	$+$
$+$	$\times$	$-$	$=$	$-$	$+$	$\div$	$-$	$=$	$-$
$-$	$\times$	$+$	$=$	$-$	$-$	$\div$	$+$	$=$	$-$
$-$	$\times$	$-$	$=$	$+$	$-$	$\div$	$-$	$=$	$+$

## EXERCISE 3E Multiplying and dividing integers

1 Use the diagrams shown to find the result for each problem.

a   
 $(+3) \times (+4)$

b   
 $(+2) \times (-7)$

### EXAMPLE 3E-1

#### Multiplying integers

Calculate:

a  $-4 \times +7$

b  $-9 \times -5$ .

#### THINK

- a Ignore the signs and calculate  $4 \times 7$ . (28)  
 Decide whether the result will be positive or negative. Signs are different so result is negative.
- b Ignore the negative signs and calculate  $9 \times 5$ . (45)  
 Decide whether the result will be positive or negative. Signs are the same so result is positive.

#### WRITE

a  $-4 \times +7$   
 $= -28$

b  $-9 \times -5$   
 $= +45$  (or 45)

2 Calculate:

a  $+3 \times -4$

b  $+7 \times +5$

c  $-6 \times +2$

d  $-3 \times -9$

e  $-1 \times +8$

f  $+4 \times -6$

g  $-8 \times -9$

h  $+2 \times +10$

i  $-4 \times +11$

j  $+7 \times -1$

k  $-2 \times -4$

l  $+9 \times +5$

3 a What happens when you divide integers? Dividing is the inverse operation to multiplying. Copy and complete each statement.

i  $+2 \times +3 = \underline{\quad}$  so  $\underline{\quad} \div +2 = +3$  or  $\underline{\quad} \div +3 = +2$

ii  $+2 \times -3 = \underline{\quad}$  so  $\underline{\quad} \div +2 = -3$  or  $\underline{\quad} \div -3 = +2$

iii  $-2 \times +3 = \underline{\quad}$  so  $\underline{\quad} \div -2 = +3$  or  $\underline{\quad} \div +3 = -2$

iv  $-2 \times -3 = \underline{\quad}$  so  $\underline{\quad} \div -2 = -3$  or  $\underline{\quad} \div -3 = -2$

b Does division follow the same pattern as multiplication to give positive or negative results?

c Is the result positive or negative for each of these division types?

i positive  $\div$  positive

ii negative  $\div$  negative

iii positive  $\div$  negative

iv negative  $\div$  positive

**EXAMPLE 3E-2****Dividing integers**

Calculate:

**a**  $-30 \div -5$

**b**  $+48 \div -6$ .

**THINK**

- a** Ignore the signs and calculate  $30 \div 5$ . (6) Decide whether the result will be positive or negative. Signs are the same so result is positive.
- b** Ignore the signs and calculate  $48 \div 6$ . (8) Decide whether the result will be positive or negative. Signs are different so result is negative.

**WRITE**

**a**  $-30 \div -5$   
 $= +6$  (or 6)

**b**  $+48 \div -6$   
 $= -8$

**4** Calculate:

**a**  $+15 \div -3$

**b**  $-32 \div -8$

**c**  $+24 \div +4$

**d**  $-63 \div +9$

**e**  $-17 \div -1$

**f**  $+48 \div -6$

**g**  $-56 \div +8$

**h**  $-81 \div -9$

**i**  $+42 \div -7$

**j**  $-12 \div +1$

**k**  $-20 \div -2$

**l**  $+72 \div -8$

**5** Calculate:

**a**  $5 \times -7$

**b**  $-9 \div 3$

**c**  $-28 \div -4$

**d**  $-10 \times 6$

**e**  $36 \div -9$

**f**  $-15 \times -4$

**g**  $-100 \div 20$

**h**  $18 \times -2$

**i**  $-3 \times -30$

**j**  $25 \div -25$

**k**  $-1 \times 38$

**l**  $-45 \div -1$

**6** Copy and complete each statement.

**a**  $-15 \times \underline{\quad} = 45$

**b**  $56 \div \underline{\quad} = -8$

**c**  $\underline{\quad} \times -4 = 36$

**d**  $\underline{\quad} \div -25 = -4$

**e**  $-140 \div \underline{\quad} = 2$

**f**  $\underline{\quad} \times 16 = -48$

**7** Copy and complete each multiplication table.

**a**

$\times$	-2	-1	0	+1	+2
-4	8			-4	
-2					
0			0		
+2					
+4		-4			
+6					

**b**

$\times$		-20	-10		20
7				70	
			-50		
0					
-2	50				
		80			
					-120

**EXAMPLE 3E-3****Dividing integers written as a fraction**

Calculate  $\frac{28}{-7}$ .

**THINK**

- 1 Write the fraction as a division problem.
- 2 Ignore the signs and calculate  $28 \div 7$ . Decide whether the result will be positive or negative. Signs are different so result is negative.

**WRITE**

$$\begin{aligned} & \frac{28}{-7} \\ & = 28 \div -7 \\ & = -4 \end{aligned}$$

- 8 Calculate each of these.

a  $\frac{-48}{6}$       b  $\frac{60}{-5}$       c  $\frac{-51}{-3}$       d  $\frac{+150}{+10}$       e  $\frac{-200}{-25}$       f  $\frac{57}{-19}$

- 9 A short way of writing  $-1 \times -2$  is  $-(-2)$ . Write each problem as the product of two numbers and then calculate the result.

a  $-(-7)$       b  $-(+12)$       c  $-(-88)$   
 d  $-(25)$       e  $-(8-5)$       f  $-(-3+2)$

**EXAMPLE 3E-4****Multiplying three integers**

Calculate  $-3 \times -5 \times -2$ .

**THINK**

- 1 Work from left to right. Multiply the first two numbers.  
( $-3 \times -5 = +15$ )
- 2 Multiply the result (+15) by the third number (-2).

**WRITE**

$$\begin{aligned} & -3 \times -5 \times -2 \\ & = +15 \times -2 \\ & = -30 \end{aligned}$$

- 10 Calculate:

a  $-4 \times +3 \times -5$       b  $+3 \times -2 \times +7$       c  $+6 \times -1 \times +9$   
 d  $+8 \times +2 \times -3$       e  $-5 \times -2 \times -13$       f  $-7 \times 3 \times -4$   
 g  $4 \times 5 \times -8$       h  $-11 \times 5 \times -2$       i  $9 \times -4 \times 5$

- 11 Calculate:

a  $-2 \times 2 \times -3 \times 3$       b  $20 \div -2 \div -2 \div -1$   
 c  $-4 \times -1 \times 3 \times -2$       d  $-18 \div -2 \times 4 \div -6$   
 e  $9 \times -2 \div 3 \times -5$       f  $-7 \times -4 \div 2 \times -3$

**12** Copy and complete each set of problems. Describe the pattern you see.

<b>a</b> $2 \times 5 = \underline{\quad}$	<b>b</b> $-3 \times 5 = \underline{\quad}$	<b>c</b> $-4 \times -5 = \underline{\quad}$	<b>d</b> $5 \times -5 = \underline{\quad}$
$2 \times 4 = \underline{\quad}$	$-3 \times 4 = \underline{\quad}$	$-4 \times -4 = \underline{\quad}$	$5 \times -4 = \underline{\quad}$
$2 \times 3 = \underline{\quad}$	$-3 \times 3 = \underline{\quad}$	$-4 \times -3 = \underline{\quad}$	$5 \times -3 = \underline{\quad}$
$2 \times 2 = \underline{\quad}$	$-3 \times 2 = \underline{\quad}$	$-4 \times -2 = \underline{\quad}$	$5 \times -2 = \underline{\quad}$
$2 \times 1 = \underline{\quad}$	$-3 \times 1 = \underline{\quad}$	$-4 \times -1 = \underline{\quad}$	$5 \times -1 = \underline{\quad}$
$2 \times 0 = \underline{\quad}$	$-3 \times 0 = \underline{\quad}$	$-4 \times 0 = \underline{\quad}$	$5 \times 0 = \underline{\quad}$
$2 \times -1 = \underline{\quad}$	$-3 \times -1 = \underline{\quad}$	$-4 \times 1 = \underline{\quad}$	$5 \times 1 = \underline{\quad}$
$2 \times -2 = \underline{\quad}$	$-3 \times -2 = \underline{\quad}$	$-4 \times 2 = \underline{\quad}$	$5 \times 2 = \underline{\quad}$
$2 \times -3 = \underline{\quad}$	$-3 \times -3 = \underline{\quad}$	$-4 \times 3 = \underline{\quad}$	$5 \times 3 = \underline{\quad}$
$2 \times -4 = \underline{\quad}$	$-3 \times -4 = \underline{\quad}$	$-4 \times 4 = \underline{\quad}$	$5 \times 4 = \underline{\quad}$
$2 \times -5 = \underline{\quad}$	$-3 \times -5 = \underline{\quad}$	$-4 \times 5 = \underline{\quad}$	$5 \times 5 = \underline{\quad}$

**13 a** Calculate:

**i**  $+8 \times 0$

**ii**  $-15 \times 0$

**iii**  $0 \times 65$

**iv**  $0 \times -72$

**b** What do you notice when you multiply any integer by zero?

**c** Calculate:

**i**  $0 \div -9$

**ii**  $0 \div +26$

**iii**  $-11 \div 0$

**iv**  $39 \div 0$

**d** What do you notice when you divide zero by any integer?

**e** What happens when you try to divide any number by zero?

What does this result mean?

**14 a** List five examples of pairs of integers that multiply to give:

**i**  $-24$

**ii**  $18$ .

**b** List five examples of pairs of integers that divide to give:

**i**  $-4$

**ii**  $5$ .

**c** List three examples of pairs of integers that multiply to give 0.

**d** List three examples of pairs of integers that divide to give 0.

**e** List five examples of a set of three integers that multiply to give:

**i**  $-48$

**ii**  $120$ .

**NOTE** For some of your examples, include at least one negative integer.

**15** Find two integers that have:

**a** a sum of  $-9$  and a product of  $18$

**b** a sum of  $4$  and a product of  $-45$

**c** a sum of  $-5$  and a product of  $-66$

**d** a sum of  $10$  and a product of  $-56$

**e** a difference of  $-2$  and a product of  $24$

**f** a difference of  $-8$  and a product of  $9$

**g** a difference of  $18$  and a product of  $-32$ .

**16** Astro the dog has escaped from his yard. His owners decide to search along the road which runs east–west outside the house. This number line can be used to represent the road outside the family’s house, with the house at zero.

**a** Tyler and Rhys walk 120 m west from the house and call Astro’s name.

What is their position on the number line?

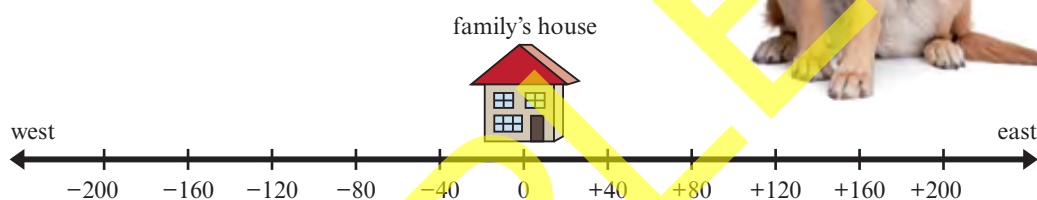
**b** Sophie and Imogen walk 80 m east from the house and call Astro’s name.

What is their position on the number line?

**c** What is the distance between the pairs of searchers?

**d** Imogen now walks a further 30 m east.

What is her new position on the number line?



**e** Rhys now walks 160 m east. What is his new position on the number line?

**f** What is the distance between:

**i** Sophie and Rhys?

**ii** Tyler and Imogen?

**g** Natalie walks twice as far as Sophie but in the same direction as Tyler. What is her position on the number line?

**h** Hayden walks one-third of the distance walked by Tyler and in the same direction. What is his position on the number line?

**i** A neighbour has found Astro in her yard, 60 m west of the family’s house.

Who is closest to Astro? How far and in which direction does he or she need to walk to collect the dog?

**j** If each member of the family now walks home, calculate the distance each person walked from the start of the search.

**17** Write a problem that contains three integers and two multiplication signs to give a result of  $-42$ .

**18** Write a problem that contains three integers, one multiplication sign and one division sign to give a result of 8.

**19** Write a problem that contains four different integers to give a result of  $-1000$ .

### Reflect

How do you identify whether the result of multiplying or dividing integers is positive, negative or zero?

# 3F Operations with directed numbers

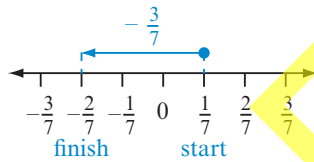
## Start thinking!

You can add or subtract positive and negative fractions and decimals in the same way as integers.

1 Look at  $(+\frac{1}{7}) + (-\frac{3}{7})$ .

a Explain why this can be simplified to  $\frac{1}{7} - \frac{3}{7}$ .

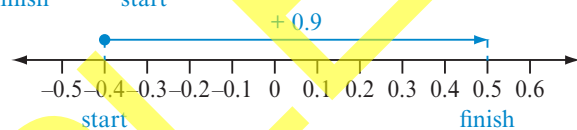
b Use this number line to calculate the result.



2 Look at  $(-0.4) - (-0.9)$ .

a Write it as a simpler equivalent problem.

b Use this number line to calculate the result.



3 You can also multiply or divide positive or negative fractions and decimals in the same way as integers. Copy these statements and use the words *positive* and *negative* to complete them.

To decide whether the result is positive or negative, remember that:

a positive  $\times$  positive = \_\_\_\_

b positive  $\div$  positive = \_\_\_\_

c positive  $\times$  negative = \_\_\_\_

d positive  $\div$  negative = \_\_\_\_

e negative  $\times$  positive = \_\_\_\_

f negative  $\div$  positive = \_\_\_\_

g negative  $\times$  negative = \_\_\_\_

h negative  $\div$  negative = \_\_\_\_

## KEY IDEAS

- ▶ Directed numbers include integers (positive and negative whole numbers and zero), positive and negative fractions, and positive and negative decimals. They have size and direction.
- ▶ You can use a number line to add or subtract directed numbers. Start at the position of the first number and then move left or right the number of units indicated by the second number. The final position is the result.
- ▶ To add a positive number or subtract a negative number, move to the right (positive direction) along the number line. Remember that  $+$  ( $+$ ) and  $-$  ( $-$ ) simplify to  $+$ .
- ▶ To add a negative number or subtract a positive number, move to the left (negative direction) along the number line. Remember that  $+$  ( $-$ ) and  $-$  ( $+$ ) simplify to  $-$ .
- ▶ When multiplying or dividing two numbers with the same sign, the result is positive.
- ▶ When multiplying or dividing two numbers with different signs, the result is negative.

## EXERCISE 3F Operations with directed numbers

- 1 Draw a number line with a scale from  $-10$  to  $10$  and show the position of each directed number.

**a**  $-3$       **b**  $+2.5$       **c**  $-\frac{1}{2}$       **d**  $5$       **e**  $+7\frac{3}{4}$       **f**  $-4.2$

### EXAMPLE 3F-1

#### Adding or subtracting directed numbers

Calculate:

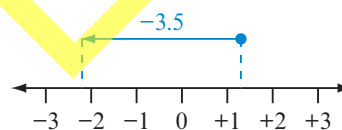
**a**  $(+1.3) + (-3.5)$       **b**  $(-\frac{2}{5}) - (-\frac{3}{10})$ .

#### THINK

- a** 1 Write as a simpler equivalent problem.
- 2 Locate  $1.3$  on a number line. Since the operation is  $-$ , move to the left (negative direction).
- 3 Write the result.
- b** 1 Write equivalent fractions with the same denominator ( $10$ ).
- 2 Write as a simpler equivalent problem.
- 3 Locate  $-\frac{4}{10}$  on a number line. Since the operation is  $+$ , move to the right (positive direction).
- 4 Write the result.

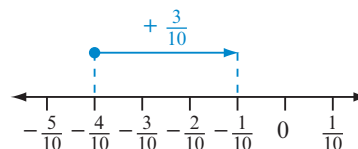
#### WRITE

**a**  $(+1.3) + (-3.5)$   
 $= 1.3 - 3.5$



$$(+1.3) + (-3.5) = -2.2$$

**b**  $(-\frac{2}{5}) - (-\frac{3}{10})$   
 $= (-\frac{4}{10}) - (-\frac{3}{10})$   
 $= -\frac{4}{10} + \frac{3}{10}$



$$(-\frac{2}{5}) - (-\frac{3}{10}) = -\frac{1}{10}$$

- 2 Calculate:

**a**  $(+3.1) + (-5.6)$       **b**  $(-7.4) + (+1.8)$       **c**  $(+2.9) - (-4.3)$   
**d**  $(-6.5) - (+3.8)$       **e**  $(-0.7) - (-0.5)$       **f**  $(+6.2) - (+4.9)$   
**g**  $(-5.7) + (+9.1)$       **h**  $(+4.3) + (-2.6)$       **i**  $(+3.9) - (-1.4)$

- 3 Calculate:

**a**  $(+\frac{1}{7}) - (-\frac{3}{7})$       **b**  $(-\frac{2}{3}) + (-\frac{5}{3})$       **c**  $(+\frac{5}{8}) - (+\frac{7}{8})$   
**d**  $(-\frac{4}{5}) + (+\frac{2}{5})$       **e**  $(-\frac{1}{3}) + (-\frac{4}{3})$       **f**  $(-\frac{3}{4}) - (+\frac{1}{2})$   
**g**  $(-\frac{1}{3}) - (-\frac{5}{9})$       **h**  $(+\frac{2}{5}) + (-\frac{3}{2})$       **i**  $(-\frac{1}{2}) + (-\frac{2}{3})$



**EXAMPLE 3F-2****Multiplying or dividing directed fractions**

Calculate:

**a**  $-\frac{9}{20} \times -\frac{8}{15}$

**b**  $-\frac{4}{7} \div \frac{3}{14}$

**THINK**

- a** 1 Look for common factors. Divide 9 and 15 by 3 and divide 8 and 20 by 4.
- 2 Multiply the fractions and decide whether the result will be positive or negative.
- b** 1 Change to a multiplication problem.
- 2 Look for common factors. Divide 7 and 14 by 7.
- 3 Multiply the fractions and decide whether the result will be positive or negative.
- 4 Write as a mixed number.

**WRITE**

**a**  $-\frac{9}{20} \times -\frac{8}{15}$

$$= -\frac{9^{\cancel{3}}}{20^{\cancel{5}}} \times -\frac{8^{\cancel{2}}}{15^{\cancel{5}}}$$

$$= -\frac{3}{5} \times -\frac{2}{5}$$

$$= \frac{6}{25} \text{ (or } +\frac{6}{25}\text{)}$$

**b**  $-\frac{4}{7} \div \frac{3}{14}$

$$= -\frac{4}{7} \times \frac{14}{3}$$

$$= -\frac{4}{\cancel{7}_1} \times \frac{14^{\cancel{2}}}{3}$$

$$= -\frac{8}{3}$$

$$= -2\frac{2}{3}$$

4 Calculate:

**a**  $-\frac{6}{11} \times -\frac{2}{3}$

**b**  $+\frac{4}{5} \times -\frac{5}{12}$

**c**  $-\frac{15}{8} \times +\frac{16}{25}$

**d**  $+\frac{12}{35} \times +\frac{7}{9}$

**e**  $-\frac{5}{6} \div +\frac{10}{7}$

**f**  $+\frac{4}{9} \times -\frac{5}{12}$

**g**  $-\frac{15}{8} \times +\frac{16}{25}$

**h**  $+\frac{10}{9} \div -\frac{27}{12}$

**i**  $-\frac{7}{15} \div -\frac{21}{10}$

**EXAMPLE 3F-3****Multiplying or dividing directed decimals**

Calculate:

**a**  $-4.3 \times -6.1$

**b**  $+25.2 \div -7$

**THINK**

- a** 1 Ignore the signs and calculate  $4.3 \times 6.1$ .
- 2 Decide whether the result will be positive or negative and write your answer.
- b** 1 Ignore the signs and calculate  $25.2 \div 7$ .
- 2 Decide whether the result will be positive or negative and write your answer.

**WRITE**

**a**  $4.3 \times 6.1$   
 $= 26.23$

$$-4.3 \times -6.1 = +26.23 \text{ (or } 26.23\text{)}$$

**b**  $\frac{3.6}{7} \overline{)25.2}$

$$+25.2 \div -7 = -3.6$$

5 Calculate:

a  $-5.8 \times -6$

b  $-46.7 \times +3$

c  $+18.5 \times -4.3$

d  $-34.2 \div +9$

e  $-4.35 \div -0.5$

f  $+0.648 \div -0.2$

6 Calculate:

a  $-\frac{4}{5} + \frac{3}{5}$

b  $\frac{2}{7} - \frac{5}{7}$

c  $-\frac{3}{11} - \frac{6}{11}$

d  $-\frac{4}{3} + \frac{1}{6}$

e  $\frac{2}{7} - \frac{1}{2}$

f  $-\frac{3}{4} + \frac{7}{8}$

7 Calculate:

a  $-5.23 + 3.72$

b  $17.6 - 19.3$

c  $-53.1 - 25.8$

d  $5.104 - 12.6$

e  $0.378 - 0.783$

f  $-11.35 + 15.27$

g  $5.2 - 8.4 + 7.6$

h  $-6.9 + 2.3 + 9.1$

i  $4.55 - 2.15 - 3.8$

j  $-15.7 - 12.1 + 9.6$

k  $\frac{2}{3} - \frac{4}{3} + \frac{1}{3}$

l  $-\frac{3}{4} + \frac{1}{4} + \frac{1}{2}$

m  $\frac{2}{3} - \frac{5}{6} - \frac{1}{2}$

n  $-\frac{3}{5} - \frac{1}{4} + \frac{7}{10}$

o  $\frac{5}{9} - \frac{2}{3} + \frac{1}{2}$

8 Copy and complete each magic square.

a	-4	6	-8
	-6		2
			0

b	-6.8		
		3.4	
	6.8		13.6

c			$-\frac{1}{3}$
	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$

**NOTE** Remember that in a magic square, the sum of the three numbers in each row, column and diagonal is the same.

9 Calculate each problem. Remember to change each mixed number to an improper fraction first.

a  $1\frac{2}{3} - 2\frac{1}{3}$

b  $-4\frac{3}{5} - 1\frac{1}{10}$

c  $-2\frac{3}{4} + 3\frac{2}{3}$

d  $-4\frac{1}{2} \times 3\frac{1}{3}$

e  $-2\frac{1}{6} \div -3\frac{1}{4}$

f  $7\frac{2}{3} \div -5\frac{1}{9}$

**EXAMPLE 3F-4**

Using order of operations with directed numbers

Calculate: a  $3.2 \times 4 - 7.1 \times 2$       b  $-\frac{2}{3} - \frac{1}{8} \div \frac{3}{4}$ .**THINK**

- a** 1 Work out the multiplication before the subtraction.  $3.2 \times 4 = 12.8$  and  $7.1 \times 2 = 14.2$ .
- 2 Work out the result.
- b** 1 Work out the division before the subtraction. Change the division to a multiplication.
- 2 Look for common factors. Divide 4 and 8 by 4.
- 3 Work out the multiplication.
- 4 Subtract fractions using equivalent fractions.
- 5 Work out the result.

**WRITE**

$$\begin{aligned} \text{b } 3.2 \times 4 - 7.1 \times 2 &= 12.8 - 14.2 \\ &= -1.4 \\ \text{c } -\frac{2}{3} - \frac{1}{8} \div \frac{3}{4} &= -\frac{2}{3} - \frac{1}{8} \times \frac{4}{3} \\ &= -\frac{2}{3} - \frac{1}{8_2} \times \frac{4^1}{3} \\ &= -\frac{2}{3} - \frac{1}{6} \\ &= -\frac{4}{6} - \frac{1}{6} \\ &= -\frac{5}{6} \end{aligned}$$

10 Calculate:

- a  $7 - 2 \times 8$                       b  $-1 + 16 \div 16$   
 c  $28 - 3 \times (15 - 4)$             d  $-6 + 5 \times 3 - 2$   
 e  $11 - (4 \times 2 - 19)$               f  $(-3 \times 7 + 1) \div (8 - 12)$

11 Calculate:

- a  $-0.5 \times 0.3 + 1.4$                 b  $1.26 \div 3 - 0.68$   
 c  $3.7 + 6.2 \times -0.9$                 d  $4.1 \times 5 - 5.6 \times 4$   
 e  $-6.4 \div 3.2 \times 0.5$                 f  $2.4 \times (15.3 - 18.3)$

12 Calculate:

- a  $-\frac{5}{9} \times \frac{3}{5} - \frac{2}{3}$                       b  $-\frac{5}{6} \div \frac{10}{9} \times -\frac{4}{7}$   
 c  $\frac{1}{2} + \frac{4}{7} \div -\frac{2}{5}$                       d  $\frac{4}{9} \times \frac{3}{8} - \frac{5}{3} \times \frac{7}{10}$   
 e  $-\frac{5}{2} \times (\frac{7}{15} - \frac{3}{5})$                       f  $-\frac{2}{3} \times \frac{6}{7} \div 2\frac{2}{3}$

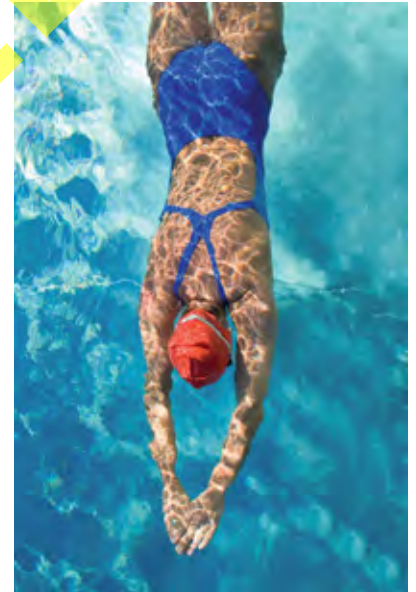
13 When standing with outstretched arms above her head, the height of an Olympic diver is 2.3 m. She dives from the 10-m platform and touches the bottom of the pool with her outstretched fingers. The depth of the pool is 5.5 m.

- a If the pool surface is the reference point for zero, write the maximum height of the diver above the pool surface as a directed number.  
 b Write the depth of the pool as a directed number.  
 c Write a subtraction problem to calculate the vertical distance covered by the diver. (Hint: find the difference between the two positions.)  
 d What vertical distance has the diver covered?

14 The daily maximum and minimum temperatures at Mt Buller were recorded over a week.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Maximum temperature [°C]	8.3	4.1	-3.0	-2.4	-1.3	0.4	2.4
Minimum temperature [°C]	1.6	-2.2	-5.3	-5.5	-3.2	-0.9	0.2

- a Calculate the difference between the minimum and maximum temperatures for each day.  
 b Which day had the biggest range of temperatures?  
 c Calculate the average (or mean) of:  
   i the maximum temperatures  
   ii the minimum temperatures.  
 d What is the difference between the average minimum and average maximum temperature?



**NOTE** Average (or mean) temperature is calculated by adding all the relevant values and dividing the total by the number of values.

- 15 For the ice cream and hot chocolate shown, calculate the difference between the two temperatures. Show the problem you used to obtain your answer.



- 16 This incomplete statement shows some deposits and withdrawals made to Martin's bank account.

Date	Reference	Transaction	Balance
30 April	-	-	+\$289.60
2 May	Tony's newsagent	+\$132.50	
15 May	Movie place	-\$17.45	
21 May	Bicycle city	-\$500.00	
24 May	Tony's newsagent	+\$230.00	
26 May	DD online music store	-\$38.95	
27 May	Sports and stuff Pty Ltd		+\$69.80

- a Is a transaction of +\$132.50 a deposit or a withdrawal?
- b Is a transaction of -\$17.45 a deposit or a withdrawal?
- c What does it mean if the balance in Martin's account is:
- positive?
  - negative?
- d Find the account balance after the transaction made on 21 May.
- e Find the balance after the transaction made on 24 May.
- f What transaction is made on 27 May so that the account balance is +\$69.80?
- g Was Martin's account overdrawn at any stage? What penalty do banks have for when an account is overdrawn?



- 17 List three examples of pairs of directed numbers where the sum of the numbers is zero.
- 18 List three examples of pairs of directed numbers that add to:
- 6.5
  - $-\frac{1}{2}$ .
- 19 List three examples of pairs of directed numbers where:
- the sum of the numbers is negative and the product of the numbers is positive
  - the sum and the product of the numbers are negative.
- 20 If the sum of two numbers is zero, will the product of the numbers be positive or negative? Explain.

### Reflect

What do you need to remember when performing operations on directed numbers?

# 3G Powers of directed numbers

## Start thinking!

- 1 a Write  $2^3$  in expanded form.  
b Calculate the basic numeral.
- 2 a Write  $(-2)^3$  in expanded form.  
b Calculate the basic numeral.  
c Is your answer positive or negative? Explain why.
- 3 Copy and complete each row of this table.

Index form	Base	Index or power	Expanded form	Basic numeral
$5^2$	5	2	$5 \times 5$	
$(-5)^2$	-5		$-5 \times -5$	
$4^3$				64
	-4	3		

- 4 Add a further four rows to your table and complete them using  $2^4$ ,  $(-2)^4$ ,  $3^5$ ,  $(-3)^5$ .
- 5 What do you notice about your results for questions 3 and 4?
- 6 Explain why  $5^2$  and  $(-5)^2$  give the same result but  $4^3$  and  $(-4)^3$  do not.
- 7 Without calculating the result, would you expect  $11^8$  and  $(-11)^8$  to give the same basic numeral? Explain.

## KEY IDEAS

- ▶ Powers are used to show repeated multiplication. The base is the number that is repeatedly multiplied and the power or index indicates the number of times the base is written.

$$\begin{array}{ccccccc}
 (-2)^3 & = & -2 \times -2 \times -2 & = & -8 \\
 \text{index form} & & \text{expanded form} & & \text{basic numeral}
 \end{array}$$

- ▶ When multiplying two numbers with the same sign, the result is positive.
- ▶ When multiplying two numbers with different signs, the result is negative.
- ▶ If the base is negative and the power is an even number, the basic numeral will be positive.
- ▶ If the base is negative and the power is an odd number, the basic numeral will be negative.

## EXERCISE 3G Powers of directed numbers

UNDERSTANDING AND FLUENCY

- Calculate each of these.
 

<b>a</b> $5 \times 5 \times 5$	<b>b</b> $-7 \times -7$
<b>c</b> $-3 \times -3 \times -3$	<b>d</b> $-10 \times -10 \times -10 \times -10$
- Which of these six options is the expanded form of  $(-4)^5$ ?
 

<b>A</b> $4 \times 4 \times 4 \times 4 \times 4$	<b>B</b> $5 \times 5 \times 5 \times 5$
<b>C</b> $-4 \times -4 \times 4 \times 4 \times 4$	<b>D</b> $-4 \times -4 \times -4 \times -4$
<b>E</b> $-4 \times -4 \times -4 \times -4 \times -4$	<b>F</b> $-5 \times -5 \times -5 \times -5$
- Write each part of question 1 in index form.

### EXAMPLE 3G-1

#### Calculating powers of integers

Write each of these in expanded form and calculate its value.

**a**  $(-3)^2$

**b**  $(-2)^5$

#### THINK

- a** 1 Write as a repeated multiplication (expanded form).
- 2 Work out the result. Remember that  $- \times - \rightarrow +$ . The result will be positive since the power is even.
- b** 1 Write in expanded form.
- 2 Work out the result. Remember that  $- \times - \rightarrow +$  and  $+ \times - \rightarrow -$ . The result will be negative since the power is odd.

#### WRITE

$$\begin{aligned} \mathbf{a} \quad (-3)^2 &= -3 \times -3 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad (-2)^5 &= -2 \times -2 \times -2 \times -2 \times -2 \\ &= 4 \times -2 \times -2 \times -2 \\ &= -8 \times -2 \times -2 \\ &= 16 \times -2 \\ &= -32 \end{aligned}$$

- Write each of these in expanded form and calculate its value.
 

<b>a</b> $(-9)^2$	<b>b</b> $(+8)^2$	<b>c</b> $(-6)^3$	<b>d</b> $(+7)^3$
<b>e</b> $(-5)^4$	<b>f</b> $(+1)^4$	<b>g</b> $(+4)^5$	<b>h</b> $(-10)^5$
<b>i</b> $(+2)^6$	<b>j</b> $(-3)^6$	<b>k</b> $(-2)^7$	<b>l</b> $(-1)^8$
- Without calculating each result, predict whether the basic numeral is positive or negative.
 

<b>a</b> $(-2)^9$	<b>b</b> $(-15)^6$	<b>c</b> $(+6)^7$	<b>d</b> $(-100)^4$
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**EXAMPLE 3G-2****Calculating powers of directed numbers**

Write each of these in expanded form and calculate its value.

**a**  $(-0.2)^3$

**b**  $(-\frac{2}{3})^4$

**THINK**

- a 1** Write in expanded form.  
**2** Work out the result. Remember that  $- \times - \rightarrow +$  and  $+ \times - \rightarrow -$ . A negative result is expected since the power is odd.
- b 1** Write in expanded form.  
**2** Work out the result. A positive result is expected since the power is even.

**WRITE**

$$\begin{aligned} \mathbf{a} \quad (-0.2)^3 &= -0.2 \times -0.2 \times -0.2 \\ &= 0.04 \times -0.2 \\ &= -0.008 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad (-\frac{2}{3})^4 &= -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \\ &= \frac{4}{9} \times -\frac{2}{3} \times -\frac{2}{3} \\ &= -\frac{8}{27} \times -\frac{2}{3} \\ &= \frac{16}{81} \end{aligned}$$

- 6** Write each of these in expanded form and calculate its value.

**a**  $(-1.2)^2$

**b**  $(+0.9)^2$

**c**  $(-0.4)^3$

**d**  $(-0.6)^4$

**e**  $(-0.2)^5$

**f**  $(-0.1)^7$

**g**  $(-\frac{1}{2})^3$

**h**  $(-\frac{5}{9})^2$

**i**  $(-\frac{4}{7})^3$

**j**  $(-\frac{1}{3})^4$

**k**  $(-\frac{2}{3})^5$

**l**  $(-1\frac{1}{2})^6$

- 7** Would the basic numeral for  $(-\frac{5}{19})^{13}$  be positive or negative? What about for  $(-28.5)^{20}$ ?

- 8** Arrange each list of numbers in ascending order.

**a**  $(+5)^4, (-2)^3, (-3)^2, (-4)^5$

**b**  $(-1)^{10}, 10^2, (-7)^2, (-2)^7$

**c**  $(-0.3)^2, (0.2)^3, (-0.2)^2, (-0.1)^3$

**d**  $(-\frac{1}{3})^2, (-\frac{1}{2})^5, (-\frac{1}{2})^4, (-\frac{1}{3})^3$

- 9** Write each of these multiplications in index form.

**a**  $-5 \times -5 \times -5 \times -9 \times -9 \times -9 \times -9$

**b**  $-4 \times -4 \times -4 \times -4 \times -4 \times -4 \times 3 \times 3 \times 3$

**c**  $7 \times 7 \times -6 \times -6 \times -6 \times -6 \times -6 \times -6$

**d**  $-8 \times -8 \times -8 \times -8 \times -10 \times -10$

- 10** Write each of these in expanded form and calculate its value.

**a**  $(-2)^3 \times (-4)^2$

**b**  $(-5)^2 \times 3^4$

**c**  $(-10)^3 \times (-2)^5$

**d**  $(-3)^5 \times (-1)^2$

**e**  $(-6)^2 \times (-2)^3$

**f**  $(-1)^4 \times 3^2$

**g**  $(-7)^2 \times (-1)^5$

**h**  $(-3)^3 \times (-2)^2$

**i**  $(-1)^3 \times (-2)^5$

- 11** Calculate each of these by first working out the value of any numbers written in index form.

**a**  $(-3)^4 + (-4)^3$     **b**  $(-10)^4 + (-7)^2$     **c**  $(-8)^2 - (-2)^5$   
**d**  $(-2)^3 - (-1)^9$     **e**  $(-1)^{13} \times (-5)^3$     **f**  $(-10)^2 \div (-5)^2$   
**g**  $6^4 \div 3^3$     **h**  $(-0.1)^3 + (-0.4)^2$     **i**  $(-0.2)^3 - (-1.1)^2$   
**j**  $(-\frac{1}{3})^2 + (-\frac{1}{2})^3$     **k**  $(-\frac{1}{2})^5 - (-\frac{3}{4})^2$     **l**  $(-0.3)^3 \times (-0.1)^4$   
**m**  $(-1.2)^2 \div (-0.6)^2$     **n**  $(-\frac{2}{3})^3 \times (-\frac{1}{2})^2$     **o**  $(-\frac{3}{10})^2 \div (-\frac{2}{5})^3$

- 12 a** Calculate each of these by first writing in expanded form.

**i**  $(-1)^1$     **ii**  $(-1)^2$     **iii**  $(-1)^3$   
**iv**  $(-1)^4$     **v**  $(-1)^5$     **vi**  $(-1)^6$

- b** Which of these problems give a positive result?  
**c** Which of these problems give a negative result?  
**d** Explain your observations.

- e** Predict the result for:

**i**  $(-1)^{13}$     **ii**  $(-1)^{32}$     **iii**  $(-1)^{100}$   
**iv**  $(-1)^{203}$     **v**  $(-1)^{188}$     **vi**  $(-1)^{555}$

- 13 a** Calculate:

**i**  $(-3)^2$     **ii**  $(-3)^3$     **iii**  $(-3)^5$ .

- b** Multiply your answers to part **a i** and **ii** to calculate  $(-3)^2 \times (-3)^3$ .

- c** How does this compare to your result for  $(-3)^5$ ?

- d** Calculate:

**i**  $(-2)^3$     **ii**  $(-2)^5$     **iii**  $(-2)^8$ .

- e** Multiply your answers to part **d i** and **ii** to calculate  $(-2)^3 \times (-2)^5$ .

- f** How does this compare to your result for  $(-2)^8$ ?

- g** What pattern can you use to make multiplication problems like this easier to calculate? (Hint: remember the index laws in Chapter 1.)

- h** Copy and complete the table shown below. The first two rows have been completed.

	Calculation	Simplified calculation	Basic numeral
	$(-3)^2 \times (-3)^3$	$(-3)^5$	-243
	$(-2)^3 \times (-2)^5$	$(-2)^8$	256
<b>i</b>	$(-4)^3 \times (-4)^2$		
<b>ii</b>	$(-2)^4 \times (-2)^3$		
<b>iii</b>	$(-0.1)^5 \times (-0.1)^3$		
<b>iv</b>	$(-\frac{1}{2})^3 \times (-\frac{1}{2})^2$		

- i** What must be the same for this pattern (or index law) to work? (Hint: does it work for  $(-3)^2 \times (-2)^5$ ?)



**14 a** Calculate:

**i**  $(-3)^6$     **ii**  $(-3)^4$     **iii**  $(-3)^2$ .

**b** Divide your answers to part **a i** and **ii** to calculate  $(-3)^6 \div (-3)^4$ .

**c** How does this compare to your result for  $(-3)^2$ ?

**d** Calculate:

**i**  $(-2)^7$     **ii**  $(-2)^4$     **iii**  $(-2)^3$ .

**e** Divide your answers to part **d i** and **ii** to calculate  $(-2)^7 \div (-2)^4$ .

**f** How does this compare to your result for  $(-2)^3$ ?

**g** What pattern can you use to make division problems like this easier to calculate?

**h** Copy and complete the table shown at right. The first two rows have been completed.

**i** What must be the same for this pattern (or index law) to work? (Hint: does it work for  $(-3)^6 \div (-2)^4$ ?)

	Calculation	Simplified calculation	Basic numeral
	$(-3)^6 \div (-3)^4$	$(-3)^2$	9
	$(-2)^7 \div (-2)^4$	$(-2)^3$	-8
<b>i</b>	$(-5)^8 \div (-5)^5$		
<b>ii</b>	$(-6)^9 \div (-6)^7$		
<b>iii</b>	$(-0.2)^{10} \div (-0.2)^6$		
<b>iv</b>	$(-\frac{4}{5})^{12} \div (-\frac{4}{5})^9$		

**15** The basic numeral for a number raised to the power of 0 is 1. For example,  $2^0 = 1$ .

**a** Calculate  $(-2)^3$  and use this value to calculate  $(-2)^3 \div (-2)^3$ .

**b** Subtract the powers to simplify  $(-2)^3 \div (-2)^3$ . Keep the result in index form.

**c** Does this relationship work for negative numbers? Explain.

### EXAMPLE 3G-3

#### Using index laws with directed numbers

Use the index laws to first simplify and then calculate each problem.

**a**  $(-2)^8 \div (-2)^6 \times (-2)^3$

**b**  $(-5)^7 \times (-5)^3 \div (-5)^8$

#### THINK

- a**
- 1 Work from left to right. To simplify  $(-2)^8 \div (-2)^6$ , subtract the powers. ( $8 - 6 = 2$ )
  - 2 To simplify  $(-2)^2 \times (-2)^3$ , add the powers. ( $2 + 3 = 5$ )
  - 3 Calculate the result.
- b**
- 1 Work from left to right. To simplify  $(-5)^7 \times (-5)^3$ , add the powers. ( $7 + 3 = 10$ )
  - 2 To simplify  $(-5)^{10} \div (-5)^8$ , subtract the powers. ( $10 - 8 = 2$ )
  - 3 Calculate the result.

#### WRITE

**a**

$$\begin{aligned} & (-2)^8 \div (-2)^6 \times (-2)^3 \\ &= (-2)^2 \times (-2)^3 \\ &= (-2)^5 \\ &= -32 \end{aligned}$$

**b**

$$\begin{aligned} & (-5)^7 \times (-5)^3 \div (-5)^8 \\ &= (-5)^{10} \div (-5)^8 \\ &= (-5)^2 \\ &= 25 \end{aligned}$$

**16** Use the index laws to first simplify and then calculate each problem.

- |  |   |
|--|---|
| <b>a</b> $(-2)^9 \times (-2)^5 \div (-2)^{11}$ | <b>b</b> $(-6)^5 \div (-6)^4 \times (-6)^2$       |
| <b>c</b> $(-10)^8 \times (-10)^5 \div (-10)^9$ | <b>d</b> $(-4)^{10} \times (-4)^3 \div (-4)^{13}$ |
| <b>e</b> $(-3)^7 \div (-3)^7 \times (-3)^3$    | <b>f</b> $(-7)^5 \times (-7)^3 \div (-7)^6$       |
| <b>g</b> $(-5)^4 \times (-5)^3 \div (-5)^7$    | <b>h</b> $(-3)^4 \times (-3)^2 \div (-3)^5$       |

**17** Squaring two different numbers can produce the same result.

For example,  $3^2 = 9$  and  $(-3)^2 = 9$ .

- Which two numbers, when squared (raised to the power of 2), give 25? Why are there two numbers?
- Is there a number that, when squared, gives  $-25$ ? Explain.
- How many different numbers can be cubed (raised to the power of 3) to give 8?
- How many different numbers can be cubed (raised to the power of 3) to give  $-8$ ?
- Comment on your results to parts **a–d**.
- Predict how many numbers can be raised to the power of 4 to give the same positive result. (Try it for 16.) Repeat for a negative result.
- Similarly, predict what would happen for powers of 5, 6 and so on. Provide examples to support your answer.

**18** What number(s), when squared, give(s) each result?

- |             |             |              |
|-------------|-------------|--------------|
| <b>a</b> 49 | <b>b</b> 81 | <b>c</b> 1   |
| <b>d</b> 4  | <b>e</b> 64 | <b>f</b> 100 |

**19** What number(s), when cubed, give(s) each result?

- |               |                 |                  |
|---------------|-----------------|------------------|
| <b>a</b> 27   | <b>b</b> $-125$ | <b>c</b> 64      |
| <b>d</b> $-1$ | <b>e</b> $-64$  | <b>f</b> $-1000$ |

**20** Consider  $(+3)^2$ ,  $(-3)^2$ ,  $3^2$  and  $-3^2$ .

- Which of these produce the same result?
- Explain why  $(-3)^2$  is different from  $-3^2$ .
- Decide whether each pair produces the same result.
 

<b>i</b> $(-4)^3$ and $-4^3$	<b>ii</b> $(-2)^4$ and $-2^4$
<b>iii</b> $(-1)^6$ and $-1^6$	<b>iv</b> $(-10)^5$ and $-10^5$
- Describe the general pattern you have seen.

**21** Calculate each problem. Remember to use the correct order of operations.

- $9^2 \div 3 - 5^2 \times 4$
- $-8.2 - 2^2 + (1.1)^2$
- $(-\frac{2}{3})^3 + \frac{4}{9}$

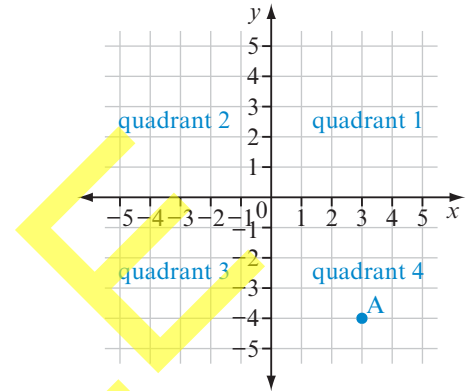
### Reflect

What is different about raising a negative number to a power compared to raising a positive number to a power?

# 3H The Cartesian plane

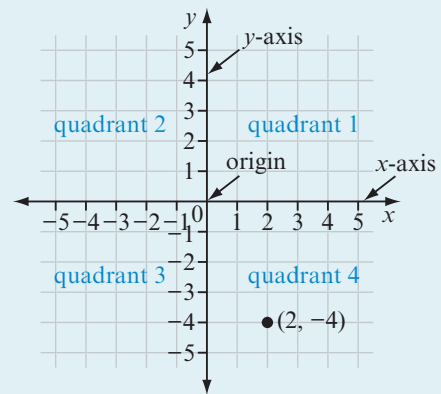
## Start thinking!

- Copy the diagram of the **Cartesian plane** shown.
  - What is the name given to:
    - the horizontal number line?
    - the vertical number line?
  - The reference point for the positive and negative numbers on the scale is called the **origin**. What are the coordinates of the origin?
- A point A has been drawn on the Cartesian plane.
  - What is the  $x$ -coordinate of this point?
  - What is the  $y$ -coordinate of this point?
  - List the coordinates of point A.
  - Which **quadrant** is point A in?
- Point B has the Cartesian coordinates  $(-2, -3)$ .
  - Write the  $x$ -coordinate of point B.
  - Write the  $y$ -coordinate of point B.
  - Plot point B on your Cartesian plane. This means to draw a dot at  $(-2, -3)$  and label it as B.
  - Which quadrant is point B in?
- Plot a point in each quadrant of your Cartesian plane and then list the quadrant number and the coordinates of each point.



## KEY IDEAS

- ▶ A Cartesian plane is formed when a horizontal number line ( $x$ -axis) and a vertical number line ( $y$ -axis) intersect at right angles. The point where they intersect is called the origin.
- ▶ The number lines in a Cartesian plane are called axes (singular axis). They divide the plane into four quadrants.
- ▶ A pair of Cartesian coordinates describes a point on the Cartesian plane. The horizontal coordinate ( $x$ -coordinate) is always listed first, followed by a comma, then the vertical coordinate ( $y$ -coordinate).
- ▶ The  $x$ -coordinate states how many units left or right to move along the  $x$ -axis from the origin and the  $y$ -coordinate states how many units to move up or down parallel to the  $y$ -axis. For example,  $(2, -4)$  means to move 2 units right from the origin and 4 units down.

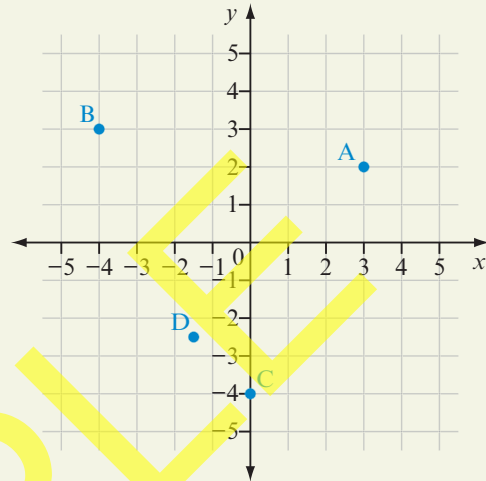


## EXERCISE 3H The Cartesian plane

### EXAMPLE 3H-1

#### Writing the Cartesian coordinates of a point

Write the coordinates of points A, B, C and D.



#### THINK

- 1 For point A, identify the  $x$ -coordinate (3) and the  $y$ -coordinate (2). Write the  $x$ -coordinate first.
- 2 Repeat step 1 for points B, C and D. Point C has an  $x$ -coordinate of 0, as you don't move any units left or right from the origin to locate it.

#### WRITE

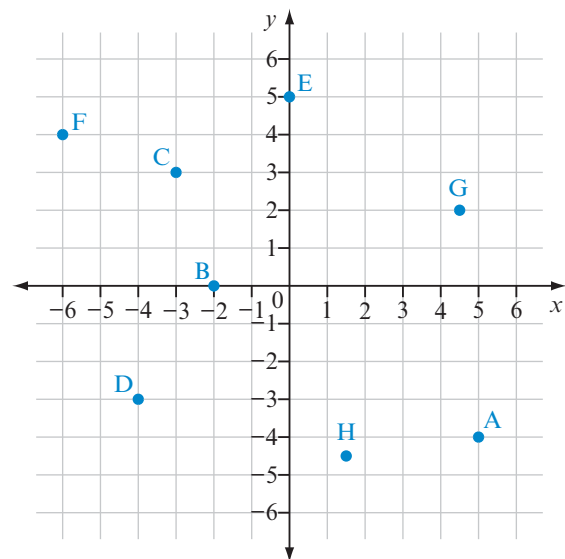
A is at (3, 2).

B is at (-4, 3).

C is at (0, -4).

D is at  $(-1\frac{1}{2}, -2\frac{1}{2})$  or (-1.5, -2.5)

- 1 Write the coordinates of points A to H shown on this Cartesian plane.
- 2 List the quadrant or the axis of the Cartesian plane on which each point in question 1 is located.



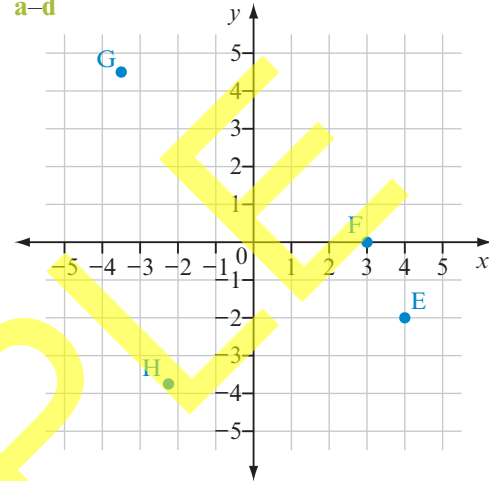
**EXAMPLE 3H-2****Plotting a point on the Cartesian plane**

Plot these points on a Cartesian plane. Label each point with its letter.

- a**  $E(4, -2)$                       **b**  $F(3, 0)$   
**c**  $G(-3\frac{1}{2}, 4\frac{1}{2})$                 **d**  $H(-2.25, -3.75)$

**THINK**

- a** Point E has  $x$ -coordinate 4 and  $y$ -coordinate  $-2$ . Move 4 units right along the  $x$ -axis from the origin and 2 units down parallel to the  $y$ -axis. Plot the point at this position and label it E.
- b** Repeat this method for point F.  $(3, 0)$  is 3 units right and 0 units up from the origin.
- c** Repeat this method for point G.  $(-3\frac{1}{2}, 4\frac{1}{2})$  is  $3\frac{1}{2}$  units left and  $4\frac{1}{2}$  units up from the origin.
- d** Repeat this method for point H.  $(-2.25, -3.75)$  is 2.25 units left and 3.75 units down from the origin.

**WRITE****a-d**

- 3** Copy the set of  $x$ - and  $y$ -axes shown in question 1.

Plot these points on the Cartesian plane. Label each point with its letter.

- a**  $I(-2, 5)$                       **b**  $J(0, -2)$                       **c**  $K(-4, -5)$   
**d**  $L(6, 0)$                       **e**  $M(3, -4)$                       **f**  $N(-4.5, 1.5)$   
**g**  $O(2, 2)$                       **h**  $P(0, 3)$                       **i**  $Q(-3.5, -1.5)$

- 4** List the quadrant or the axis of the Cartesian plane on which each point in question 3 is located.

- 5** Another way of listing the coordinates of a set of points is in a table of values. Copy and complete each table of values.

<b>a</b>	x-coordinate	-6	-4		0	2	
	y-coordinate	-3	-2	-1		1	
	coordinates	$[-6, -3]$	$[-4, -2]$	$[-2, -1]$	$[0, 0]$		$[4, 2]$ $[6, 3]$

<b>b</b>	x-coordinate	-2	-1		1		3
	y-coordinate	-7			-1	1	3
	coordinates	$[-2, -7]$	$[-1, -5]$	$[0, -3]$		$[2, 1]$	$[4, 5]$

**EXAMPLE 3H-3****Plotting points from a table of values**

For this table of values:

$x$	-6	-4	-2	0	2	4	6
$y$	-3	-2	-1	0	1	2	3

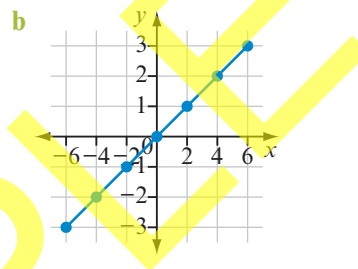
- list the coordinates for each point
- plot each point on a Cartesian plane and join them with a smooth line
- describe whether the points lie on a curved or straight line.

**THINK**

- Write the coordinates for each point. Write the  $x$ -coordinate first.
- Plot the points and join them in order.

**WRITE**

- $(-6, -3), (-4, -2), (-2, -1), (0, 0), (2, 1), (4, 2), (6, 3)$



- Describe the line you have drawn.
- The points lie on a straight line.

- 6** For each table of values:
- list the coordinates for each point
  - plot each point on a Cartesian plane and join them with a smooth line
  - describe whether the points lie on a curved or straight line.

**a**

$x$	-3	-2	-1	0	1	2	3
$y$	5	0	-3	-4	-3	0	5

**b**

$x$	-2	-1	0	1	2	3	4
$y$	-5	-4	-3	-2	-1	0	1

**c**

$x$	-2	-1	0	1	2	3	4
$y$	8	6	4	2	0	-2	-4

**d**

$x$	-6	-5	-1	0	2	3	6
$y$	-10	$-4\frac{1}{2}$	$7\frac{1}{2}$	8	6	$3\frac{1}{2}$	-10

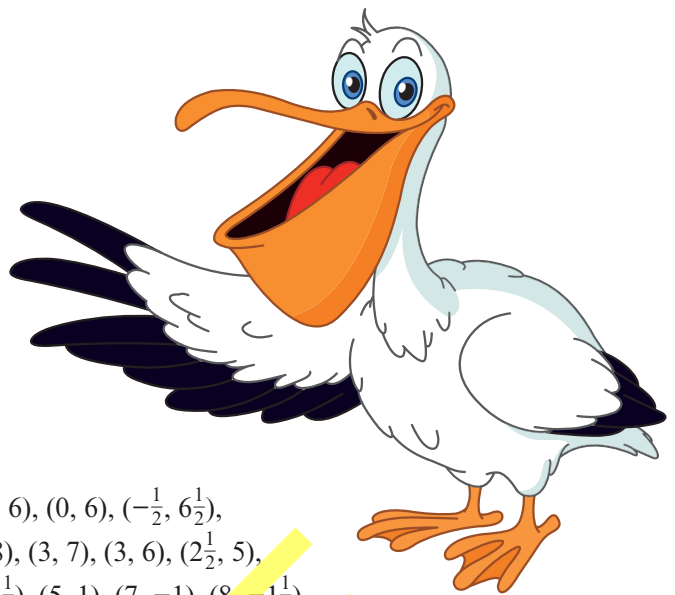
**NOTE** Look at the lowest and highest  $x$  values to work out the scale needed along the  $x$ -axis of the Cartesian plane. Repeat for the  $y$ -axis using the lowest and highest  $y$  values.

- 7** Draw a Cartesian plane with axes extending from  $-5$  to  $5$ . Plot each point and join the points together in the order shown. Describe the shape you have created.
- $(4, 1), (-4, 1), (2.5, -3.5), (0, 4), (-2.5, -3.5), (4, 1)$

- 8 Draw a Cartesian plane with the horizontal axis extending from  $-10$  to  $10$  and the vertical axis extending from  $-8$  to  $9$ . Use 1-cm grid paper or graph paper to make plotting easier. Plot each of these points and join them, in order, with smooth, slightly curved lines.

Join  $(-5, 5\frac{1}{2}), (-4, 6\frac{1}{2}), (-3, 6\frac{1}{2}), (-1, 6), (0, 6), (-\frac{1}{2}, 6\frac{1}{2}), (0, 8), (\frac{1}{2}, 7\frac{1}{2}), (\frac{1}{4}, 7\frac{3}{4}), (1, 8\frac{1}{4}), (2, 8), (3, 7), (3, 6), (2\frac{1}{2}, 5), (2, 3), (2, 2), (2\frac{1}{4}, 1\frac{1}{2}), (3, 1\frac{3}{4}), (4, 1\frac{1}{2}), (5, 1), (7, -1), (8, -1\frac{1}{2}), (9, -3), (8, -3), (9, -4), (7, -3\frac{1}{2}), (6, -4), (5\frac{1}{2}, -5), (5\frac{1}{4}, -5), (5\frac{1}{2}, -6), (6, -5\frac{1}{2}), (6\frac{1}{2}, -6), (5\frac{1}{2}, -6\frac{1}{2}), (5, -8), (4\frac{1}{2}, -7\frac{1}{2}), (3, -8), (3\frac{1}{2}, -7), (3, -7), (5, -6), (5, -5), (4\frac{1}{2}, -4\frac{1}{2}), (4, -5), (4, -5\frac{1}{2}), (4\frac{1}{2}, -5\frac{1}{2}), (3, -6), (1\frac{1}{2}, -7), (1\frac{1}{2}, -6\frac{1}{2}), (\frac{1}{2}, -6\frac{1}{2}), (1, -6), (1, -5\frac{1}{2}), (3\frac{1}{2}, -5\frac{1}{2}), (3\frac{1}{2}, -5), (2, -4), (\frac{1}{2}, -2\frac{1}{2}), (-2, -3), (-2\frac{1}{2}, -2\frac{1}{2}), (-4, -2\frac{3}{4}), (-5, -2), (-7, -1\frac{1}{2}), (-8, -1), (-9, -\frac{1}{2}), (-5, 0), (-9\frac{1}{2}, 1\frac{1}{2}), (-6, 1), (-9\frac{1}{2}, 3), (-9, 3\frac{1}{2}), (-3, 1), (-4, 2\frac{1}{2}), (0, 5\frac{1}{2}), (-3, 6), (-5, 5\frac{1}{2}).$

Use the image of the pelican to finish details like eyes, bill, feathers and so on.



- 9 You can make a drawing of this helicopter by joining a set of points with straight or slightly curved lines in a given order.

- a First trace the outline of the body of the helicopter using tracing paper. Cut out the outline and place it over a Cartesian plane. Choose a starting point and list the coordinates of that point. Decide which points would work best to give you the required outline when they are joined.



Now consider further points needed to draw other features of the helicopter, such as the blades and 'feet'. Produce a list of instructions that anyone could use, similar to question 8.

- b To produce an enlarged drawing of the helicopter, draw a new Cartesian plane with a larger interval between each unit mark on the axes. Follow your instructions to produce the enlarged picture.

- 10 The temperature at Cradle Mountain in Tasmania was recorded every 3 hours starting at midnight. The results are shown in the table.

Time (hours after midnight)	0	3	6	9	12	15	18	21	24
Temperature ( $^{\circ}\text{C}$ )	-3	-5	-2	3	7	5	4	0	-2

- a Draw a Cartesian plane and label the horizontal axis Time and the vertical axis Temperature.
- b Decide what scale to use for the horizontal axis. Will you need negative numbers? Explain why or why not. Mark your scale on the Time axis.
- c Decide what scale to use for the vertical axis. Mark your scale on the Temperature axis.
- d Plot the points provided in the table of values.
- e What was the coldest recorded temperature? At what time of the day was this temperature recorded?
- f What was the warmest recorded temperature? At what time of the day was this temperature recorded?
- g What was the range of recorded temperatures? (Hint: find the difference between the highest and lowest temperatures.)
- h What was the average recorded temperature?



- 11 A set of points is created by performing an operation on each  $x$ -coordinate to produce the  $y$ -coordinate.

- a Copy and complete this table of values for each operation described below.

$x$ -coordinate	-3	-2	-1	0	1	2	3
$y$ -coordinate							

- i Add 2 to each  $x$ -coordinate to produce the matching  $y$ -coordinate.
- ii Double each  $x$ -coordinate to produce the matching  $y$ -coordinate.
- iii Subtract 3 from each  $x$ -coordinate to produce the matching  $y$ -coordinate.
- iv Halve each  $x$ -coordinate to produce the matching  $y$ -coordinate.
- v Square each  $x$ -coordinate to produce the matching  $y$ -coordinate.
- b Draw a Cartesian plane with the horizontal axis extending from  $-3$  to  $3$  and the vertical axis extending from  $-6$  to  $9$ . For each table of values, plot the points and join them with a smooth line. Which line is different from the others? Can you explain why?
- 12 Each table of values in question 6 shows a set of points where one or more operations have been performed on each  $x$ -coordinate to produce the  $y$ -coordinate. What relationship has been used in each case?

### Reflect

How are positive and negative numbers used on a Cartesian plane?



# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

positive

negative

number line

integers

opposite integers

directed numbers

powers

index form

basic numeral

Cartesian plane

 $x$ -axis $y$ -axis

quadrants

 $x$ -coordinate $y$ -coordinate

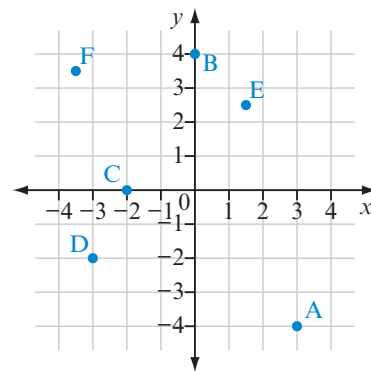
table of values

## MULTIPLE-CHOICE

- 3A** ▶ 1 Which of these is an integer between  $-3.5$  and  $3.5$ ?  
 A  $-4$                       B  $-2.5$   
 C  $-2$                         D  $5$
- 3B** ▶ 2 What integer when added to  $+12$  gives  $-7$ ?  
 A  $-19$                       B  $-5$   
 C  $+5$                         D  $+19$
- 3C** ▶ 3 What is  $(-24) - (+20)$ ?  
 A  $-4$                         B  $+4$   
 C  $-44$                       D  $+44$
- 3D** ▶ 4 What does  $(-35) - (-47)$  simplify to?  
 A  $-35 - 47$                 B  $35 - 47$   
 C  $35 + 47$                 D  $-35 + 47$
- 3D** ▶ 5 Which of these problems does *not* equal  $5$ ?  
 A  $-4 - 11 + 16 - 2 + 6$   
 B  $8 - 13 - 1 + 17 - 6$   
 C  $-7 + 5 - 9 + 22 - 6$   
 D  $5 - 12 + 3 - 7 + 6$
- 3E** ▶ 6 What is  $8 \div -2$ ?  
 A  $-16$     B  $-4$     C  $4$     D  $16$
- 3E** ▶ 7 What pair of integers gives a sum of  $-5$  and a product of  $-36$ ?  
 A  $-4$  and  $9$                 B  $-9$  and  $4$   
 C  $4$  and  $9$                  D  $-9$  and  $-4$
- 3F** ▶ 8 What is  $\frac{1}{5} - \frac{4}{5}$ ?  
 A  $-3$     B  $-\frac{3}{5}$     C  $\frac{3}{5}$     D  $\frac{5}{5}$
- 3F** ▶ 9 Which problem has a positive answer?  
 A  $(-1.2) + (-2.4)$   
 B  $(-\frac{2}{5}) \times (+\frac{1}{2})$   
 C  $(-4.9) - (-1.1)$   
 D  $(-\frac{1}{7}) \div (-\frac{8}{9})$
- 3G** ▶ 10 Which gives the largest result?  
 A  $(-2)^6$                       B  $(+6)^2$   
 C  $(-1)^{10}$                     D  $(-4)^3$
- 3G** ▶ 11 What does  $(-5)^8 \times (-5)^4$  simplify to?  
 A  $(-5)^2$                       B  $(-5)^4$   
 C  $(-5)^{12}$                     D  $(-5)^{32}$
- 3H** ▶ 12 The point with coordinates  $(-3, -8)$  is found in which quadrant on a Cartesian plane?  
 A quadrant 1                B quadrant 2  
 C quadrant 3                D quadrant 4

## SHORT ANSWER

- 3A** ▶ **1** Decide whether each statement is true or false.
- a**  $-5 > +2$       **b**  $-10 < -8$   
**c**  $0 > -3.5$       **d**  $7\frac{1}{2} < -7\frac{1}{2}$
- 3A** ▶ **2** Write each list of numbers in ascending order.
- a**  $-2, 8, 4, -4, 0, -8$   
**b**  $-19, 5\frac{1}{3}, 0, -9, -1, 5.5$
- 3B** ▶ **3** Calculate:
- a**  $(-5) + (-4)$       **b**  $(+2) + (+7)$   
**c**  $(+22) + (-34)$       **d**  $(-50) + (+69)$
- 3B** ▶ **4** Tamara's bank account shows a balance of  $-\$28$ . If she deposits  $\$150$ , what is her new account balance?
- 3C** ▶ **5** Calculate:
- a**  $(+3) - (+8)$       **b**  $(-1) - (-9)$   
**c**  $(-46) - (+35)$       **d**  $(+71) - (+53)$
- 3C** ▶ **6** Overnight, the minimum temperature in Alice Springs was  $-3^\circ\text{C}$ . By 2 pm, the temperature had climbed to a maximum of  $18^\circ\text{C}$ . Calculate the difference between the minimum and maximum temperatures.
- 3D** ▶ **7** Calculate:
- a**  $-8 + 7$       **b**  $-4 - 6$   
**c**  $17 - 25$       **d**  $-44 + 34$   
**e**  $-66 + 66$       **f**  $-50 - 50$
- 3E** ▶ **8** Calculate:
- a**  $+7 \times -8$       **b**  $-5 \times -9$   
**c**  $-36 \div +4$       **d**  $-100 \div -20$   
**e**  $-12 \times 0$       **f**  $4 \times -15$   
**g**  $\frac{18}{-6}$       **h**  $\frac{-42}{-3}$
- 3E** ▶ **9** Calculate:
- a**  $-6 \times 3 \times -10$       **b**  $20 \div -4 \times 7$   
**c**  $-18 \div -3 \times 2$       **d**  $-5 \times -2 \times -4$
- 3F** ▶ **10** Calculate:
- a**  $(-\frac{3}{4}) - (-\frac{1}{2})$       **b**  $(+\frac{2}{5}) - (+\frac{1}{3})$   
**c**  $\frac{3}{7} - 1\frac{2}{7}$       **d**  $-2\frac{1}{6} + 6\frac{2}{3}$   
**e**  $\frac{1}{2} \div \frac{7}{4} - \frac{2}{5}$       **f**  $-\frac{1}{8} + \frac{2}{3} \times -\frac{9}{8}$
- 3F** ▶ **11** Calculate:
- a**  $(+5.7) + (-6.2)$   
**b**  $(-0.9) - (-0.64)$   
**c**  $7.45 - 9.38$       **d**  $-12.5 - 11.6$   
**e**  $-3.2 \div -0.2 - 16$   
**f**  $0.7 \times -0.2 - 0.6 \times -0.3$
- 3F** ▶ **12** Calculate the average of  $-4.2, 5.6, 7.1$  and  $-9.3$ .
- 3G** ▶ **13** Calculate:
- a**  $(-2)^5$       **b**  $(-3)^2$   
**c**  $(-1)^9$       **d**  $(-10)^4$
- 3G** ▶ **14** Use the index laws to first simplify and then calculate:
- a**  $(-9)^8 \times (-9)^5 \div (-9)^{11}$   
**b**  $(-6)^7 \times (-6)^4 \div (-6)^{11}$
- 3H** ▶ **15** Write the coordinates of points A to F shown on this Cartesian plane.

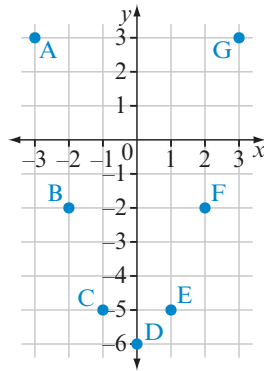


- 3H** ▶ **16** Plots these points on a Cartesian plane and then join them in order.  $(0, -1), (1, -1), (1, 1), (-1, 1), (-1, -2), (2, -2), (2, 2), (-2, 2), (-2, -3), (3, -3)$  Write the coordinates of the next four points to continue the pattern.

## NAPLAN-STYLE PRACTICE

- 1 Which number is the largest?  
 3       -3       2       -2
- 2 Which number is smaller than  $-4.5$ ?  
  $-4.3$         $-4.7$         $2\frac{1}{2}$         $-4\frac{2}{5}$
- 3 What is  $(+5) + (-9)$ ?  
 -14       -4       4       14
- 4 What is  $(-6) - (-7)$ ?  
 -13       -1       1       13
- 5 Hayden and Stacy enter a lift at the third level above ground level. They move down five levels. Where does the lift stop?  
 second level above ground level  
 second level below ground level  
 eighth level above ground level  
 eighth level below ground level
- 6 Complete this number statement.  
 $17 + \boxed{\phantom{000}} = -25$
- 7 Complete this number statement.  
 $-36 - \boxed{\phantom{000}} = -80$
- 8 A stone falls from a cliff top 50 m above water to the bottom of the sea. The water is 12 m deep. How far does the stone fall?
- 9 Bread at a temperature of  $-16^{\circ}\text{C}$  is taken out of the freezer. After 15 minutes, its temperature rises by  $24^{\circ}\text{C}$ . What is the new temperature of the bread?  
  $-40^{\circ}\text{C}$         $-8^{\circ}\text{C}$         $8^{\circ}\text{C}$         $40^{\circ}\text{C}$
- 10 Complete this number statement.  
 $-4 \times \boxed{\phantom{000}} = 12$
- 11 A number is multiplied by  $-5$  and the answer is  $-30$ .  
 The original number is
- 12 Which pair of numbers has a positive sum and a negative product?  
  $-6, 5$         $-8, -3$         $-4, 7$         $2, 9$
- 13 A ski resort recorded these temperatures:  $-5^{\circ}\text{C}$ ,  $2^{\circ}\text{C}$ ,  $4^{\circ}\text{C}$ ,  $-3^{\circ}\text{C}$ ,  $-1^{\circ}\text{C}$ . What is the average of these temperatures?
- 14 Transactions for Chloe's bank account are shown below.
- | Date   | Reference | Transaction | Balance   |
|--------|-----------|-------------|-----------|
| 1 May  | -         | -           | +\$124.80 |
| 4 May  | Deposit   | +\$75.00    |           |
| 9 May  | Music     | -\$42.75    |           |
| 15 May | Shoes     | -\$149.95   |           |
| 26 May | Movies    | -\$52.20    |           |
| 31 May | Deposit   | +\$150.00   |           |
- How much is in Chloe's bank account at the end of the month?
- 15 What is  $-\frac{3}{8} \times \frac{4}{9} \times -\frac{2}{7}$ ?
- 16 What is  $(-2)^{5^?}$ ?
- 17 Which of these does *not* give a negative result?  
  $(-5)^6$         $(-3)^5$         $(-2)^7$         $(-4)^3$
- 18 What is  $(-3)^{10} \div (-3)^7$ ?  
 1       -9       27       -27
- 19 A point on the Cartesian plane has the coordinates  $(-5, 3)$ . What is the  $x$ -coordinate?
- 20 What are the coordinates of the origin on the Cartesian plane?

Questions 21–27 refer to this figure.



21 Which point has the coordinates  $(2, -2)$ ?

- A     B  
 F     G

22 Which point has the coordinates  $(-3, 3)$ ?

23 Write the coordinates of point E.

24 Write the coordinates of point D.

25 Which points are in the third quadrant of the Cartesian plane?

26 Which point has the same  $y$ -coordinate as point A?

27 Which table of values matches the points plotted on the Cartesian plane?

Table 1

$x$	-3	-2	-1	0	1	2	3
$y$	-3	-2	-1	0	1	2	3

Table 2

$x$	3	-2	-5	-6	-5	-2	3
$y$	-3	-2	-1	0	1	2	3

Table 3

$x$	-3	-2	-1	0	1	2	3
$y$	3	2	5	6	5	2	3

Table 4

$x$	-3	-2	-1	0	1	2	3
$y$	3	-2	-5	-6	-5	-2	3

## ANALYSIS

The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Min. temp. ( $^{\circ}\text{C}$ )	0.7	-2.2	-0.9	-3.4	-5.1	-1.8	0.2
Max. temp. ( $^{\circ}\text{C}$ )	6.5	4.1	7.8	2.5	-0.6	3.9	5.3

- a Which day had:
- the highest temperature?
  - the lowest temperature?
- b Calculate the difference between the minimum and maximum temperatures for Monday.
- c Calculate the difference between the minimum and maximum temperatures for each other day.
- d Which day had the biggest range of temperatures?
- e Calculate the average of:
- the minimum temperatures (correct to one decimal place).
  - the maximum temperatures.
- f What is the difference between the average minimum and average maximum temperatures?
- g Draw a Cartesian plane with the horizontal axis labelled Day (show 1, 2, 3, ..., 7 on the scale to represent Monday to Sunday) and the vertical axis Temperature ( $^{\circ}\text{C}$ ). Think about the scale you will use on the vertical axis to represent all the temperatures listed in the table.
- h Write the information for the seven daily minimum temperatures as a set of Cartesian coordinates.
- i Plot the seven points with these coordinates on your Cartesian plane. To observe a trend, join the points with a smooth line.
- j In a different colour, plot and join points on the same Cartesian plane to show the information for the seven maximum temperatures.
- k Describe and interpret the trend you see.

# CONNECT

## Playing golf

The aim in golf is to hit a ball from the tee to the hole in the least number of strokes. Every golf course is rated by the number of strokes taken by a skilled player to complete a round of 18 holes. This number of strokes is called par. A golfer's score is the number of strokes they take that is more or less than par.

Integers can be used to describe these scores. A score of zero means that a golfer achieved par. Scores under par can be described using negative integers and scores over par can be described using positive integers. For example, a score of +5 means that the golfer's total number of strokes is five more than par (five over par).

Leo is a young golfer planning to become a professional golf player. During the 10 rounds of the summer competition season, Leo's scores were:

+2, +1, +4, -3, -2, -1, +2, +1, -4, -1.

The information that follows will help you decide whether he should apply.

Each hole of a golf course also has its own par. This table shows par for each of the 18 holes at Leo's golf course.

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Par	3	3	4	4	5	4	5	4	4	4	3	4	5	4	3	4	5	4

The total number of strokes for a par round can be worked out from this table. In order to be accepted, Leo needs to have an average number of strokes per round in the summer competition of less than 73.



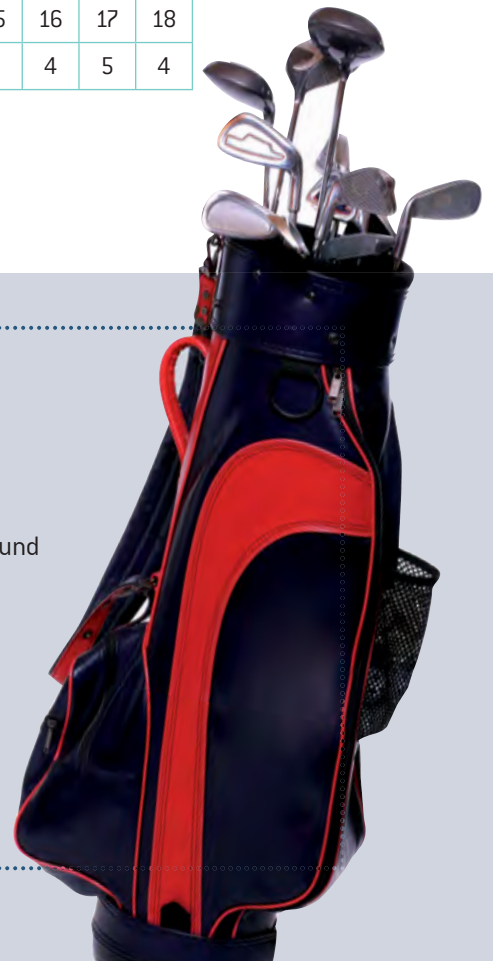
### Your task

To investigate Leo's chances of turning professional, carry out these steps:

- decide which round produced Leo's best score
- calculate his average score for the summer competition
- calculate the number of strokes he used to complete the course in each round
- decide whether he can apply to become a professional golfer
- analyse his results to decide where he needs to improve
- work out what his score should be in another round of golf so that his average score for the 11 rounds is close to  $-0.25$

Include all necessary working to justify your answers.

For an extra task, design a 9-hole or 18-hole golf course, showing the tee-off positions and holes on a Cartesian plane.



You can also analyse Leo's result at each hole. For his first two rounds of the summer competition, the number of strokes at each hole is shown below.

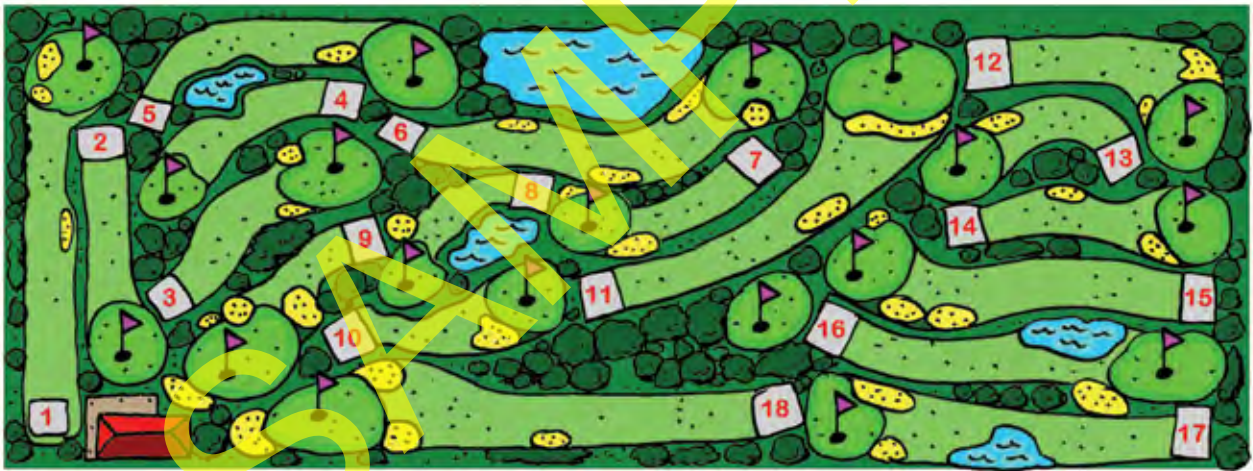
Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Round 1 results	4	3	3	5	8	4	3	6	5	4	4	3	2	5	3	5	4	3
Round 2 results	3	4	4	5	7	5	4	4	4	3	5	2	4	6	3	4	3	3

Golfers often refer to whether the number of strokes at a hole is over or under par for that hole. There are terms to describe this. Can you give a name to each result in Leo's first round?

Golf terms for number of strokes over or under par at a given hole

+3 triple bogey  
 +2 double bogey  
 +1 bogey  
 0 par  
 -1 birdie  
 -2 eagle  
 -3 albatross

As an extension, design a game where you and other players take turns rolling a die to simulate the number of strokes taken at each hole for a round of golf. Describe the rules (including any limitations or assumptions) and then play the game. Who is the best 'golfer' in your group of players? Express your total number of strokes for the round as a score over or under par using the information for Leo's golf course described earlier. Compare your score to those obtained by Leo.



Complete the **3 CONNECT** worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a golf brochure
- a player performance report
- a PowerPoint presentation
- other (check with your teacher).

